

104 PHYS

Ch. 22

Electric Fields



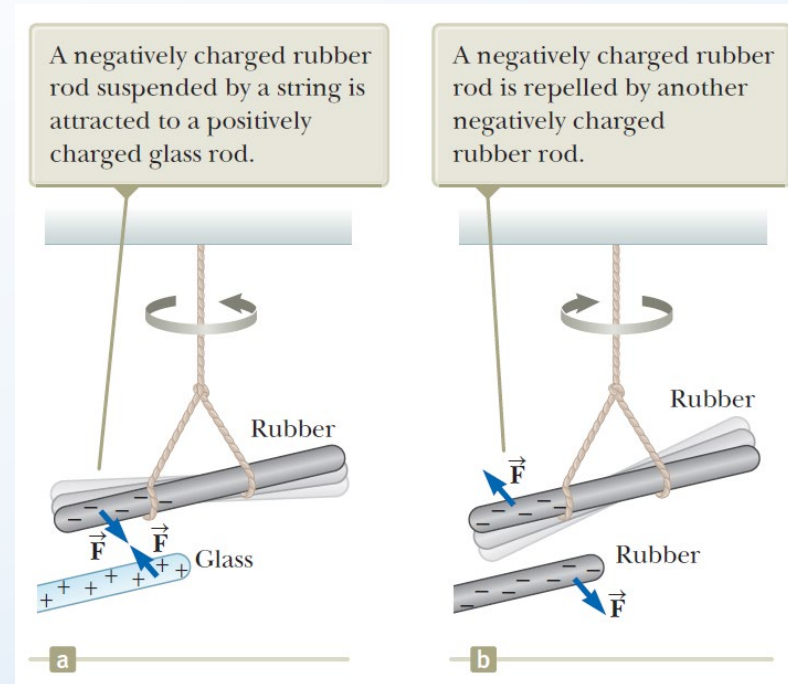
Contents

sec.	22.01	Properties of Electric Charges
sec.	22.02	Charging Objects by Induction
sec.	22.03	Coulomb's Law
sec.	22.04	Analysis Model: Particle in a Field (Electric)
sec.	22.05	Electric Field Lines
sec.	22.06	Motion of Charged Particles in a Uniform Electric Field

- Charges of the same sign repel one another and charges with opposite signs attract one another.
- Electric charge is always conserved in an isolated system.

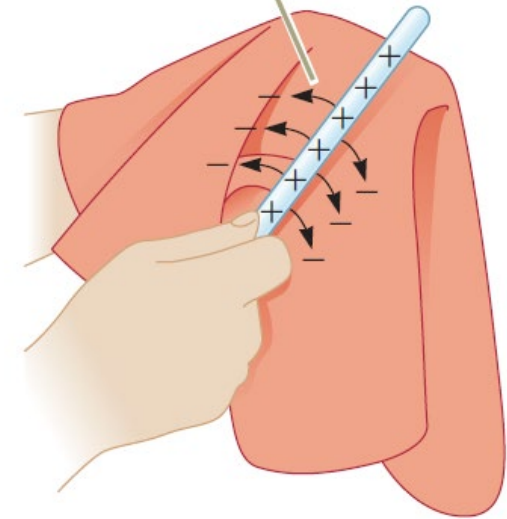


This young woman is enjoying the effects of electrically charging her body. Each individual hair on her head becomes charged and exerts a repulsive force on the other hairs, resulting in the “stand-up” hairdo seen here.

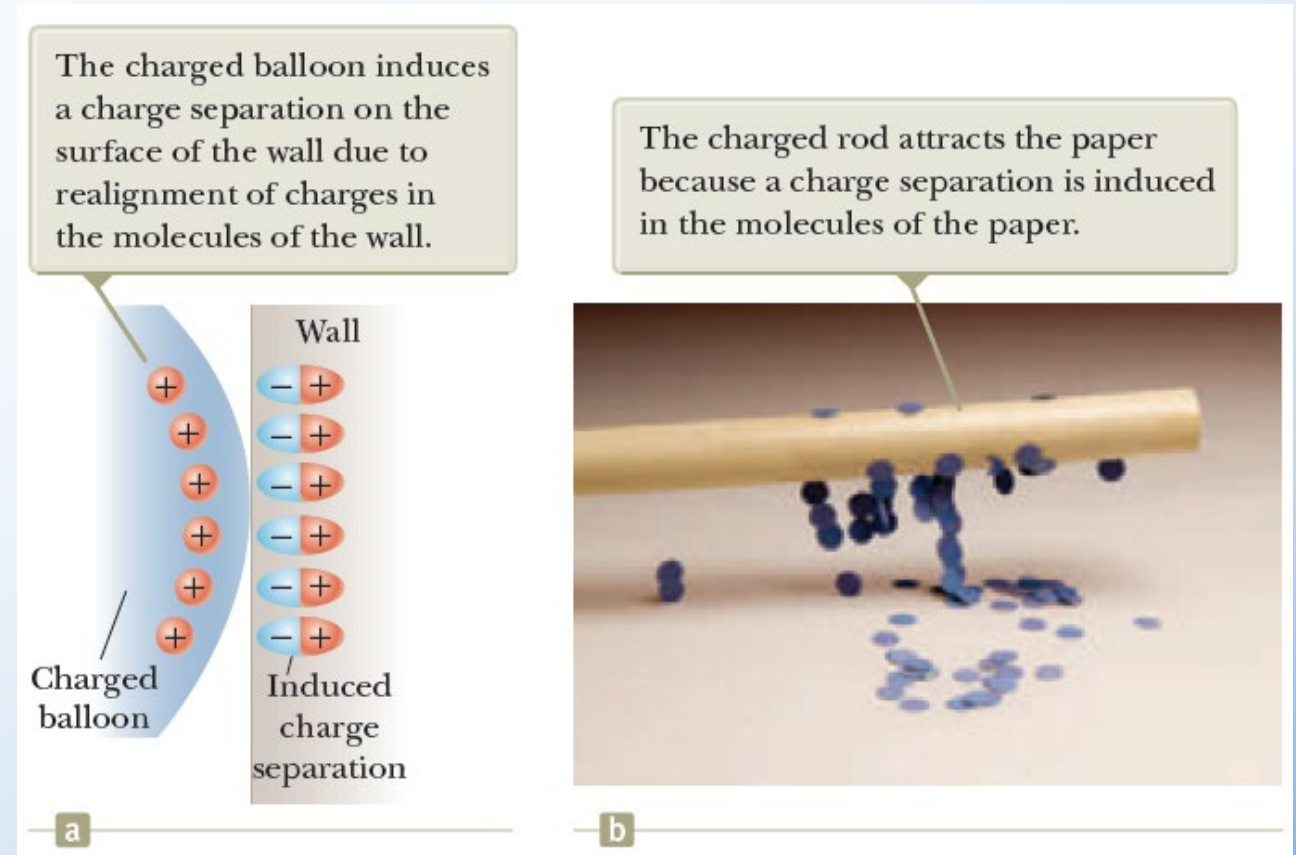
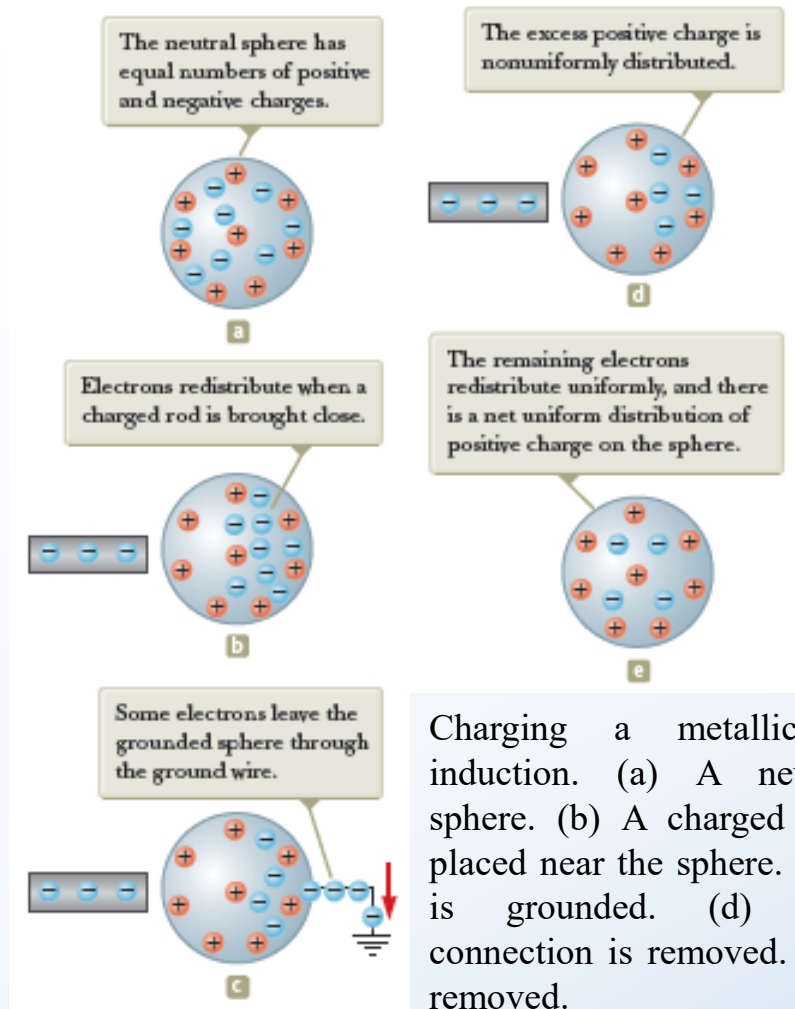


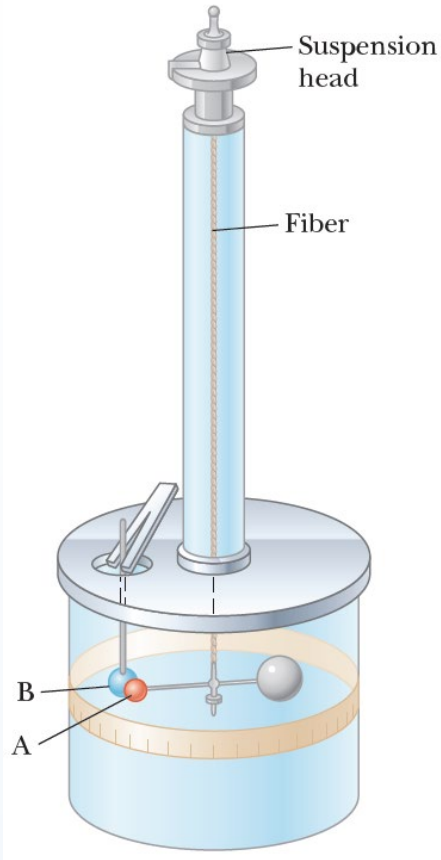
The electric force between (a) oppositely charged objects and (b) like-charged objects.

Because of conservation of charge, each electron adds negative charge to the silk and an equal positive charge is left on the glass rod.



When a glass rod is rubbed with silk, electrons are transferred from the glass to the silk.





Coulomb's balance, used to establish the inverse-square law for the electric force.

From Coulomb's experiments, The **electric force** F_e :

- is inversely proportional to the square of the separation r between the particles and directed along the line joining them.
- is proportional to the product of the charges q_1 and q_2 on the two particles.
- is attractive if the charges are of opposite sign and repulsive if the charges have the same sign.
- is a conservative force.

we can express **Coulomb's law** as:

$$F_e = k_e \frac{|q_1||q_2|}{r^2}$$

where k_e is a constant called the Coulomb constant and has the value:

$$k_e = 8.9875 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2$$

This constant is also written in the form

$$k_e = \frac{1}{4\pi\epsilon_0}$$

where the constant ϵ_0 is known as the permittivity of free space and has the value:

$$\epsilon_0 = 8.8542 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2$$

The smallest unit of charge e known in nature is the charge on an electron ($-e$) or a proton ($+e$) and has a magnitude:

$$e = 1.60219 \times 10^{-19} \text{ C}$$

Particle	Charge (C)	Mass (kg)
Electron (e)	$-1.602\,191\,7 \times 10^{-19}$	$9.109\,5 \times 10^{-31}$
Proton (p)	$+1.602\,191\,7 \times 10^{-19}$	$1.672\,61 \times 10^{-27}$
Neutron (n)	0	$1.674\,92 \times 10^{-27}$

The charges and masses of the electron, proton, and neutron

Example 22.01

The electron and proton of a hydrogen atom are separated (on the average) by a distance of approximately 5.3×10^{-11} m. Find the magnitudes of the electric force and the gravitational force between the two particles.

$$F_e = k_e \frac{|q_e||q_p|}{r^2}$$

$$F_e = 9 \times 10^9 \times \frac{|-1.60 \times 10^{-19}| \times |1.60 \times 10^{-19}|}{(5.3 \times 10^{-11})^2}$$

$$F_e = 8.2 \times 10^{-8} \text{ N}$$

we can express Newton's law of universal gravitation as:

$$F_g = G \frac{m_1 m_2}{r^2}$$

F_g is the gravitational force acting between two objects, m_1 and m_2 are the masses of the objects, r is the distance between the centers of their masses, and G is the gravitational constant. The gravitational constant G has the value:

$$G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$$

Example 22.01

The electron and proton of a hydrogen atom are separated (on the average) by a distance of approximately 5.3×10^{-11} m. Find the magnitudes of the electric force and the gravitational force between the two particles.

$$F_e = k_e \frac{|q_e||q_p|}{r^2}$$

$$F_e = 9 \times 10^9 \times \frac{|-1.60 \times 10^{-19}| \times |1.60 \times 10^{-19}|}{(5.3 \times 10^{-11})^2}$$

$$F_e = 8.2 \times 10^{-8} \text{ N}$$

$$F_g = G \frac{m_e m_p}{r^2}$$

$$F_g = 6.67 \times 10^{-11} \times \frac{9.11 \times 10^{-31} \times 1.67 \times 10^{-27}}{(5.3 \times 10^{-11})^2}$$

$$F_g = 3.6 \times 10^{-47} \text{ N}$$

$$\frac{F_e}{F_g} = 2.28 \times 10^{39}$$

Problem 22.09

In the Bohr theory of the hydrogen atom, an electron moves in a circular orbit about a proton, where the radius of the orbit is 0.529×10^{-10} m. (a) Find the electric force between the two. (b) If this force causes the centripetal acceleration of the electron, what is the speed of the electron?

(a)

$$F_e = k_e \frac{|q_e||q_p|}{r^2}$$

$$F_e = 8.99 \times 10^9 \times \frac{|-1.6 \times 10^{-19}| \times |1.6 \times 10^{-19}|}{(0.529 \times 10^{-10})^2}$$

$$F_e = 8.22 \times 10^{-8} \text{ N}$$

The magnitude of the centripetal force F_c on an object of mass m moving at tangential speed v along a path with radius of curvature r is:

$$F_c = m \frac{v^2}{r}$$

Problem 22.09

In the Bohr theory of the hydrogen atom, an electron moves in a circular orbit about a proton, where the radius of the orbit is 0.529×10^{-10} m. (a) Find the electric force between the two. (b) If this force causes the centripetal acceleration of the electron, what is the speed of the electron?

