King Saud University
College of Science
Physics & Astronomy Dept.

PHYS 103 (GENERAL PHYSICS)

LECTURE NO. 1

THIS PRESENTATION HAS BEEN PREPARED BY: *DR NASSR S. ALZAYED*

Chapter Outline

- 1.0 General introduction to course
- 1.1 Standards of Length, Mass, and Time
- 1.4 Dimensional Analysis
- 1.5 Conversion of Units

Physics Sections and Exercises

Ch	Title	Section	Hours	Exercises and problems
1	Physics and measurement	*1.1, IA, 1.5*	2	13,15,21,25,31
2	Motion in One Dimension	2.1, 2.2, 2.3, 2.5,2.6	4	4,5,11,15,16,20,21,22,23,25,27,28,29,32,33,40,42,43,46,48,51,52
3	Vectors	3.1--to--3 *A*	3	1,4,19,21,27,30,31,33,39,49,50
4	Motion in Two Dimensions	4.1--to--4.5	5	1,3,5,6,8,14,15,17,19,20,22,23,25,29
5	The Laws of Motion	5.1--to--5.8	5	3,7,11,16,18,24,25,26,28,30,31,37,41,44,45,46,68
6	Circular Motion and Other Applications of Newton's Laws	6.1	2	1,2,5,7,59
7	Energy and Energy Transfer	7.2--to--7.8	5	1,4,7,13,14,15,16,19,21,24,25,26,28,31,32,33,35,37,40
8	Potential Energy	8.1--to--8.5	5	2,5,6,11,13,17,31,33,36,38,42,55,57,59,60
9	Linear Momentum and Collisions	9.1--to--9*A*	5	1,2,4,5,7,8,9,10,13,15,16,17,18,21,25,27,32,33,35
10	Rotation of a Rigid Object About a Fixed Axis	10.1--to--1O.8	6	1,3,5,6,8,12,13,16,17,18,20,21,31,35,37,46,70,71

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Welcome to 103 Physics

- Importance of the course
- Directions on how to get maximum benefit of the course
- Talk about attendance, participation and office hours
- Short information about the LMS and how to make it effective and useful.
- Little about the textbook and online resources.
- Solving Problems Tips.

1.1 Standards of Length, Mass, and Time

- In mechanics, there are three basic quantities: *length*, *mass*, and *time*
- All other quantities in mechanics can be expressed in terms of these three.
- In 1960, an international committee established a set of standards for the fundamental quantities of science. It is called the SI (Système International)
- *In the SI: Units of length* ***meter***