(a)

$$F_e = k_e \frac{|q_e||q_p|}{r^2}$$

$$F_e = 8.99 \times 10^9 \times \frac{|-1.6 \times 10^{-19}| \times |1.6 \times 10^{-19}|}{(0.529 \times 10^{-10})^2}$$

$$F_e = 8.22 \times 10^{-8} \text{ N}$$

(b)

$$F_c = F_e$$

$$F_c = m_e \frac{v^2}{r}$$

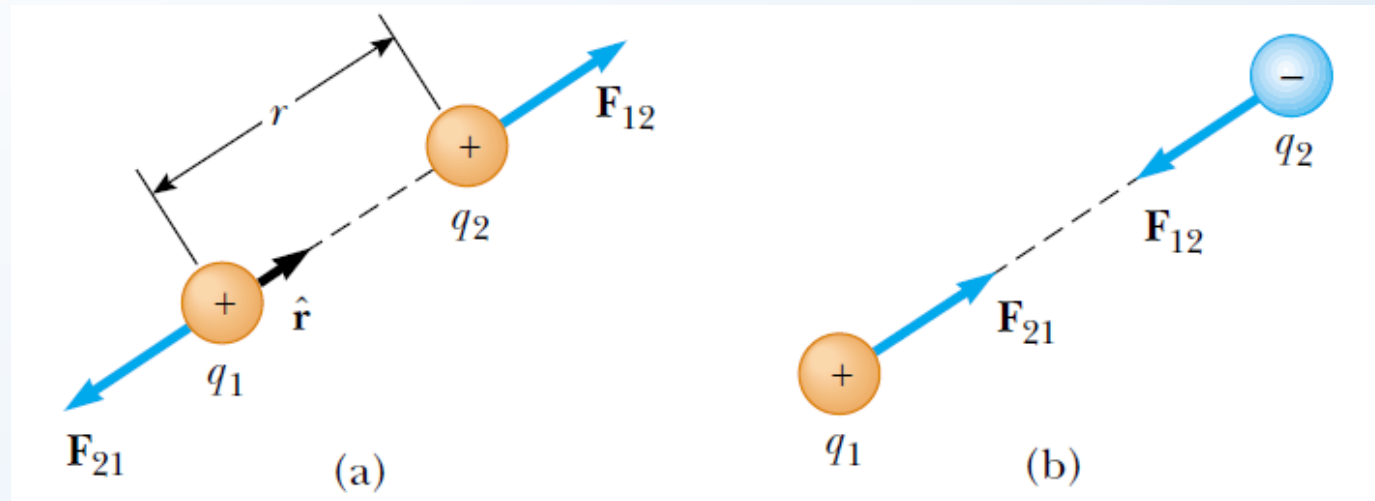
$$v = \sqrt{\frac{r}{m_e} F_c}$$

$$v = \sqrt{\frac{0.529 \times 10^{-10}}{9.11 \times 10^{-31}} \times 8.22 \times 10^{-8}}$$

$$v = 2.19 \times 10^6 \text{ m/s}$$

Coulomb's law expressed in vector form for the electric force exerted by a charge q_1 on a second charge q_2 , written $\vec{F}_{12}$, is:

$$\vec{F}_{12} = k_e \frac{|q_1||q_2|}{r^2} \hat{r}$$



Two point charges separated by a distance r exert a force on each other that is given by Coulomb's law. The force $\vec{F}_{21}$ exerted by q_2 on q_1 is equal in magnitude and opposite in direction to the force $\vec{F}_{12}$ exerted by q_1 on q_2 . (a) When the charges are of the same sign, the force is repulsive. (b) When the charges are of opposite signs, the force is attractive.

Example 22.02

Consider three point charges located at the corners of a right triangle as shown in the following figure, where $q_1 = q_3 = 5.0 \mu\text{C}$, $q_2 = -2.0 \mu\text{C}$, $a = 0.1 \text{ m}$. Find the resultant force exerted on q_3 .

$$F_{13} = k_e \frac{|q_1||q_3|}{(\sqrt{2}a)^2}$$

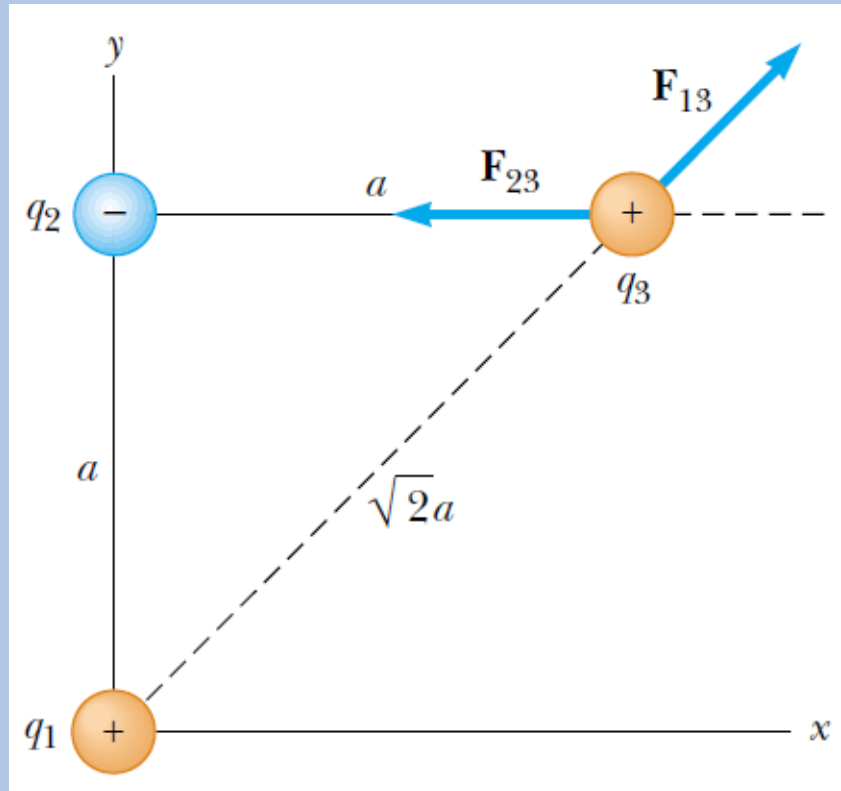
$$F_{13} = 9 \times 10^9 \times \frac{|5 \times 10^{-6}| \times |5 \times 10^{-6}|}{2(0.1)^2}$$

$$F_{13} = 11.24 \text{ N}$$

$$F_{23} = k_e \frac{|q_2||q_3|}{a^2}$$

$$F_{23} = 9 \times 10^9 \times \frac{|-2 \times 10^{-6}| \times |5 \times 10^{-6}|}{(0.1)^2}$$

$$F_{23} = 9 \text{ N}$$



The force exerted by q_1 on q_3 is $\vec{F}_{13}$. The force exerted by q_2 on q_3 is $\vec{F}_{23}$. The resultant force $\vec{F}_3$ exerted on q_3 is the vector sum $\vec{F}_{13} + \vec{F}_{23}$.

Example 22.02

Consider three point charges located at the corners of a right triangle as shown in the following figure, where $q_1 = q_3 = 5.0 \mu\text{C}$, $q_2 = -2.0 \mu\text{C}$, $a = 0.1 \text{ m}$. Find the resultant force exerted on q_3 .

$$F_{3x} = F_{13x} + F_{23x}$$

$$F_{3x} = F_{13} \cos(\theta) + F_{23x}$$

$$F_{3x} = F_{13} \cos(45) + F_{23x}$$

$$F_{3x} = 11.24 \cos(45) + (-9)$$

$$F_{3x} = -1.05 \text{ N}$$

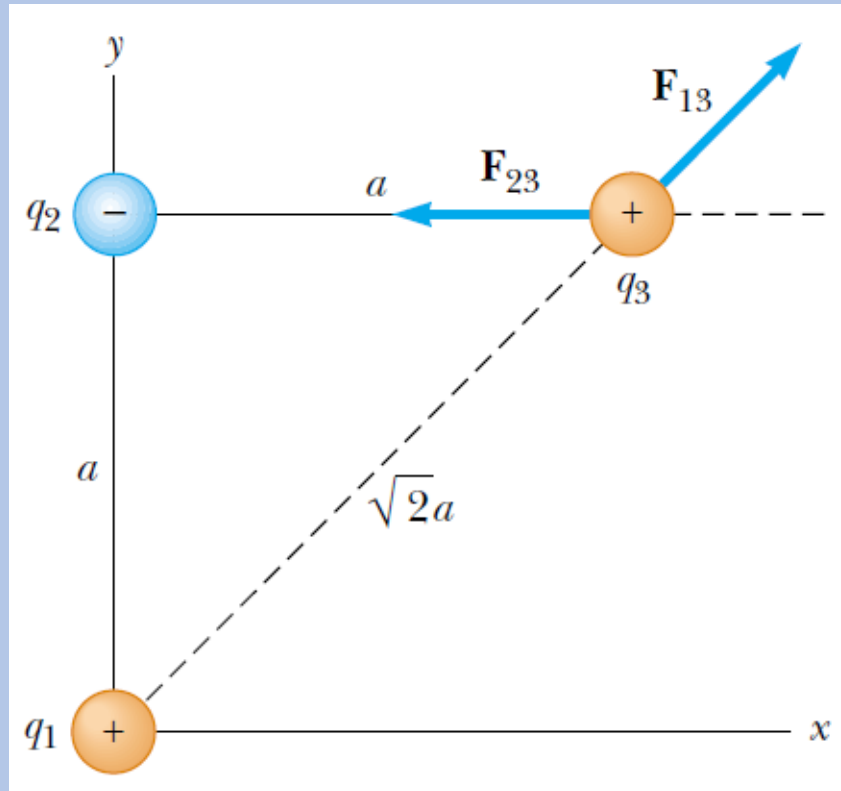
$$F_{3y} = F_{13y} + F_{23y}$$

$$F_{3y} = F_{13} \sin(\theta) + 0$$

$$F_{3y} = F_{13} \sin(45) + 0$$

$$F_{3y} = 11.24 \sin(45)$$

$$F_{3y} = 7.95 \text{ N}$$



The force exerted by q_1 on q_3 is $\vec{F}_{13}$. The force exerted by q_2 on q_3 is $\vec{F}_{23}$. The resultant force $\vec{F}_3$ exerted on q_3 is the vector sum $\vec{F}_{13} + \vec{F}_{23}$.

Example 22.02

Consider three point charges located at the corners of a right triangle as shown in the following figure, where $q_1 = q_3 = 5.0 \mu\text{C}$, $q_2 = -2.0 \mu\text{C}$, $a = 0.1 \text{ m}$. Find the resultant force exerted on q_3 .

$$F_3 = \sqrt{(F_{3x})^2 + (F_{3y})^2}$$

$$F_3 = \sqrt{(-1.05)^2 + (7.95)^2}$$

$$F_3 = 8.02 \text{ N}$$

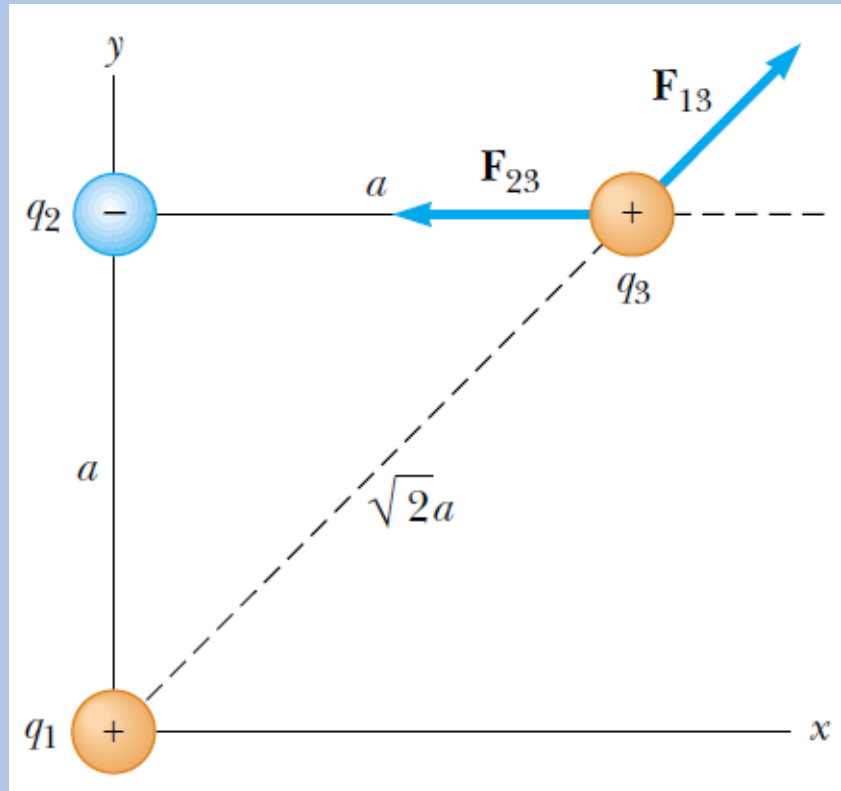
$$\varphi = \tan^{-1} \frac{F_{3y}}{F_{3x}}$$

$$\varphi = \tan^{-1} \frac{7.95}{-1.05}$$

$$\varphi = 97.5^\circ$$

or

$$\vec{F}_3 = (-1.05 \hat{i} + 7.95 \hat{j}) \text{ N}$$



y		x
F_x negative	F_x positive	
F_y positive	F_y positive	
F_x negative	F_x positive	
F_y negative	F_y negative	

The force exerted by q_1 on q_3 is $\vec{F}_{13}$. The force exerted by q_2 on q_3 is $\vec{F}_{23}$. The resultant force $\vec{F}_3$ exerted on q_3 is the vector sum $\vec{F}_{13} + \vec{F}_{23}$.

Example 22.03

Three point charges lie along the x axis as shown in the following figure. The positive charge $q_1 = 15 \mu\text{C}$ is at $x = 2 \text{ m}$, the positive charge $q_2 = 6 \mu\text{C}$ is at the origin, and the resultant force acting on q_3 is zero. What is the x coordinate of q_3 ?

$$F_{13} = k_e \frac{|q_1||q_3|}{(2-x)^2} \quad ; \quad F_{23} = k_e \frac{|q_2||q_3|}{x^2}$$

$$k_e \frac{|q_1||q_3|}{(2-x)^2} = k_e \frac{|q_2||q_3|}{x^2}$$

$$x^2|q_1| = (2-x)^2|q_2|$$

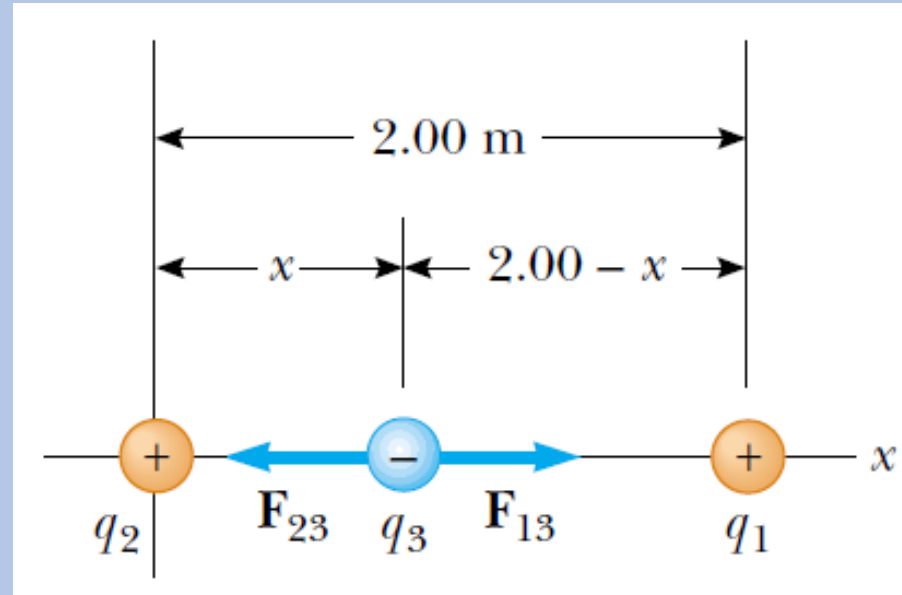
$$x^2(15 \times 10^{-6}) = (2-x)^2(6 \times 10^{-6})$$

$$\sqrt{15}x = 2\sqrt{6} - \sqrt{6}x$$

$$(\sqrt{15} + \sqrt{6})x = 2\sqrt{6}$$

$$x = \frac{2\sqrt{6}}{\sqrt{15} + \sqrt{6}}$$

$$x = 0.775 \text{ m}$$



Three point charges are placed along the x axis. If the resultant force acting on q_3 is zero, then the force F_{13} exerted by q_1 on q_3 must be equal in magnitude and opposite in direction to the force F_{23} exerted by q_2 on q_3 .



Photomath



Socratic

Question 22.01

Additional question

Two protons in an atomic nucleus are typically separated by a distance of 2×10^{-15} m. The electric repulsion force between the protons is huge, but the attractive nuclear force is even stronger and keeps the nucleus from bursting apart. What is the magnitude of the electric force between two protons separated by 2×10^{-15} m?

$$F_e = k_e \frac{|q_p||q_p|}{r^2}$$

$$F_e = 8.99 \times 10^9 \times \frac{|1.60 \times 10^{-19}| \times |1.60 \times 10^{-19}|}{(2 \times 10^{-15})^2}$$

$$F_e = 57.5 \text{ N}$$

Question 22.02

Additional question

Three point charges are located at the corners of an equilateral triangle as shown in the following figure. Calculate the resultant electric force on the $7.0 \mu\text{C}$ charge.

$$F_{13} = k_e \frac{|q_1||q_3|}{r^2}$$

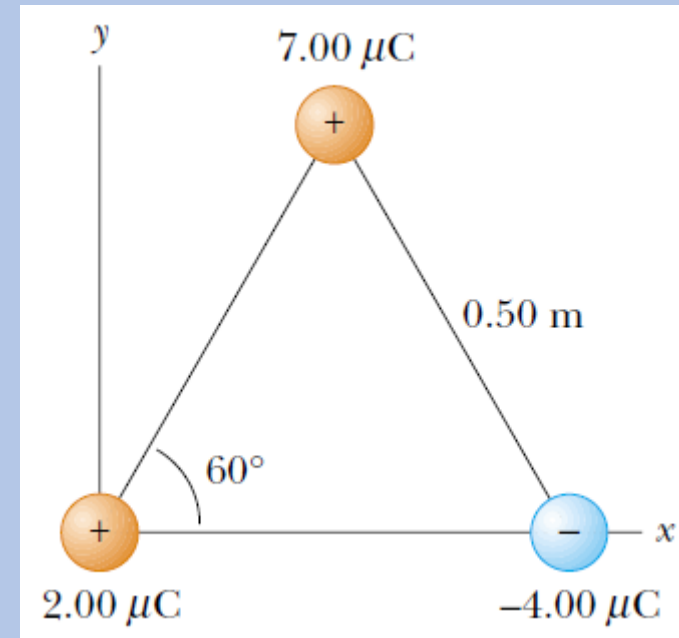
$$F_{13} = 8.99 \times 10^9 \times \frac{|2 \times 10^{-6}| \times |7 \times 10^{-6}|}{(0.5)^2}$$

$$F_{13} = 0.503 \text{ N}$$

$$F_{23} = k_e \frac{|q_2||q_3|}{r^2}$$

$$F_{23} = 8.99 \times 10^9 \times \frac{|-4 \times 10^{-6}| \times |7 \times 10^{-6}|}{(0.5)^2}$$

$$F_{23} = 1.01 \text{ N}$$



Question 22.02

Additional question

Three point charges are located at the corners of an equilateral triangle as shown in the following figure. Calculate the resultant electric force on the $7.0 \mu\text{C}$ charge.

$$F_{3x} = F_{13x} + F_{23x}$$

$$F_{3x} = F_{13} \cos(\theta_1) + F_{23} \cos(\theta_2)$$

$$F_{3x} = 0.503 \cos(60) + 1.01 \cos(-60)$$

$$F_{3x} = 0.252 + 0.505$$

$$F_{3x} = 0.757 \text{ N}$$

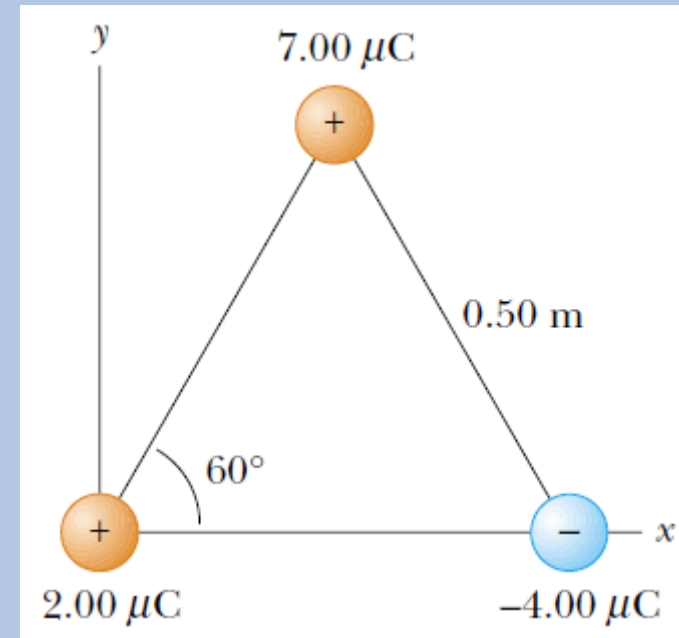
$$F_{3y} = F_{13y} + F_{23y}$$

$$F_{3y} = F_{13} \sin(\theta_1) + F_{23} \sin(\theta_2)$$

$$F_{3y} = 0.503 \sin(60) + 1.01 \sin(-60)$$

$$F_{3y} = 0.436 - 0.875$$

$$F_{3y} = -0.439 \text{ N}$$



Question 22.02

Additional question

Three point charges are located at the corners of an equilateral triangle as shown in the following figure. Calculate the resultant electric force on the $7.0 \mu\text{C}$ charge.

$$F_3 = \sqrt{(F_{3x})^2 + (F_{3y})^2}$$

$$F_3 = \sqrt{(0.757)^2 + (-0.439)^2}$$

$$F_3 = 0.875 \text{ N}$$

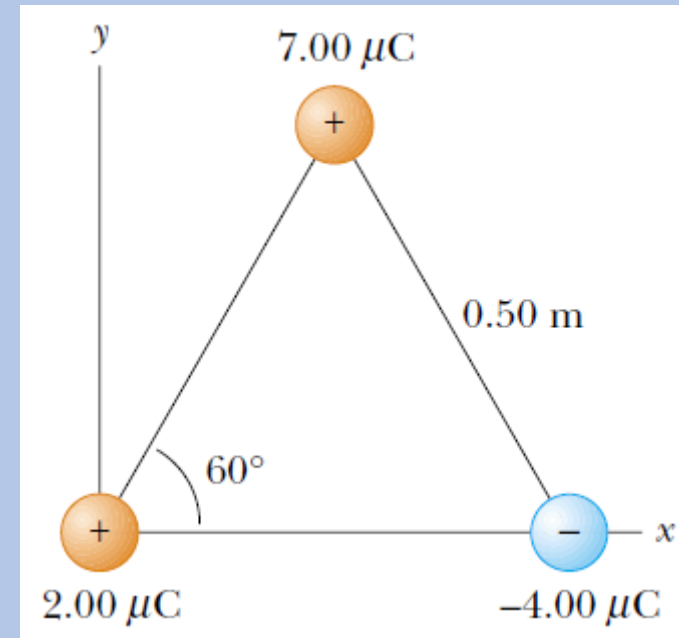
$$\varphi = \tan^{-1} \frac{F_{3y}}{F_{3x}}$$

$$\varphi = \tan^{-1} \frac{-0.439}{0.757}$$

$$\varphi = 329.89^\circ \text{ (or } -30.11^\circ)$$

or

$$\vec{F}_3 = (0.757\hat{i} - 0.439\hat{j}) \text{ N}$$



Problem 22.07

Additional problem

Two small beads having positive charges $3q$ and q are fixed at the opposite ends of a horizontal, insulating rod, extending from the origin to the point $x = d$. As shown in the following figure, a third small charged bead is free to slide on the rod. At what position is the third bead in equilibrium? Can it be in stable equilibrium?

$$F_{3qQ} = k_e \frac{|3q||Q|}{x^2} \quad ; \quad F_{qQ} = k_e \frac{|q||Q|}{(d-x)^2}$$

$$k_e \frac{|3q||Q|}{x^2} = k_e \frac{|q||Q|}{(d-x)^2}$$

$$3(d-x)^2 = x^2$$

$$\sqrt{3}(d-x) = x$$

$$x + \sqrt{3}x = \sqrt{3}d$$

$$(1 + \sqrt{3})x = \sqrt{3}d$$

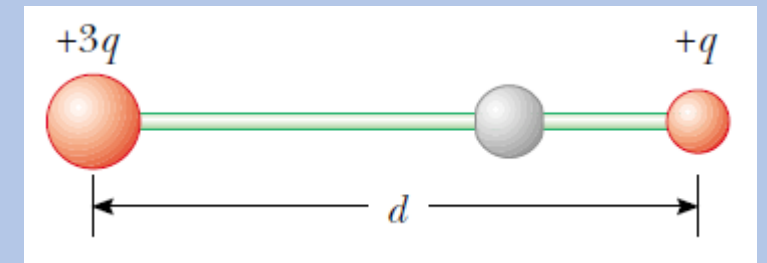
$$x = \frac{\sqrt{3}}{1 + \sqrt{3}} d$$

This gives an equilibrium position of the third bead of:

$$x = 0.635d$$

The equilibrium is stable if the third bead has positive charge

Why?



Problem 22.36

Two known charges, $-12.0 \mu\text{C}$ and $45.0 \mu\text{C}$, and an unknown charge are located on the x axis. The charge $-12.0 \mu\text{C}$ is at the origin, and the charge $45.0 \mu\text{C}$ is at $x = 15 \text{ cm}$. The unknown charge is to be placed so that each charge is in equilibrium under the action of the electric forces exerted by the other two charges. Is this situation possible? Is it possible in more than one way? Find the required location, magnitude, and sign of the unknown charge.

***Hint: which is the correct location of the unknown charge that satisfies its equilibrium.**

The equilibrium of the unknown charge requires:

$$k_e \frac{|-12 \times 10^{-6}| |Q|}{d^2} = k_e \frac{|45 \times 10^{-6}| |Q|}{(d + 0.15)^2}$$

$$4(d + 0.15)^2 = 15d^2$$

$$2(d + 0.15) = \sqrt{15}d$$

$$\sqrt{15}d - 2d = 0.30$$

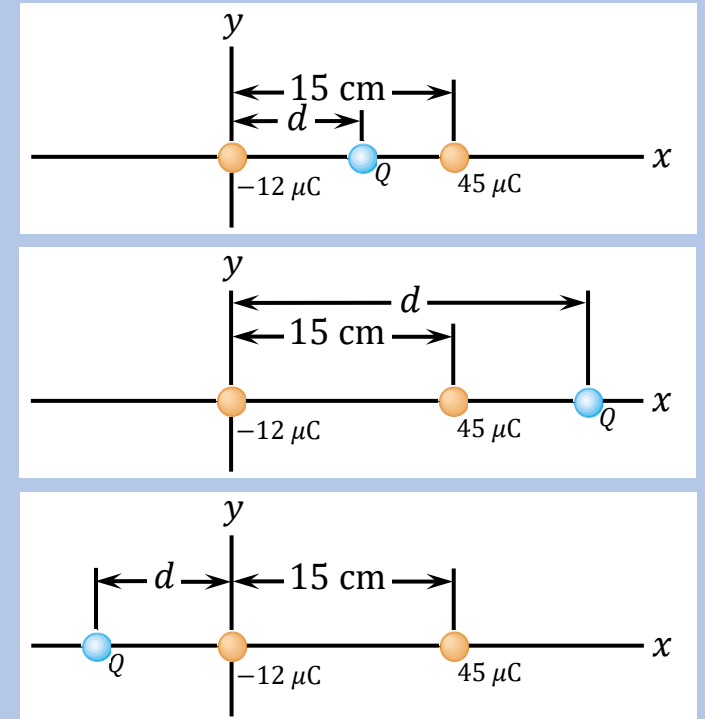
$$(\sqrt{15} - 2)d = 0.30$$

$$d = \frac{0.30}{(\sqrt{15} - 2)}$$

$$d = 0.16 \text{ m}$$

To satisfy the equilibrium of the unknown, it should be placed at the position:

$$x = -0.16 \text{ m}$$



Problem 22.36

Two known charges, $-12.0 \mu\text{C}$ and $45.0 \mu\text{C}$, and an unknown charge are located on the x axis. The charge $-12.0 \mu\text{C}$ is at the origin, and the charge $45.0 \mu\text{C}$ is at $x = 15 \text{ cm}$. The unknown charge is to be placed so that each charge is in equilibrium under the action of the electric forces exerted by the other two charges. Is this situation possible? Is it possible in more than one way? Find the required location, magnitude, and sign of the unknown charge.