  Units of mass ***kilogram***

  Units of time ***second***

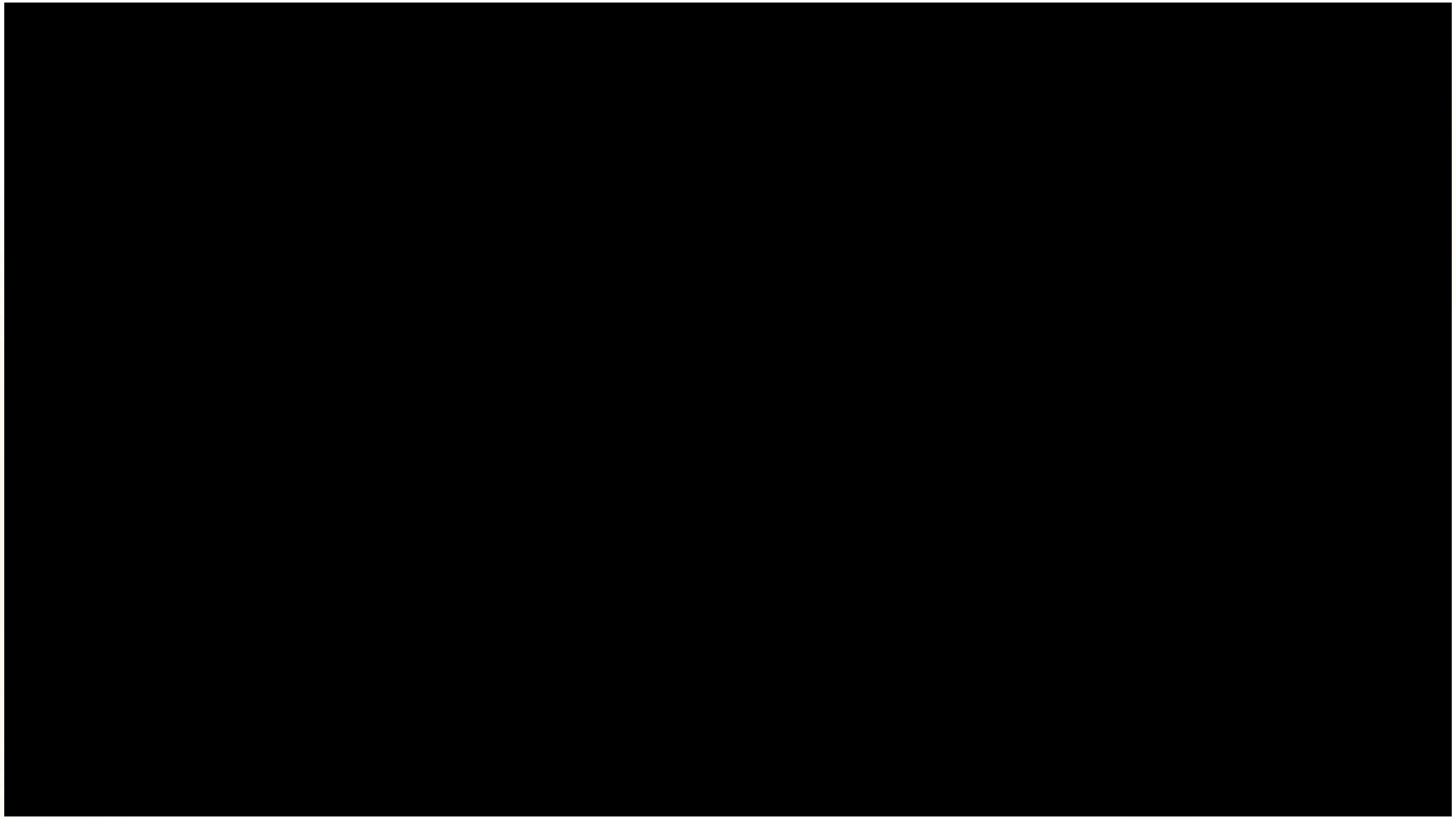
1.1 Standards of Length, Mass, and Time

- ***Length***: *SI Unit of length is: meter (m).*
- ***Mass:*** *SI Unit of mass is: kilogram (kg)*
- ***Time:*** *SI Unit of time is: second (s)*

- In many situations, you may have to derive or check a specific equation. A useful and powerful procedure called *dimensional analysis* can be used to assist in the derivation or to check your final expression.
- As a simple method: Left Hand Side must = Right Hand Side

1.1 Standards of Length, Mass, and Time: Movie

Please Click by mouse on the movie to play Then Wait …..



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1.4 Dimensional Analysis

- ***Dimension:*** *it denotes the physical nature of a quantity*
- ***Example:*** *distance: could be in meters, yards, or micrometers. But overall it is: a* ***length***
- ***Symbols we are going to use are:***
  - *dimension of length:* **[L]**
  - *dimension of mass:* **[M]**
  - *dimension of time:* **[T]**

Units of Area, Volume, Velocity, Speed, and Acceleration

System	Area (L^2)	Volume (L^3)	Speed (L/T)	Acceleration (L/T^2)
SI	m^2	m^3	m/s	m/s^2
U.S. customary	ft^2	ft^3	ft/s	ft/s^2

1.4 Dimensional Analysis

- ***Example:*** *Use dimensional analysis to check the equation:* $x = \frac{1}{2}at^2$

Solution:

$$L = \frac{L}{\cancel{T}^2} \cdot \cancel{T}^2 = L$$

- ***Example:*** *Show that v=at is dimensionally correct.*
- ***Solution:***

$$\text{L.H.S.: } [v] = \frac{L}{T}$$

$$\text{R.H.S.: } [at] = \frac{L}{T^{\cancel{2}}} \cancel{T} = \frac{L}{T}$$

$$\therefore \text{ L.H.S} = \text{R.H.S}$$

- *Hence the equation is dimensionally correct*

1.4 Dimensional Analysis (Quiz)

Quick Quiz 1.2 True or False: Dimensional analysis can give you the numerical value of constants of proportionality that may appear in an algebraic expression.

- False.
- Dimensional Analysis can only provide information about the **DIMENSIONS** of a constant.
- Numerical value cannot be calculated this way.

CommandButtc

1.5 Conversion of Units

- Sometimes it is necessary to convert units from one *measurement* system to *another*, or to convert *within* a system, for example, from kilometers to meters.
- Please visit this page for comprehensive list
- Examples:
  - 1 mile = 1 609 m = 1.609 km
  - 1 ft = 0.304 8 m = 30.48 cm
  - 1 m = 39.37 in. = 3.281 ft
  - 1 in. = 0.025 4 m = 2.54 cm (exactly)

Lecture Summary

- The three fundamental physical quantities of mechanics are length, mass, and time, which in the SI system have the units meters (m), kilograms (kg), and seconds (s), respectively.
- The method of dimensional analysis is very powerful in solving physics problems.
- Dimensions can be treated as algebraic quantities. By making estimates and performing order-of-magnitude calculations, you should be able to approximate the answer to a problem when there is not enough information available to completely specify an exact solution.

Please read the attachment ….

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CHAPTER 2

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2.0 Motion in One Dimension

- From everyday experience we recognize that motion represents a continuous change in the position of an object.
- In physics we can categorize motion into three types: ***translational***, ***rotational***, and ***vibrational***.
- A car moving down a highway is an example of ***translational*** motion, the Earth's spin on its axis is an example of ***rotational*** motion, and the back-and-forth movement of a pendulum is an example of ***vibrational*** motion.
- In this and the next few chapters, we are concerned only with ***translational*** motion. (Later in the course we shall discuss rotational and vibrational motions.)

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2.1 Position, Velocity, and Speed

- When a particle moves from its (initial) position x_i to its (final) position x_f ; it has what we call: a displacement.
- Displacement is defined as follows:

$$\Delta x = x_f - x_i \qquad (2.1)$$

- There are 3 cases:

$\Delta x > 0$ or $x_f > x_i$:motion to the right $\rightarrow$

$\Delta x < 0$ or $x_f < x_i$:motion to the left $\leftarrow$

$\Delta x = 0$ or $x_f = x_i$: object returned to its initial position, or there was no motion.

- *Selecting (right) as +tive and (left) as –tive is a convention in this course and we shall stick to it all over the course.*

2.1 : The average velocity

- *It is defined as particle's displacement Δx divided by the time interval Δt during which that displacement occurs:*

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t} \qquad (2.2)$$

- There are 3 cases:

$\Delta x > 0 \rightarrow \bar{v} > 0$:motion to the right $\rightarrow$

$\Delta x < 0 \rightarrow \bar{v} < 0$:motion to the left $\leftarrow$

$\Delta x = 0 \rightarrow \bar{v} = 0$: object returned to its initial position, or there was no motion.

In the next slide; we show how to calculate $\bar{v}$ from charts. Take 2 points and draw a line between them and find the slope of that line. The first point (e.g. A) represents x_i (initial position) while the second point (e.g B) represents the final position or x_f.

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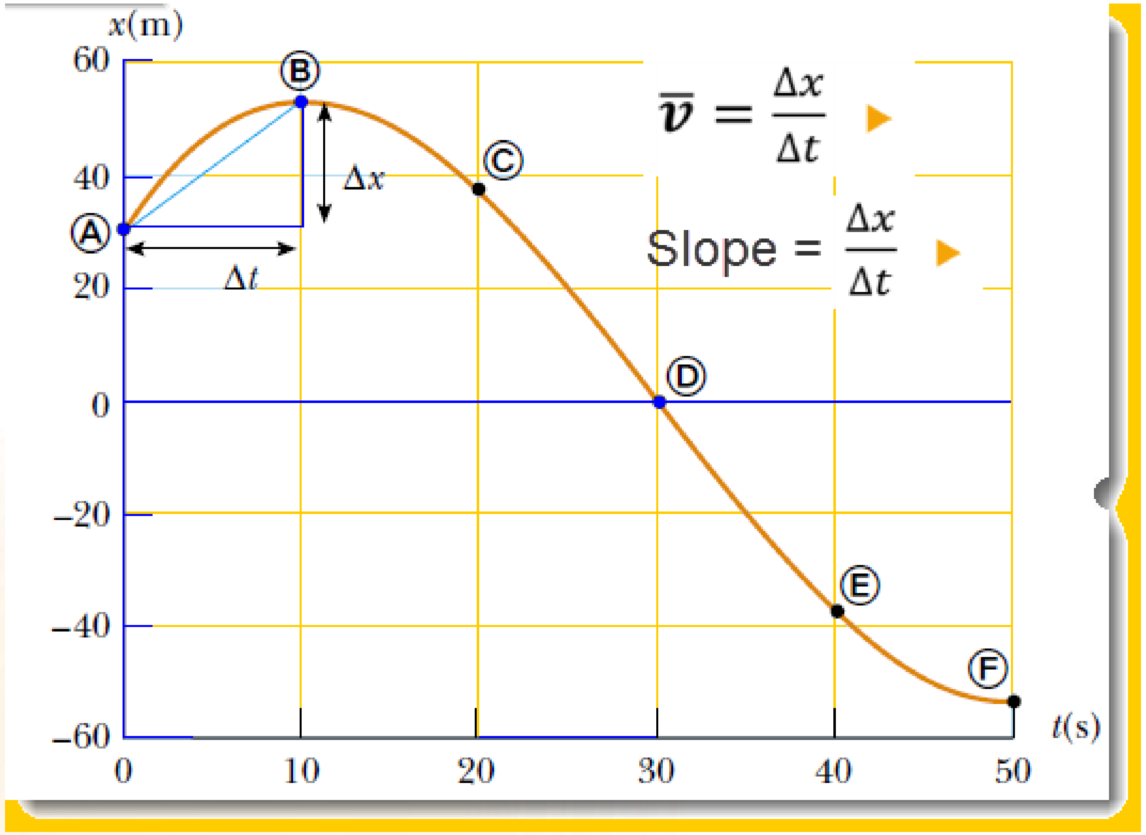
2.1 : The average velocity



This grapgh is called: Position–time graph for the motion of the "particle."