To find the magnitude, and sign of the unknown charge

The equilibrium of the $-12.0 \mu\text{C}$ charge requires:

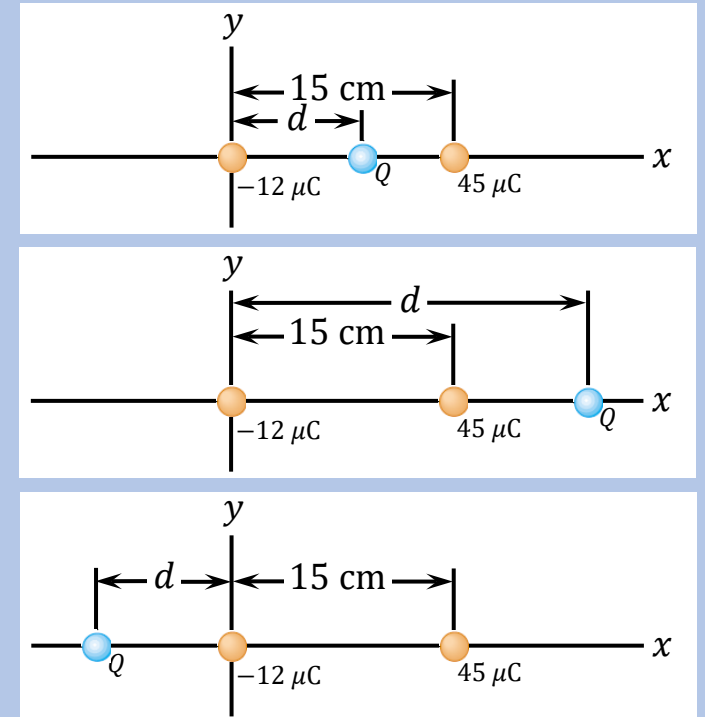
$$k_e \frac{|Q| |-12 \times 10^{-6}|}{(0.16)^2} = k_e \frac{|45 \times 10^{-6}| |-12 \times 10^{-6}|}{(0.15)^2}$$

$$(0.15)^2 Q = (0.16)^2 \times 45 \times 10^{-6}$$

$$Q = \frac{0.0256 \times 45 \times 10^{-6}}{0.0225}$$

$$Q = 5.12 \times 10^{-5} \text{ C}$$

$$Q = 51.2 \mu\text{C} \quad ; \text{ positive charge}$$



- An **electric field** is said to exist in the region of space around a charged object—the source charge.
- When another charged object—the test charge—enters this electric field, an electric force acts on it.

The **electric field** vector E at a point in space is defined as the electric force F_e acting on a positive test charge q_0 placed at that point divided by the test charge:

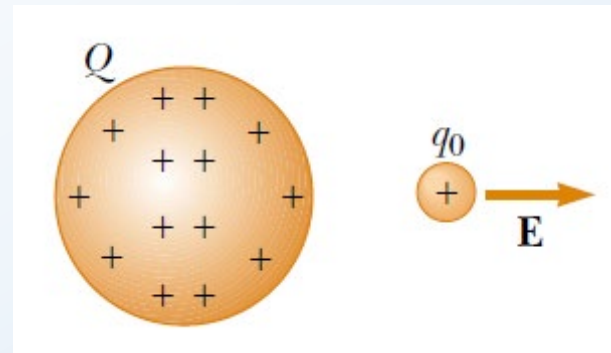
$$\vec{E} = \frac{\vec{F}_e}{q_0}$$

or the equation can be rearranged as:

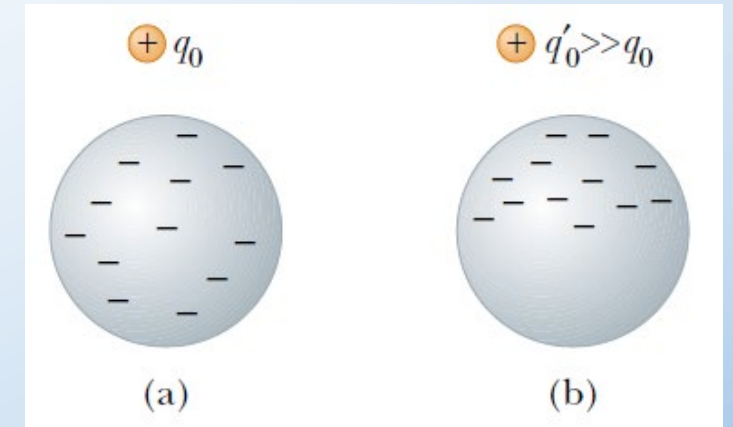
$$\vec{F}_e = q\vec{E}$$

where we have used the general symbol q for a charge.

This equation gives us the force on a charged particle placed in an electric field.



A small positive test charge q_0 placed near an object carrying a much larger positive charge Q experiences an electric field E directed as shown.



(a) For a small enough test charge q_0 , the charge distribution on the sphere is undisturbed. (b) When the test charge q_0 , is greater, the charge distribution on the sphere is disturbed as the result of the proximity of q_0 .

- The vector E has the SI units of newtons per coulomb (N/C).
- The electric field does not depend on the existence of the test charge—it is established solely by the source charge.

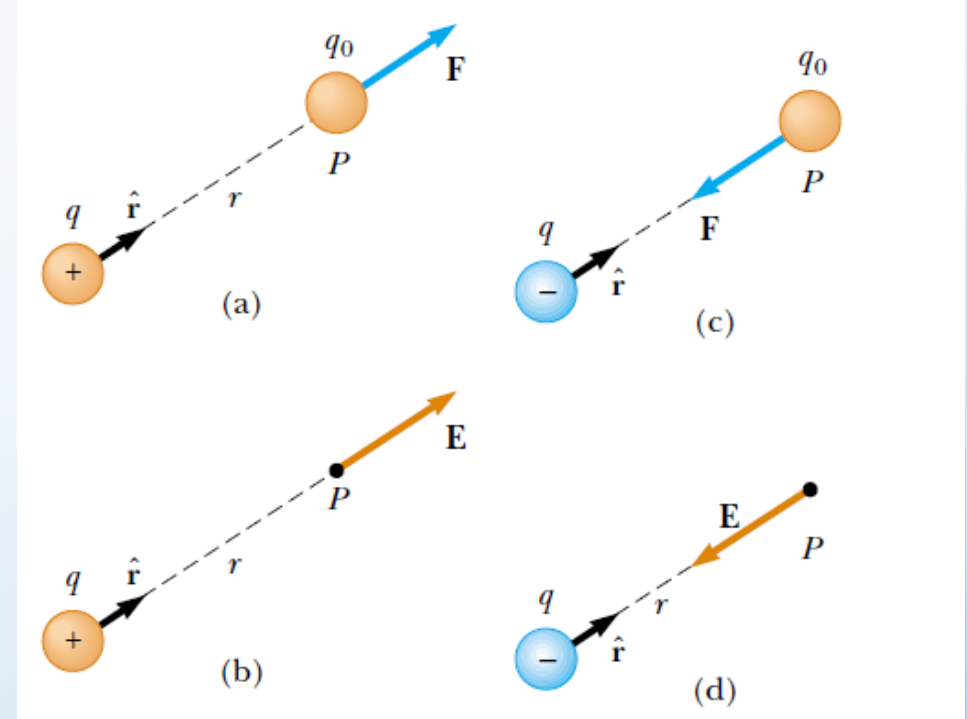
According to Coulomb's law, the electric force vector $\vec{F}_e$ exerted by q on the test charge q_0 is $\vec{F}_e = (k_e q q_0 / r^2) \hat{r}$

The electric field at P , the position of the test charge, is defined by $\vec{E} = \vec{F}_e / q_0$, we find that at P , the electric field created by q is:

$$\vec{E} = k_e \frac{q}{r^2} \hat{r}$$

If the source charge q is positive, the electric force on the test charge q_0 is directed away from the source charge q , so the electric field at point P is directed away from q .

If the source charge q is negative, the electric force on the test charge q_0 is directed toward the source charge q , so the electric field at point P is directed toward the source charge q .



A test charge q_0 at point P is a distance r from a point charge q . (a) If q is positive, then the force on the test charge is directed away from q . (b) For the positive source charge, the electric field at P points radially outward from q . (c) If q is negative, then the force on the test charge is directed toward q . (d) For the negative source charge, the electric field at P points radially inward toward q .

Question 22.03

A charge $q_1 = 7.0 \mu\text{C}$ is located at the origin, and a second charge $q_2 = -5.0 \mu\text{C}$ is located on the x axis, 0.30 m from the origin (see the figure below). Find the electric field at the point P , which has coordinates $(0, 0.40)$ m.

$$E_1 = k_e \frac{|q_1|}{r_1^2}$$

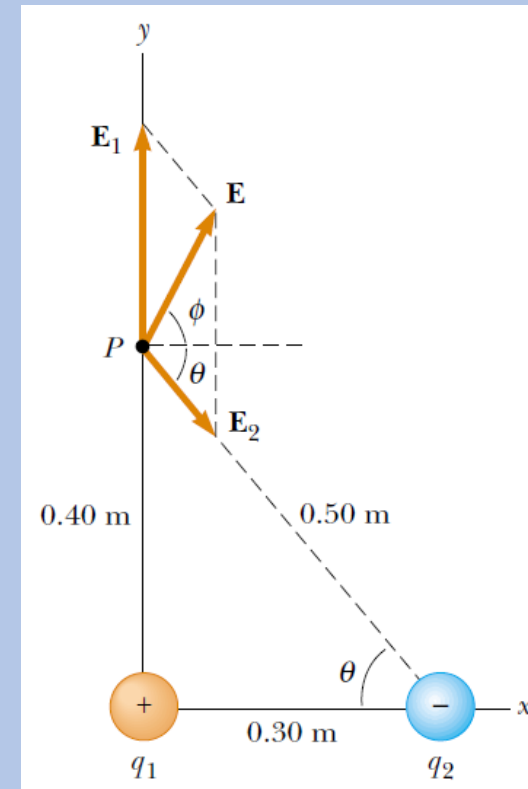
$$E_1 = 8.99 \times 10^9 \times \frac{|7.0 \times 10^{-6}|}{(0.40)^2}$$

$$E_1 = 3.93 \times 10^5 \text{ N/C}$$

$$E_2 = k_e \frac{|q_2|}{r_2^2}$$

$$E_2 = 8.99 \times 10^9 \times \frac{|-5.0 \times 10^{-6}|}{(0.50)^2}$$

$$E_2 = 1.80 \times 10^5 \text{ N/C}$$



The total electric field $\vec{E}$ at P equals the vector sum $\vec{E}_1 + \vec{E}_2$, where $\vec{E}_1$ is the field due to the positive charge q_1 and $\vec{E}_2$ is the field due to the negative charge q_2 .

Question 22.03

A charge $q_1 = 7.0 \mu\text{C}$ is located at the origin, and a second charge $q_2 = -5.0 \mu\text{C}$ is located on the x axis, 0.30 m from the origin (see the figure below). Find the electric field at the point P , which has coordinates $(0, 0.40)$ m.

$$E_x = E_{1x} + E_{2x}$$

$$E_x = 0 + E_2 \cos(\theta)$$

$$E_x = 1.80 \times 10^5 \times \frac{0.3}{0.5}$$

$$E_x = 1.08 \times 10^5 \text{ N/C}$$

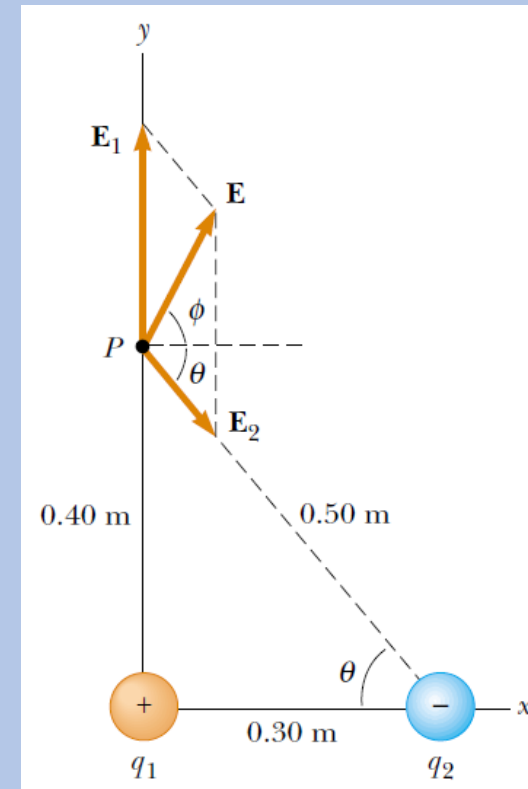
$$E_y = E_{1y} + E_{2y}$$

$$E_y = E_1 + E_2 \sin(\theta)$$

$$E_y = (3.93 \times 10^5) - \left(1.80 \times 10^5 \times \frac{0.4}{0.5}\right)$$

$$E_y = 3.93 \times 10^5 - 1.44 \times 10^5$$

$$E_y = 2.49 \times 10^5 \text{ N/C}$$



The total electric field $\vec{E}$ at P equals the vector sum $\vec{E}_1 + \vec{E}_2$, where $\vec{E}_1$ is the field due to the positive charge q_1 and $\vec{E}_2$ is the field due to the negative charge q_2 .

Question 22.03

A charge $q_1 = 7.0 \mu\text{C}$ is located at the origin, and a second charge $q_2 = -5.0 \mu\text{C}$ is located on the x axis, 0.30 m from the origin (see the figure below). Find the electric field at the point P , which has coordinates (0, 0.40) m.

$$E = \sqrt{(E_x)^2 + (E_y)^2}$$

$$E = \sqrt{(1.08 \times 10^5)^2 + (2.49 \times 10^5)^2}$$

$$E = 2.71 \times 10^5 \text{ N/C}$$

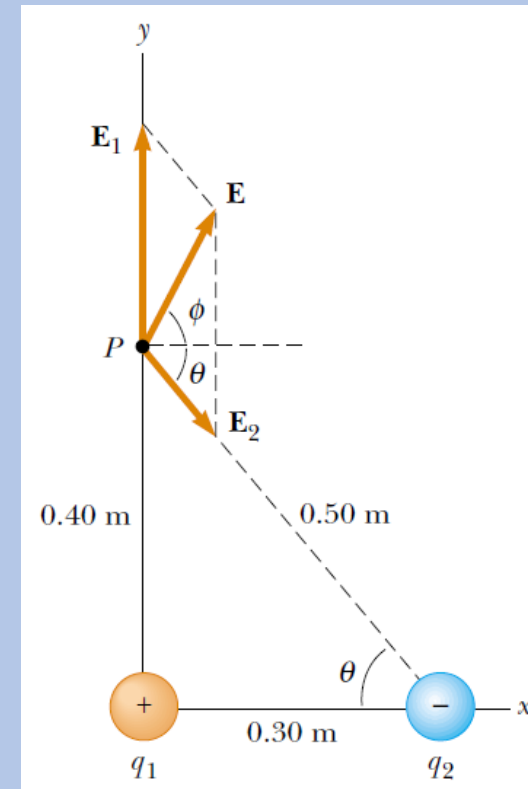
$$\phi = \tan^{-1} \frac{E_y}{E_x}$$

$$\phi = \tan^{-1} \frac{2.49 \times 10^5}{1.08 \times 10^5}$$

$$\phi = 66.55^\circ$$

or

$$\vec{E} = (1.08 \times 10^5 \hat{i} + 2.49 \times 10^5 \hat{j}) \text{ N/C}$$



The total electric field $\vec{E}$ at P equals the vector sum $\vec{E}_1 + \vec{E}_2$, where $\vec{E}_1$ is the field due to the positive charge q_1 and $\vec{E}_2$ is the field due to the negative charge q_2 .

Question 22.04

An object having a net charge of $24.0 \mu\text{C}$ is placed in a uniform electric field of 610 N/C directed vertically. What is the mass of this object if it “floats” in the field?

$$F_e = mg$$

$$qE = mg$$

$$m = \frac{qE}{g}$$

$$m = \frac{24.0 \times 10^{-6} \times 610}{9.8}$$

$$m = 0.00149 \text{ kg}$$

or

$$m = 1.49 \text{ g}$$

Question 22.05

In the following figure, determine the point (other than infinity) at which the electric field is zero.

$$E_1 = k_e \frac{|q_1|}{d^2} \quad ; \quad E_2 = k_e \frac{|q_2|}{(d + 1.00)^2}$$

$$k_e \frac{|q_1|}{d^2} = k_e \frac{|q_2|}{(d + 1.00)^2}$$

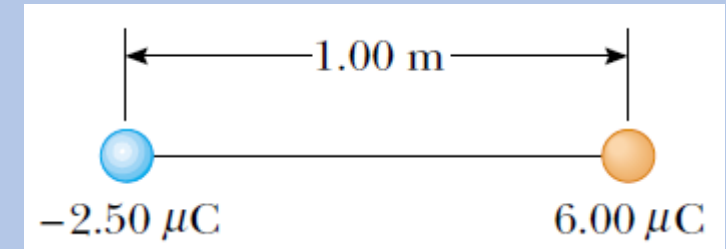
$$\frac{|-2.50 \times 10^{-6}|}{d^2} = \frac{|6.00 \times 10^{-6}|}{(d + 1.00)^2}$$

$$(d + 1.00)^2 = 2.4d^2$$

$$d + 1.00 = 1.55d$$

which yields:

$$d = 1.82 \text{ m} \quad \text{to the left of the } -2.50 \mu\text{C charge}$$



Problem 22.21

Additional problem

Three point charges are arranged as shown in the following figure. (a) Find the vector electric field that the 6.00 nC and -3.00 nC charges together create at the origin. (b) Find the vector force on the 5.00 nC charge.

(a) $E_1 = k_e \frac{|q_1|}{r_1^2}$

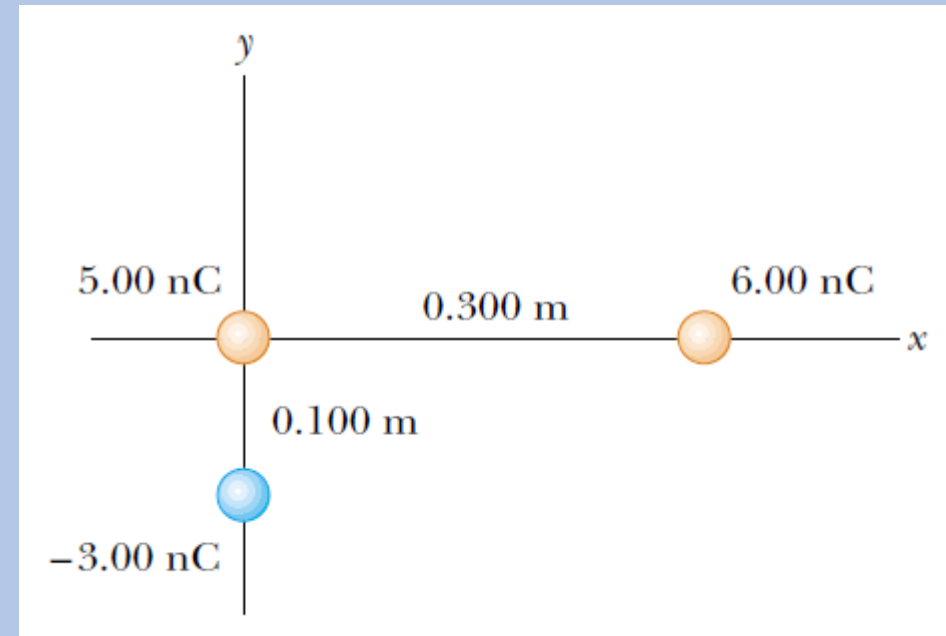
$$E_1 = 8.99 \times 10^9 \times \frac{|-3.00 \times 10^{-9}|}{(0.100)^2}$$

$$E_1 = 2.7 \times 10^3 \text{ N/C}$$

 $E_2 = k_e \frac{|q_2|}{r_2^2}$

$$E_2 = 8.99 \times 10^9 \times \frac{|6.00 \times 10^{-9}|}{(0.300)^2}$$

$$E_2 = 5.99 \times 10^2 \text{ N/C}$$



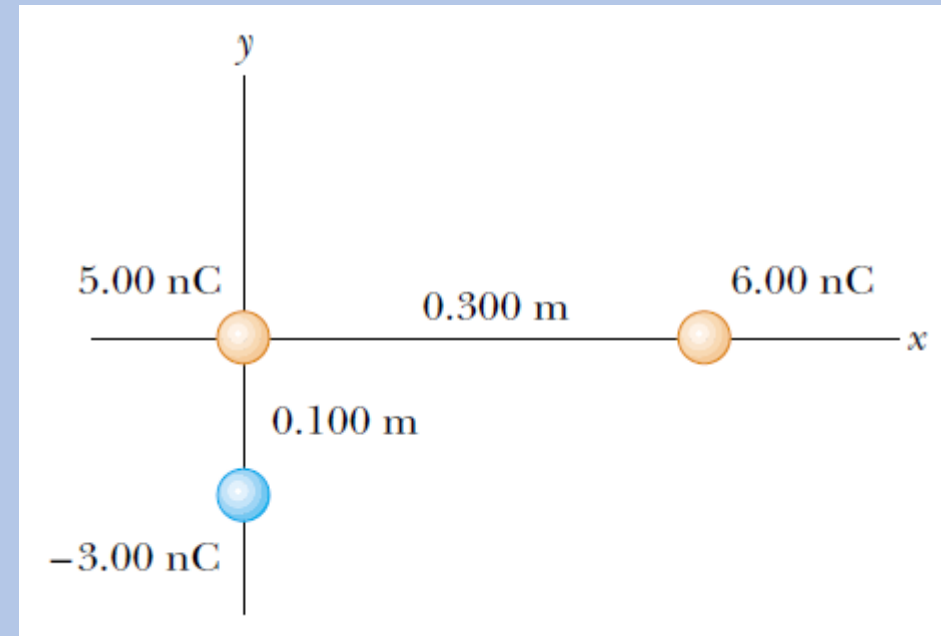
Problem 22.21

Additional problem

Three point charges are arranged as shown in the following figure. (a) Find the vector electric field that the 6.00 nC and -3.00 nC charges together create at the origin. (b) Find the vector force on the 5.00 nC charge.

(a) $E_x = E_{1x} + E_{2x}$
 $E_x = 0 + E_2 \cos(\theta_2)$
 $E_x = 5.99 \times 10^2 \times \cos(180)$
 $E_x = -5.99 \times 10^2 \text{ N/C}$

 $E_y = E_{1y} + E_{2y}$
 $E_y = E_1 \sin(\theta_1) + 0$
 $E_y = 2.7 \times 10^3 \times \sin(270)$
 $E_y = -2.7 \times 10^3 \text{ N/C}$



Problem 22.21

Additional problem

Three point charges are arranged as shown in the following figure. (a) Find the vector electric field that the 6.00 nC and -3.00 nC charges together create at the origin. (b) Find the vector force on the 5.00 nC charge.

$$(a) \quad E = \sqrt{(E_x)^2 + (E_y)^2}$$

$$E = \sqrt{(-5.99 \times 10^2)^2 + (-2.7 \times 10^3)^2}$$

$$E = 2.77 \times 10^3 \text{ N/C}$$

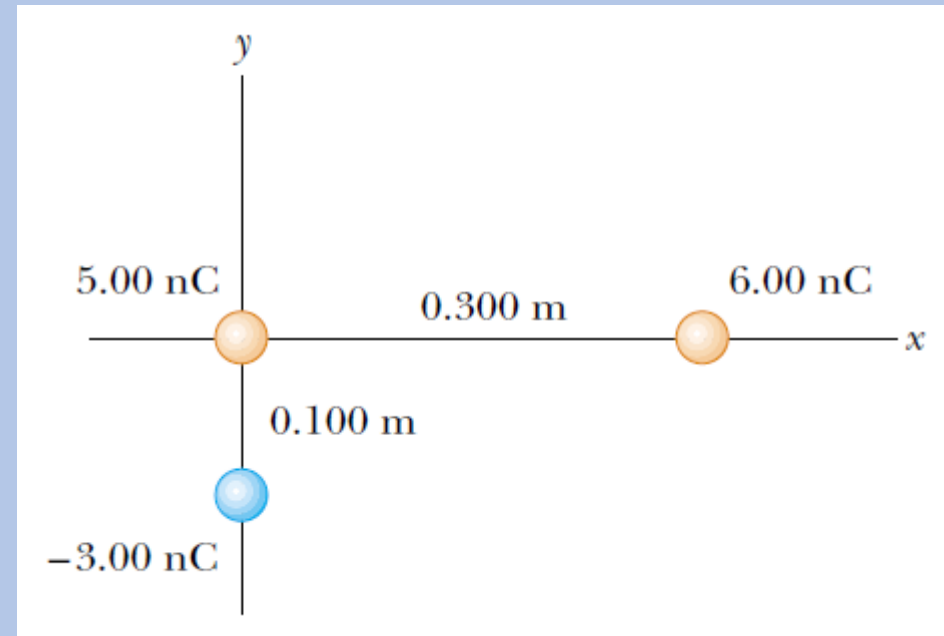
$$\varphi = \tan^{-1} \frac{E_y}{E_x}$$

$$\varphi = \tan^{-1} \frac{-2.7 \times 10^3}{-5.99 \times 10^2}$$

$$\varphi = 257.49^\circ \quad !!$$

or

$$\vec{E} = (-5.99 \times 10^3 \hat{i} - 2.7 \times 10^3 \hat{j}) \text{ N/C}$$



Problem 22.21

Additional problem

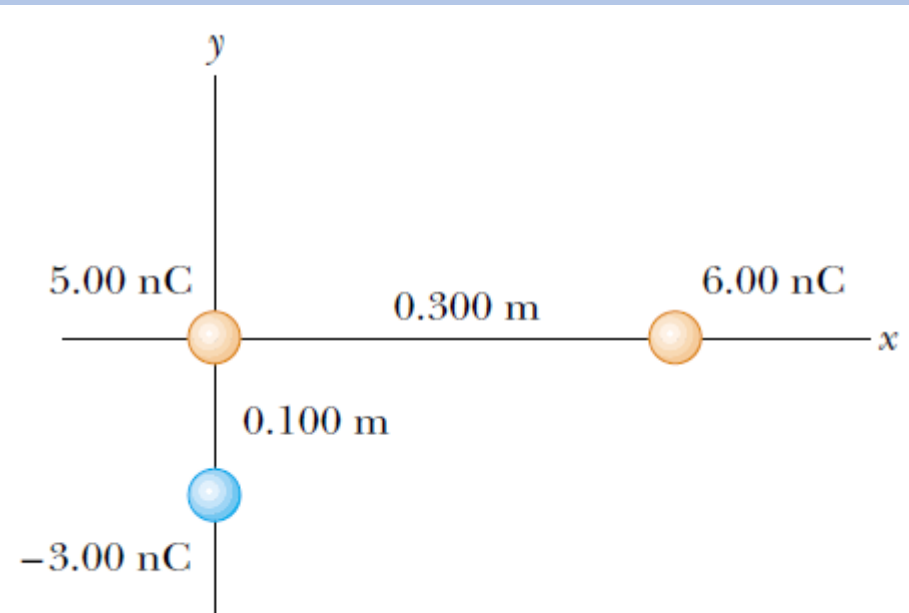
Three point charges are arranged as shown in the following figure. (a) Find the vector electric field that the 6.00 nC and -3.00 nC charges together create at the origin. (b) Find the vector force on the 5.00 nC charge.

(b) $F_3 = q_3 E$

$$F_3 = 5.00 \times 10^{-9} \times 2.77 \times 10^3$$

$$F_3 = 1.39 \times 10^{-5} \text{ N}$$

$$\varphi = 257.49^\circ$$



Question 22.06

Additional question

Four point charges are at the corners of a square of side a as shown in the following figure. (a) Determine the magnitude and direction of the electric field at the location of charge q . (b) What is the resultant force on q ?

(a)

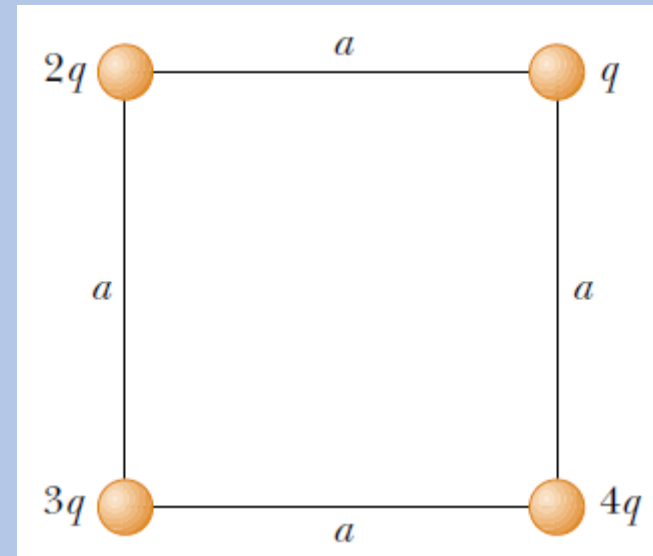
$$E_1 = k_e \frac{|q_1|}{r_1^2}$$

$$E_1 = k_e \frac{2q}{a^2}$$

$$E_2 = k_e \frac{|q_2|}{r_2^2}$$

$$E_2 = k_e \frac{3q}{(\sqrt{2}a)^2}$$

$$E_2 = k_e \frac{3q}{2a^2}$$



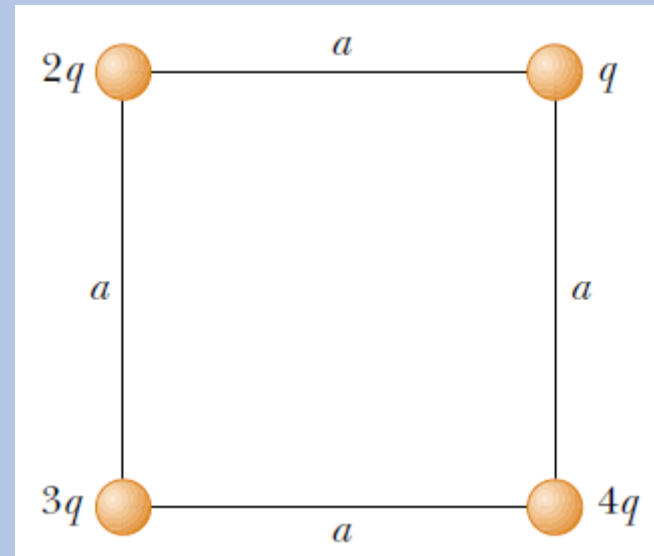
Question 22.06

Additional question

Four point charges are at the corners of a square of side a as shown in the following figure. (a) Determine the magnitude and direction of the electric field at the location of charge q . (b) What is the resultant force on q ?

(a)
$$E_3 = k_e \frac{|q_2|}{r_3^2}$$

$$E_3 = k_e \frac{4q}{a^2}$$



Question 22.06

Additional question

Four point charges are at the corners of a square of side a as shown in the following figure. (a) Determine the magnitude and direction of the electric field at the location of charge q . (b) What is the resultant force on q ?

(a)

$$E_x = E_{1x} + E_{2x} + E_{3x}$$

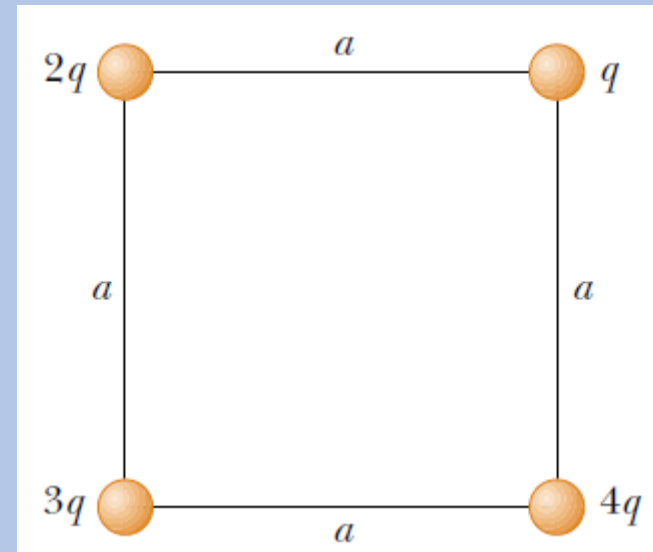
$$E_x = k_e \frac{2q}{a^2} + k_e \frac{3q}{2a^2} \cos(45) + 0$$

$$E_x = 3.06 \times \frac{k_e q}{a^2}$$

$$E_y = E_{1y} + E_{2y} + E_{3y}$$

$$E_y = 0 + k_e \frac{3q}{2a^2} \sin(45) + k_e \frac{4q}{a^2}$$

$$E_y = 5.06 \times \frac{k_e q}{a^2}$$



Question 22.06

Additional question

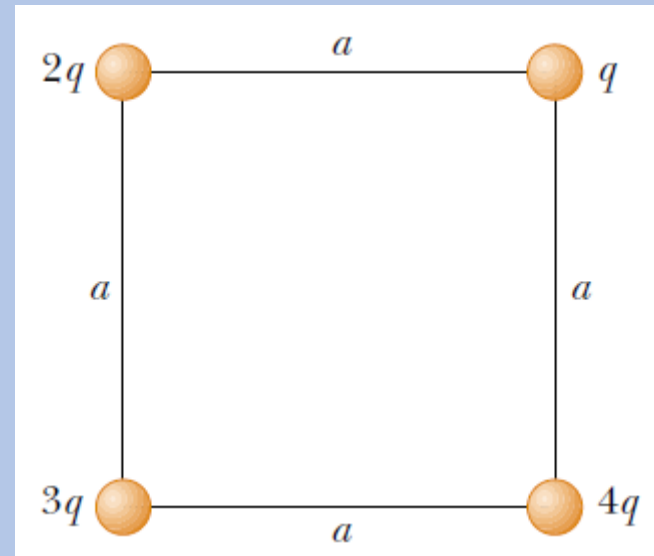
Four point charges are at the corners of a square of side a as shown in the following figure. (a) Determine the magnitude and direction of the electric field at the location of charge q . (b) What is the resultant force on q ?

$$(a) \quad E = \sqrt{(E_x)^2 + (E_y)^2} = \sqrt{\left(3.06 \times \frac{k_e q}{a^2}\right)^2 + \left(5.06 \times \frac{k_e q}{a^2}\right)^2} = 5.91 \times \frac{k_e q}{a^2}$$

$$\varphi = \tan^{-1} \frac{E_y}{E_x} = \tan^{-1} \left(\frac{5.06 \times \frac{k_e q}{a^2}}{3.06 \times \frac{k_e q}{a^2}} \right) = 58.83^\circ$$

or

$$\vec{E} = (3.06\hat{i} + 5.06\hat{j}) \frac{k_e q}{a^2}$$



Question 22.06

Additional question

Four point charges are at the corners of a square of side a as shown in the following figure. (a) Determine the magnitude and direction of the electric field at the location of charge q . (b) What is the resultant force on q ?

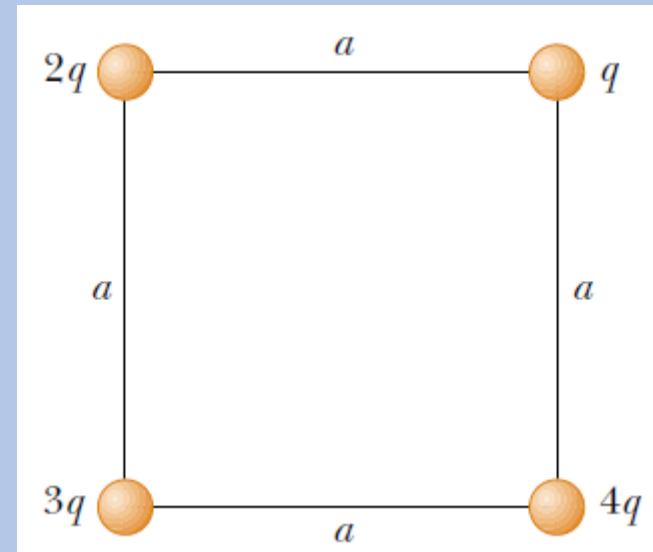
(b) $F_4 = q_4 E$

$$F_4 = q \times 5.91 \times \frac{k_e q}{a^2}$$

$$F_4 = 5.91 \times \frac{k_e q^2}{a^2}$$

$$\varphi = \tan^{-1} \frac{F_{4y}}{F_{4x}}$$

$$\varphi = 58.83^\circ$$



Problem 22.22

Additional problem

Consider the electric dipole shown in the following figure. Show that the electric field at a distant point on the $+x$ axis is $E_x \approx 4k_e \frac{qa}{x^3}$.

The electric field at any point x is:

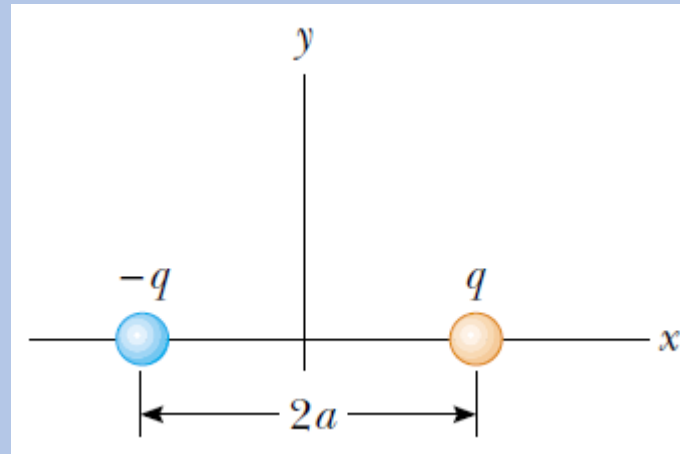
$$E_x = E_{1x} + E_{2x}$$

$$E_x = -k_e \frac{|-q|}{(x+a)^2} + k_e \frac{|q|}{(x-a)^2}$$

$$E_x = k_e \frac{q}{(x-a)^2} - k_e \frac{q}{(x+a)^2}$$

$$E_x = k_e q \frac{(x+a)^2 - (x-a)^2}{(x-a)^2(x+a)^2}$$

$$E_x = k_e q \frac{(x^2 + 2ax + a^2) - (x^2 - 2ax + a^2)}{((x-a)(x+a))^2}$$



Problem 22.22

Additional problem

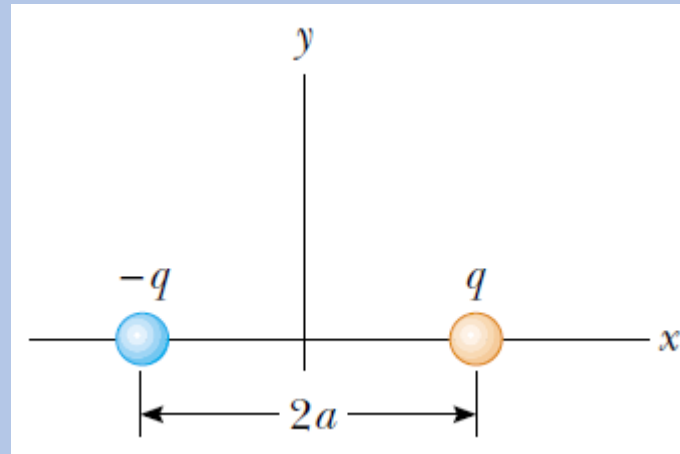
Consider the electric dipole shown in the following figure. Show that the electric field at a distant point on the $+x$ axis is $E_x \approx 4k_e \frac{qa}{x^3}$.

$$E_x = k_e q \frac{x^2 + 2ax + a^2 - x^2 + 2ax - a^2}{(x^2 - a^2)^2}$$

$$E_x = k_e q \frac{4ax}{(x^2 - a^2)^2}$$

When x is much, much greater than a , we find:

$$E_x \approx 4k_e \frac{qa}{x^3}$$



Problem 22.35

Additional problem

Three point charges are aligned along the x axis as shown in the following figure. Find the electric field at (a) the position $(2.00, 0)$ and (b) the position $(0, 2.00)$.

$$(a) E_1 = k_e \frac{|q_1|}{r_1^2}$$

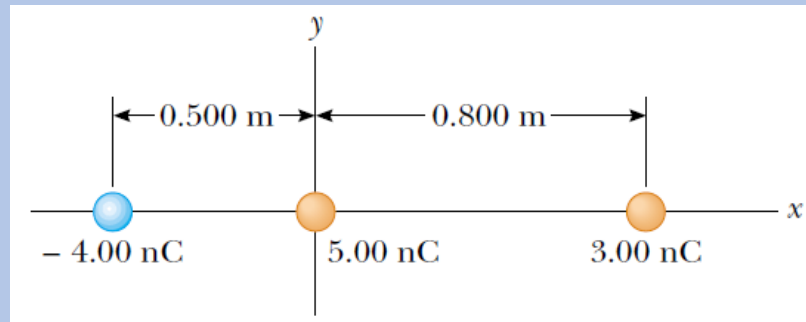
$$E_1 = 8.99 \times 10^9 \times \frac{|-4.00 \times 10^{-9}|}{(2.50)^2}$$

$$E_1 = 5.75 \text{ N/C}$$

$$E_2 = k_e \frac{|q_2|}{r_2^2}$$

$$E_2 = 8.99 \times 10^9 \times \frac{|5.0 \times 10^{-9}|}{(2.00)^2}$$

$$E_2 = 11.24 \text{ N/C}$$



Problem 22.35

Additional problem

Three point charges are aligned along the x axis as shown in the following figure. Find the electric field at (a) the position $(2.00, 0)$ and (b) the position $(0, 2.00)$.

$$E_3 = k_e \frac{|q_3|}{r_3^2}$$

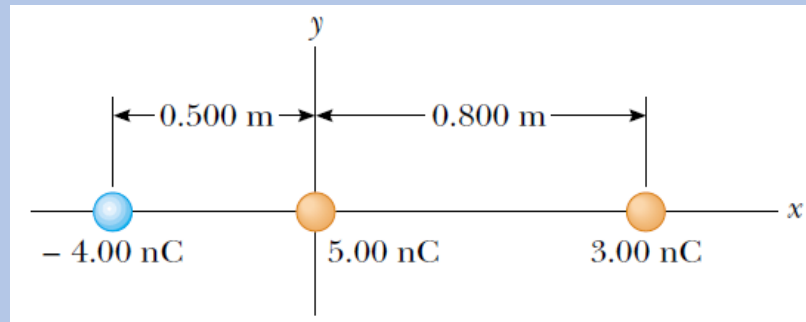
$$E_3 = 8.99 \times 10^9 \times \frac{|3.00 \times 10^{-9}|}{(1.20)^2}$$

$$E_3 = 18.73 \text{ N/C}$$

$$E = -E_1 + E_2 + E_3$$

$$E = -5.75 + 11.24 + 18.73$$

$$E = 24.22 \text{ N/C} \quad \text{in } +x \text{ direction.}$$



Problem 22.35

Additional problem

Three point charges are aligned along the x axis as shown in the following figure. Find the electric field at (a) the position $(2.00, 0)$ and (b) the position $(0, 2.00)$.

$$(b) E_1 = k_e \frac{|q_1|}{r_1^2}$$

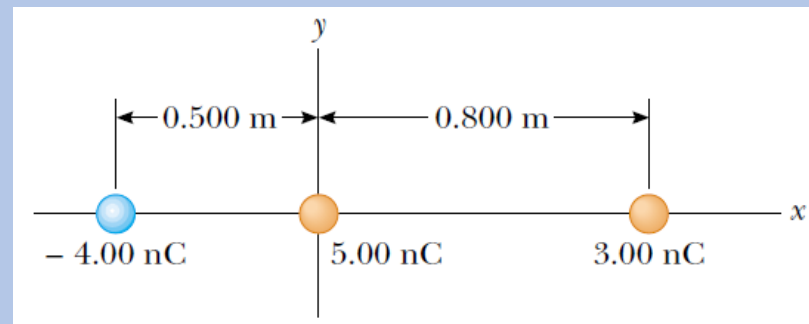
$$E_1 = 8.99 \times 10^9 \times \frac{|-4.0 \times 10^{-9}|}{(2.06)^2}$$

$$E_1 = 8.47 \text{ N/C}$$

$$E_2 = k_e \frac{|q_2|}{r_2^2}$$

$$E_2 = 8.99 \times 10^9 \times \frac{|5.0 \times 10^{-9}|}{(2.00)^2}$$

$$E_2 = 11.24 \text{ N/C}$$



Problem 22.35

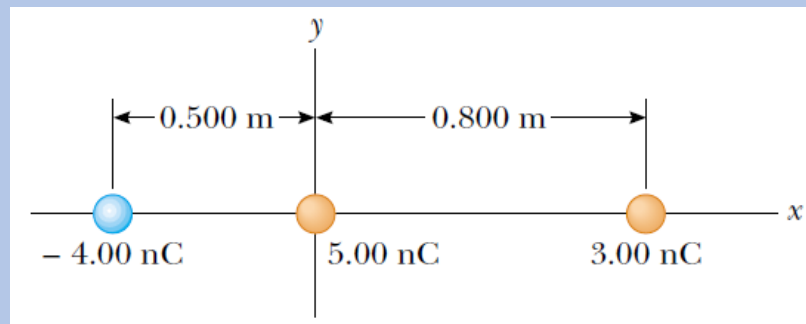
Additional problem

Three point charges are aligned along the x axis as shown in the following figure. Find the electric field at (a) the position $(2.00, 0)$ and (b) the position $(0, 2.00)$.

$$E_3 = k_e \frac{|q_3|}{r_3^2}$$

$$E_3 = 8.99 \times 10^9 \times \frac{|3.0 \times 10^{-9}|}{(2.15)^2}$$

$$E_3 = 5.83 \text{ N/C}$$



Problem 22.35

Additional problem

Three point charges are aligned along the x axis as shown in the following figure. Find the electric field at (a) the position $(2.00, 0)$ and (b) the position $(0, 2.00)$.

$$E_x = E_{1x} + E_{2x} + E_{3x}$$

$$E_x = E_1 \cos(\theta_1) + E_2 \cos(\theta_2) + E_3 \cos(\theta_3)$$

$$E_x = -\left(8.47 \times \frac{0.50}{2.06}\right) + 0 - \left(5.83 \times \frac{0.80}{2.15}\right)$$

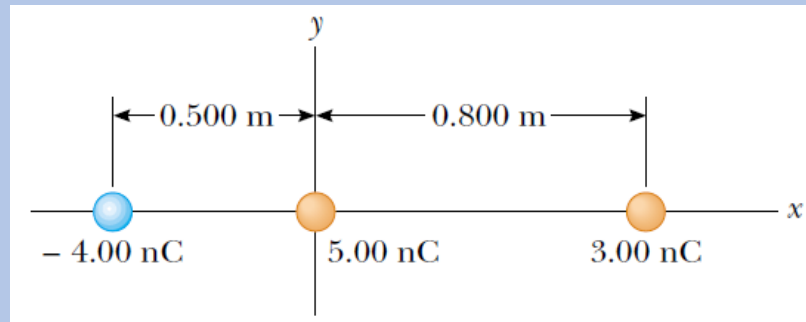
$$E_x = -4.23 \text{ N/C}$$

$$E_y = E_{1y} + E_{2y} + E_{3y}$$

$$E_y = E_1 \sin(\theta_1) + E_2 \sin(\theta_2) + E_3 \sin(\theta_3)$$

$$E_y = -\left(8.47 \times \frac{2.00}{2.06}\right) + 11.24 + \left(5.83 \times \frac{2.00}{2.15}\right)$$

$$E_y = 8.44 \text{ N/C}$$



Problem 22.35

Additional problem

Three point charges are aligned along the x axis as shown in the following figure. Find the electric field at (a) the position $(2.00, 0)$ and (b) the position $(0, 2.00)$.

$$E = \sqrt{(E_x)^2 + (E_y)^2}$$

$$E = \sqrt{(-4.23)^2 + (8.44)^2}$$

$$E = 9.44 \text{ N/C}$$

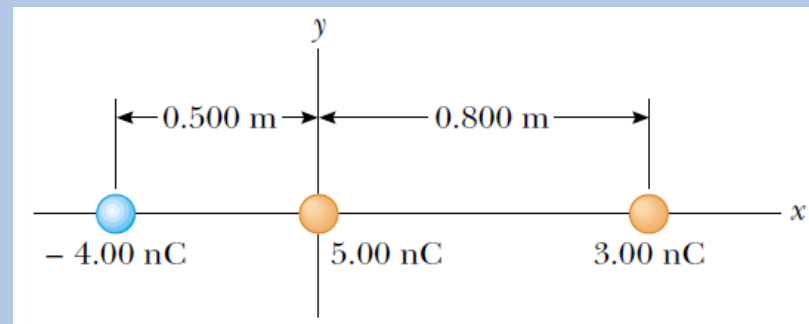
$$\varphi = \tan^{-1} \frac{E_y}{E_x}$$

$$\varphi = \tan^{-1} \frac{8.44}{-4.23}$$

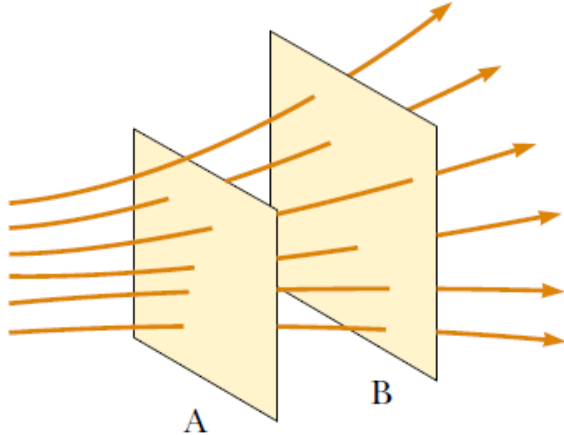
$$\varphi = 116.62^\circ \quad !!$$

or

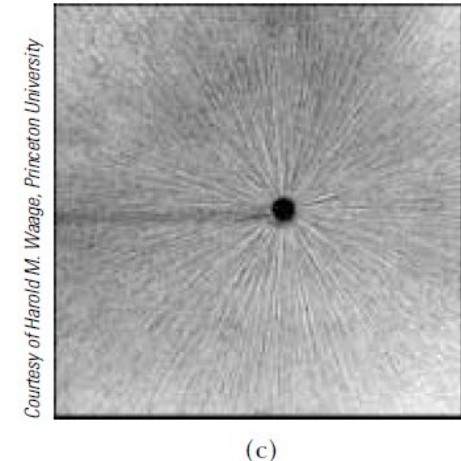
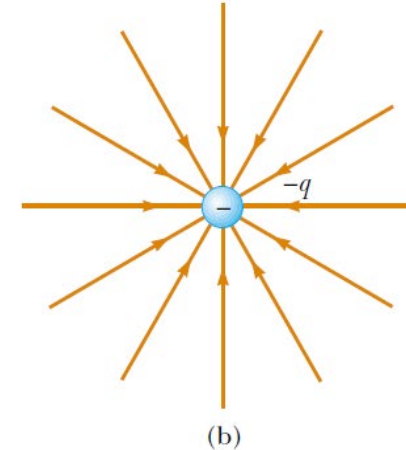
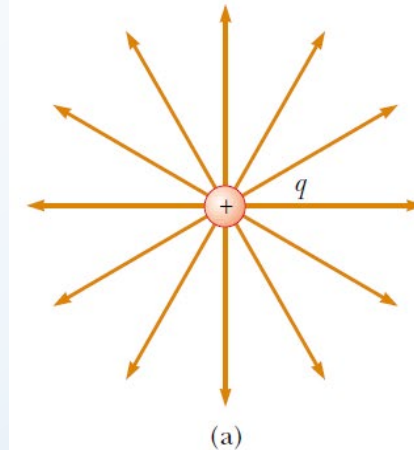
$$\vec{E} = (-4.23\hat{i} + 8.44\hat{j}) \text{ N/C}$$



- The electric field vector E is tangent to the electric field line at each point. The line has a direction, indicated by an arrowhead, that is the same as that of the electric field vector.
- The number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of the electric field in that region. Thus, the field lines are close together where the electric field is strong and far apart where the field is weak.

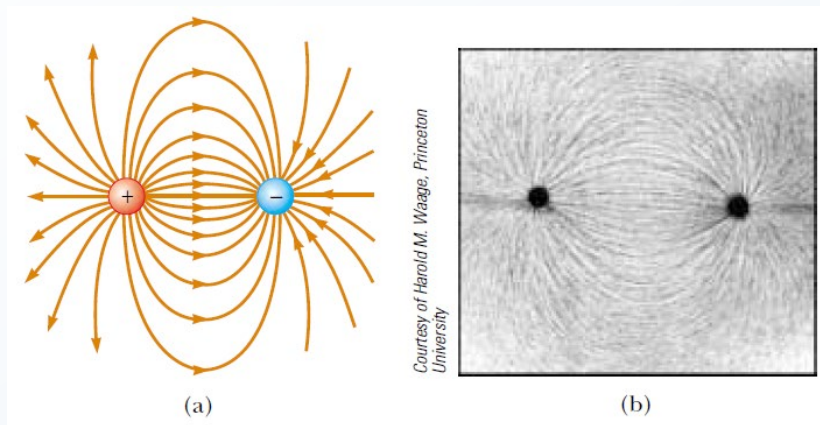


Electric field lines penetrating two surfaces. The magnitude of the field is greater on surface A than on surface B.

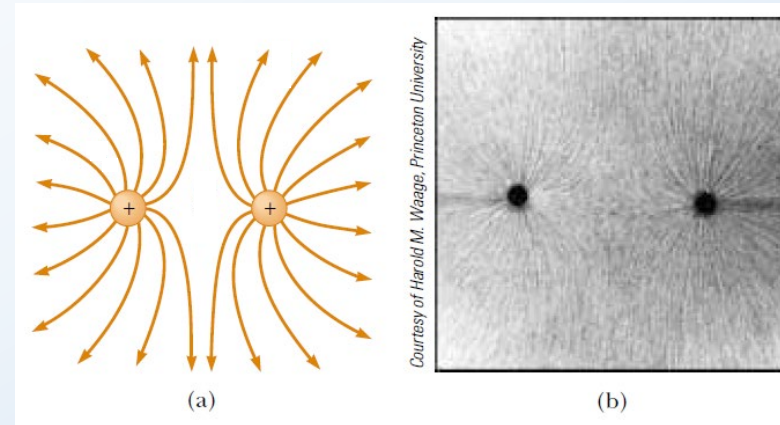


The electric field lines for a point charge. (a) For a positive point charge, the lines are directed radially outward. (b) For a negative point charge, the lines are directed radially inward. Note that the figures show only those field lines that lie in the plane of the page. (c) The dark areas are small pieces of thread suspended in oil, which align with the electric field produced by a small charged conductor at the center.

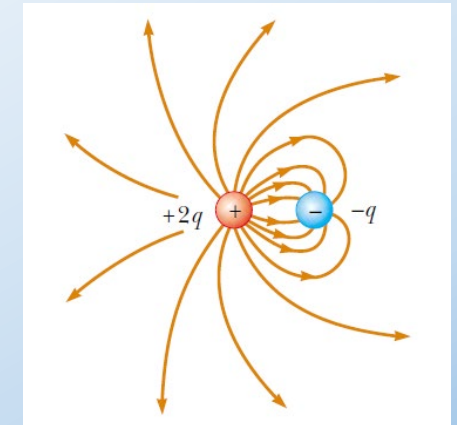
- The lines must begin on a positive charge and terminate on a negative charge. In the case of an excess of one type of charge, some lines will begin or end infinitely far away.
- The number of lines drawn leaving a positive charge or approaching a negative charge is proportional to the magnitude of the charge.
- No two field lines can cross.



(a) The electric field lines for two point charges of equal magnitude and opposite sign (an electric dipole). The number of lines leaving the positive charge equals the number terminating at the negative charge. (b) The dark lines are small pieces of thread suspended in oil, which align with the electric field of a dipole.



(a) The electric field lines for two positive point charges. (b) Pieces of thread suspended in oil, which align with the electric field created by two equal-magnitude positive charges.



The electric field lines for a point charge $+2q$ and a second point charge $-q$. Note that two lines leave $+2q$ for every one that terminates on $-q$.

Case 1:

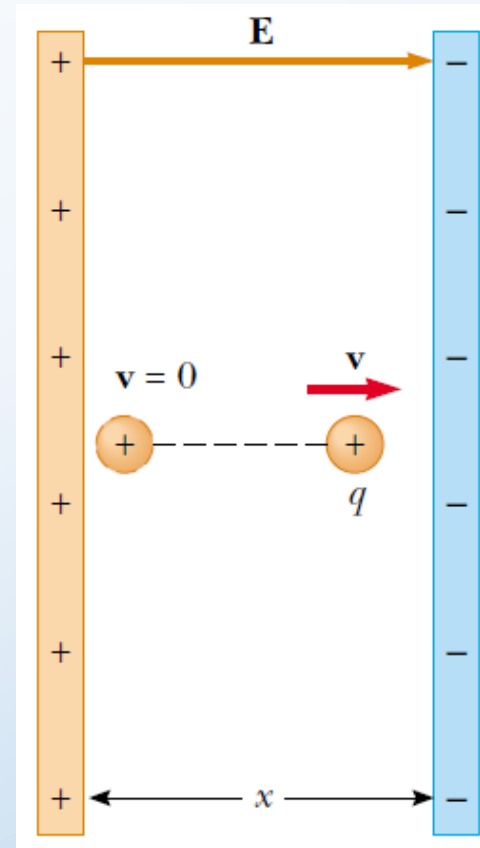
- When a particle of charge q and mass m is placed in an electric field $\vec{E}$, the electric force exerted on the charge is $q\vec{E}$.
- If this is the only force exerted on the particle, it must be the net force and causes the particle to accelerate according to Newton's second law. Thus:

$$\vec{F}_e = q\vec{E} = m\vec{a}$$

- The acceleration $\vec{a}$ of the particle is therefore:

$$\vec{a} = a_x \hat{i} = \frac{qE_x}{m} \hat{i}$$

- If $\vec{E}$ is uniform (that is, constant in magnitude and direction), then the acceleration is $\vec{a}$ constant.
- If the particle has a positive charge, its acceleration is in the direction of the electric field.
- If the particle has a negative charge, its acceleration is in the direction opposite the electric field.



A positive point charge q in a uniform electric field $\vec{E}$ undergoes constant acceleration in the direction of the field.

Question 22.07

A positive point charge q of mass m is released from rest in a uniform electric field $\vec{E}$ directed along the x axis, as shown in the following figure. Describe its motion.

The acceleration is constant and is given by $q\vec{E}/m$. The motion is simple linear motion along the x axis. Therefore, we can apply the equations of kinematics in one dimension:

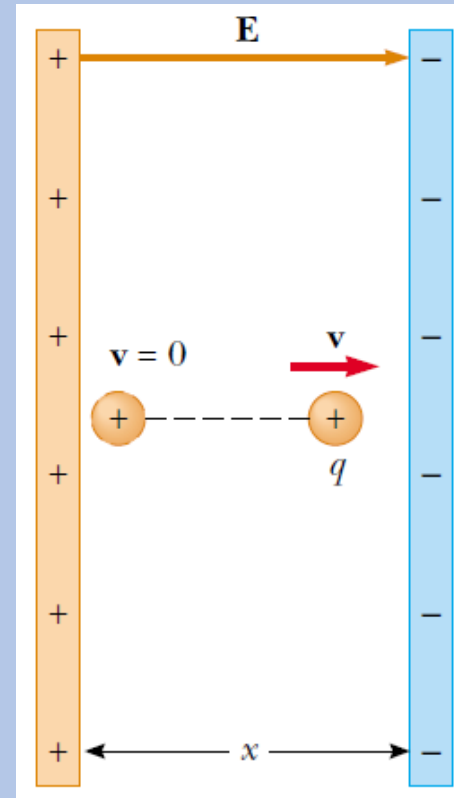
$$x_f = x_i + v_{xi}t + \frac{1}{2}a_xt^2$$

$$v_{xf} = v_{xi} + a_xt$$

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

Choosing the initial position of the charge as $x_i = 0$ and assigning $v_{xi} = 0$ because the particle starts from rest, the position of the particle as a function of time is:

$$x_f = \frac{1}{2}a_xt^2 = \frac{1}{2}\frac{qE_x}{m}t^2$$



A positive point charge q in a uniform electric field $\vec{E}$ undergoes constant acceleration in the direction of the field.

Question 22.07

A positive point charge q of mass m is released from rest in a uniform electric field $\vec{E}$ directed along the x axis, as shown in the following figure. Describe its motion.

The speed of the particle is given by:

$$v_{xf} = a_x t = \frac{qE_x}{m} t$$

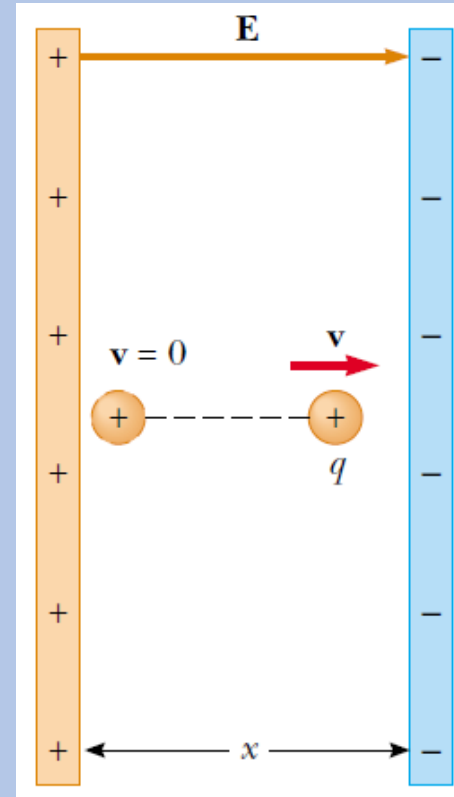
The third kinematic equation gives us:

$$v_{xf}^2 = 2a_x x_f = \frac{2qE_x}{m} x_f$$

from which we can find the kinetic energy of the charge after it has moved a distance $\Delta x = x_f - x_i$:

$$K = \frac{1}{2} m v_{xf}^2 = \frac{1}{2} m \left(\frac{2qE_x}{m} \right) \Delta x = qE_x \Delta x$$

We can also obtain this result from the work–kinetic energy theorem because the work done by the electric force is $F_{ex} \Delta x = qE_x \Delta x$ and $W = \Delta K$.



A positive point charge q in a uniform electric field $\vec{E}$ undergoes constant acceleration in the direction of the field.

Question 22.08

An electron and a proton are each placed at rest in an electric field of 520 N/C. Calculate the speed of each particle 48.0 ns after being released.

$$v_{xf} = v_{xi} + a_x t = 0 + a_x t = \frac{qE_x}{m} t$$

$$v_{exf} = -\frac{eE_x}{m_e} t \quad (\text{for electron})$$

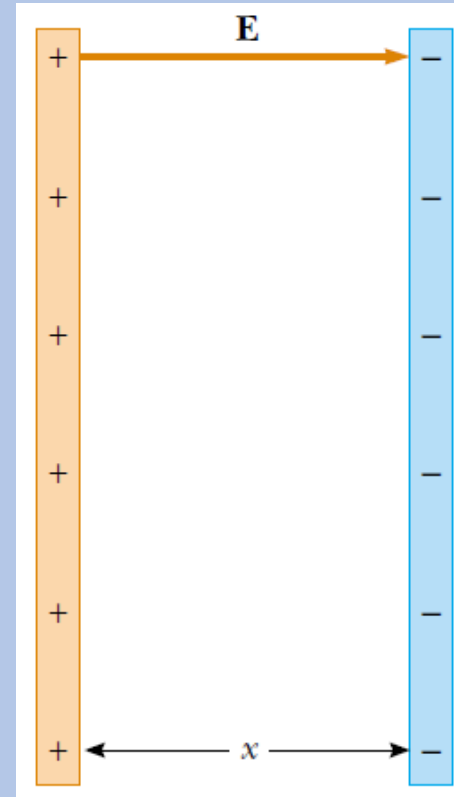
$$v_{exf} = -\frac{1.60 \times 10^{-19} \times 520}{9.11 \times 10^{-31}} \times 48.0 \times 10^{-9}$$

$$v_{exf} = -4.39 \times 10^6 \text{ m/s} \quad \text{in a direction opposite to the field}$$

$$v_{pxf} = \frac{eE_x}{m_p} t \quad (\text{for proton})$$

$$v_{pxf} = \frac{1.60 \times 10^{-19} \times 520}{1.67 \times 10^{-27}} \times 48.0 \times 10^{-9}$$

$$v_{pxf} = 2.39 \times 10^3 \text{ m/s} \quad \text{in the same direction as the field}$$



Question 22.09

The electrons in a particle beam each have a kinetic energy K . What are the magnitude and direction of the electric field that will stop these electrons in a distance d ?

The required electric field will be in the direction of motion

$$W = \Delta K$$

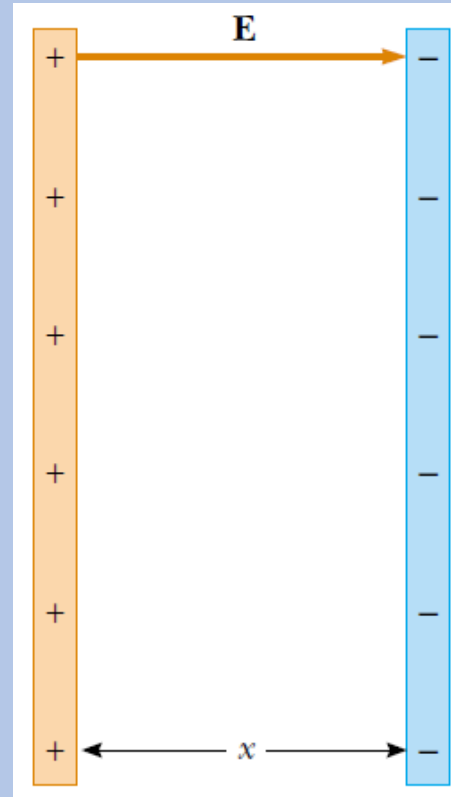
so,

$$F_{ex}d = \frac{1}{2}mv_{xi}^2 \text{ (since } v_{xf} = 0\text{)}$$

which becomes:

$$eE_xd = K$$

$$E_x = \frac{K}{ed}$$



Question 22.10

Additional question

A positively charged bead having a mass of 1.0 g falls from rest in a vacuum from a height of 5.0 m in a uniform vertical electric field with a magnitude of 1.0×10^4 N/C. The bead hits the ground at a speed of 21 m/s. Determine (a) the direction of the electric field (up or down), and (b) the charge on the bead.

(a) The acceleration is given by:

$$v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$$

$$v_{yf}^2 = 0 + 2a_y\Delta h$$

$$a_y = \frac{v_{yf}^2}{2\Delta h}$$

$$a_y = \frac{(21)^2}{2 \times 5}$$

$$a_y = 44.1 \text{ m/s}^2$$

$a_y > g \Rightarrow$ the electric field must have a downward direction

Question 22.10

Additional question

A positively charged bead having a mass of 1.0 g falls from rest in a vacuum from a height of 5.0 m in a uniform vertical electric field with a magnitude of 1.0×10^4 N/C. The bead hits the ground at a speed of 21 m/s. Determine (a) the direction of the electric field (up or down), and (b) the charge on the bead.

$$(b) \sum F_y = ma_y$$

$$mg + qE_y = ma_y$$

$$q = \frac{m}{E_y} (a_y - g)$$

$$q = \frac{1.0 \times 10^{-3}}{1.0 \times 10^4} (44.1 - 9.8)$$

$$q = 3.43 \times 10^{-6} \text{ C}$$

$$q = 3.43 \mu\text{C}$$

Case 2:

- Suppose an electron of charge $-e$ is projected horizontally into an uniform electric field $\vec{E}$ from the origin with an initial velocity $v_i \hat{i}$ at time $t = 0$.
- Because the electric field $\vec{E}$ is in the positive y direction, the acceleration of the electron is in the negative y direction. That is:

$$\vec{a} = a_y \hat{j} = -\frac{eE_y}{m_e} \hat{j}$$

- Because the acceleration is constant, we can apply the equations of kinematics in two dimensions with $v_{xi} = v_i$ and $v_{yi} = 0$. After the electron has been in the electric field for a time interval, the components of its velocity at time t are:

$$v_x = v_{xi} = \text{constant}$$

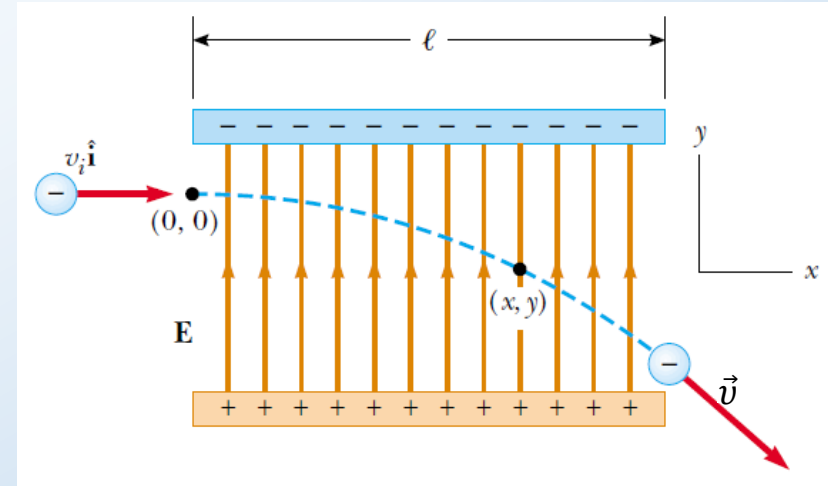
$$v_{yf} = a_y t = -\frac{eE_y}{m_e} t$$

- Its position coordinates at time t are:

$$x_f = v_{xi} t$$

$$y_f = \frac{1}{2} a_y t^2 = -\frac{1}{2} \frac{eE_y}{m_e} t^2 = -\frac{1}{2} \frac{eE_y}{m_e} \frac{x_f^2}{v_{xi}^2}$$

- We see that y_f is proportional to x_f^2 . Hence, the trajectory is a parabola.
- After the electron leaves the field, the electric force vanishes and the electron continues to move in a straight line in the direction of $\vec{v}$ with a speed $v > v_i$.



An electron is projected horizontally into a uniform electric field produced by two charged plates. The electron undergoes a downward acceleration (opposite $\vec{E}$), and its motion is parabolic while it is between the plates.

Example 22.08

An electron enters the region of a uniform electric field as shown in the following figure, with $v_i = 3.00 \times 10^6$ m/s and $E = 200$ N/C. The horizontal length of the plates is $l = 0.100$ m. (a) Find the acceleration of the electron while it is in the electric field. (b) If the electron enters the field at time $t = 0$, find the time at which it leaves the field. (c) If the vertical position of the electron as it enters the field is $y_i = 0$, what is its vertical position when it leaves the field?

$$(a) \quad a_y = -\frac{eE_y}{m_e}$$

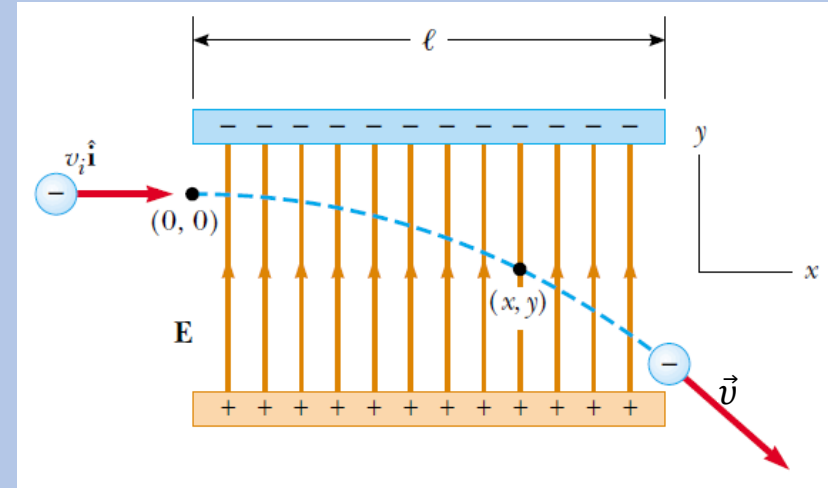
$$a_y = -\frac{1.60 \times 10^{-19} \times 200}{9.11 \times 10^{-31}}$$

$$a_y = -3.51 \times 10^{13} \text{ m/s}^2$$

$$(b) \quad x_f = v_{xi}t$$

or

$$t = \frac{x_f}{v_{xi}} = \frac{l}{v_{xi}} = \frac{0.100}{3.00 \times 10^6} = 3.33 \times 10^{-8} \text{ s}$$



An electron is projected horizontally into a uniform electric field produced by two charged plates. The electron undergoes a downward acceleration (opposite $\vec{E}$), and its motion is parabolic while it is between the plates.

Example 22.08

An electron enters the region of a uniform electric field as shown in the following figure, with $v_i = 3.00 \times 10^6$ m/s and $E = 200$ N/C. The horizontal length of the plates is $l = 0.100$ m. (a) Find the acceleration of the electron while it is in the electric field. (b) If the electron enters the field at time $t = 0$, find the time at which it leaves the field. (c) If the vertical position of the electron as it enters the field is $y_i = 0$, what is its vertical position when it leaves the field?

$$(c) \quad y_f = \frac{1}{2} a_y t^2$$

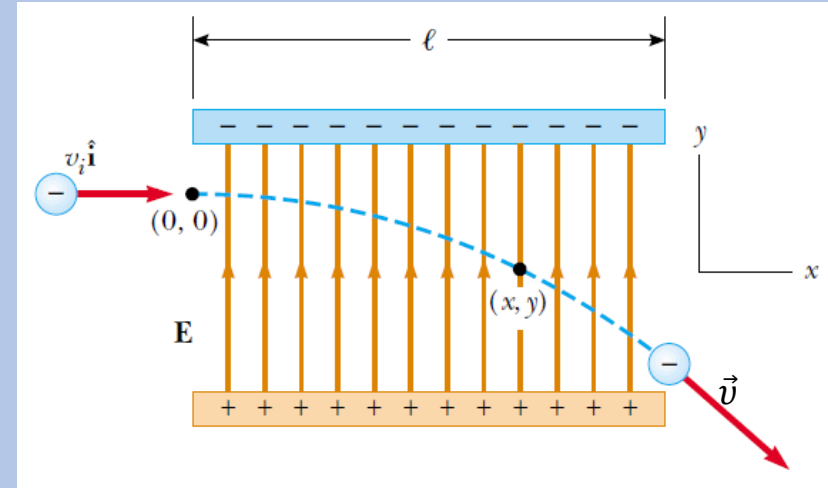
$$y_f = \frac{1}{2} (-3.51 \times 10^{13}) (3.33 \times 10^{-8})^2$$

$$y_f = -0.0195 \text{ m}$$

or

$$y_f = -1.95 \text{ cm}$$

If the electron enters just below the negative plate and the separation between the plates is less than the value we have just calculated, the electron will strike the positive plate.



An electron is projected horizontally into a uniform electric field produced by two charged plates. The electron undergoes a downward acceleration (opposite $\vec{E}$), and its motion is parabolic while it is between the plates.