- When the line is point above horizontal:
  $\rightarrow \bar{v} > 0$
- When the line is point below horizontal:
  $\rightarrow \bar{v} < 0$

In the graph:

A to B: + (Increasing x)
A to C: + (Increasing x)
A to D: - (Decreasing x)
C to D: - (Decreasing x)

2.1 : The average Speed

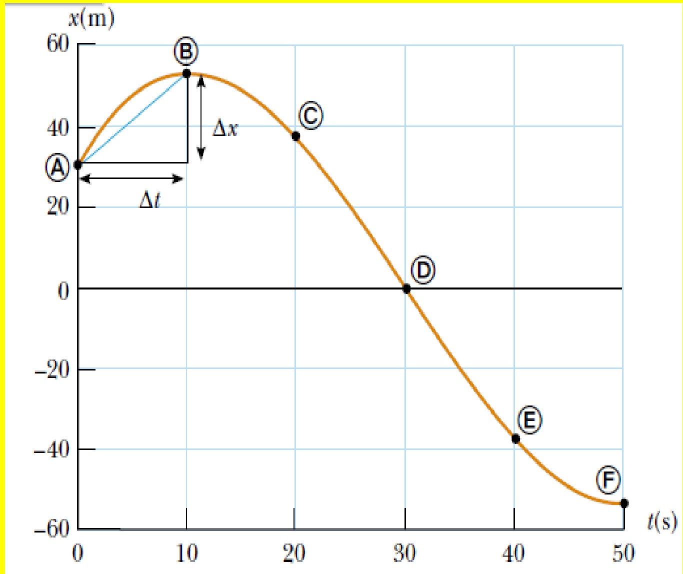
- In everyday usage, the terms speed and velocity are interchangeable. In physics, however, there is a clear distinction between these two quantities.:
- *The average speed of a particle, is defined as the total distance traveled divided by the total time interval required to travel that distance:*

$$\text{Average Speed} = \frac{\text{Total Distance}}{\text{Total Time}} \qquad (2.3)$$

- The SI unit of average speed is the same as the unit of average velocity: m/s. However, unlike average velocity, average speed has *no direction* and hence carries *no algebraic sign*. Average velocity (Eq. 2.2) is the *displacement* divided by the time interval, while average speed (Eq. 2.3) is the *distance* divided by the time interval.

Example 2.1

- Find the displacement, average velocity, and average speed of the car in Figure 2.1 between positions A and F.
- Solution:



In this example:

$$x_i = x_A = 30m \;\; and \;\; x_f = x_F = -53m$$

$$\rightarrow \Delta x = x_f - x_i = -53 - 30 = -83m$$

$$t_i = 0s \;\; and \;\; t_f = 50s$$

$$\therefore \Delta t = 50 - 0 = 50s$$

$$\therefore \bar{v} = \frac{\Delta x}{\Delta t} = \frac{-83}{50} = -1.7\, m/s$$

- For average speed: distance is 22 m (A to B) +105 m (B to F)=127 m

$$\text{Average Speed} = \frac{\text{Total Distance}}{\text{Total Time}} = \frac{127}{50} = 2.5\, m/s$$

Quiz

Quick Quiz 2.1 Under which of the following conditions is the magnitude of the average velocity of a particle moving in one dimension smaller than the average speed over some time interval? (a) A particle moves in the $+x$ direction without reversing. (b) A particle moves in the $-x$ direction without reversing. (c) A particle moves in the $+x$ direction and then reverses the direction of its motion. (d) There are no conditions for which this is true.

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Quiz-solved

$\because$ Average Velocity $= \dfrac{\text{Displacement}}{\text{time}} = \dfrac{\Delta x}{\Delta t}$

$\therefore \Delta x = x_f - x_i$

$\because$ Average speed $= \dfrac{\text{total distance}}{\text{time}} = \dfrac{d}{\text{time}}$

4 cases

① $\longrightarrow$ only $\rightarrow d = |\Delta x|$

② $\longleftarrow$ only $\rightarrow d = |\Delta x|$

③ $\longrightarrow$ then back $\leftarrow$ $\rightarrow d > |\Delta x|$

④ $\longleftarrow$ then back $\rightarrow$ $\rightarrow d > |\Delta x|$

$\therefore$ only in cases ① & ② $|\Delta x| < d \rightarrow |\bar{v}| <$ Average speed

$\therefore$ (C) is correct

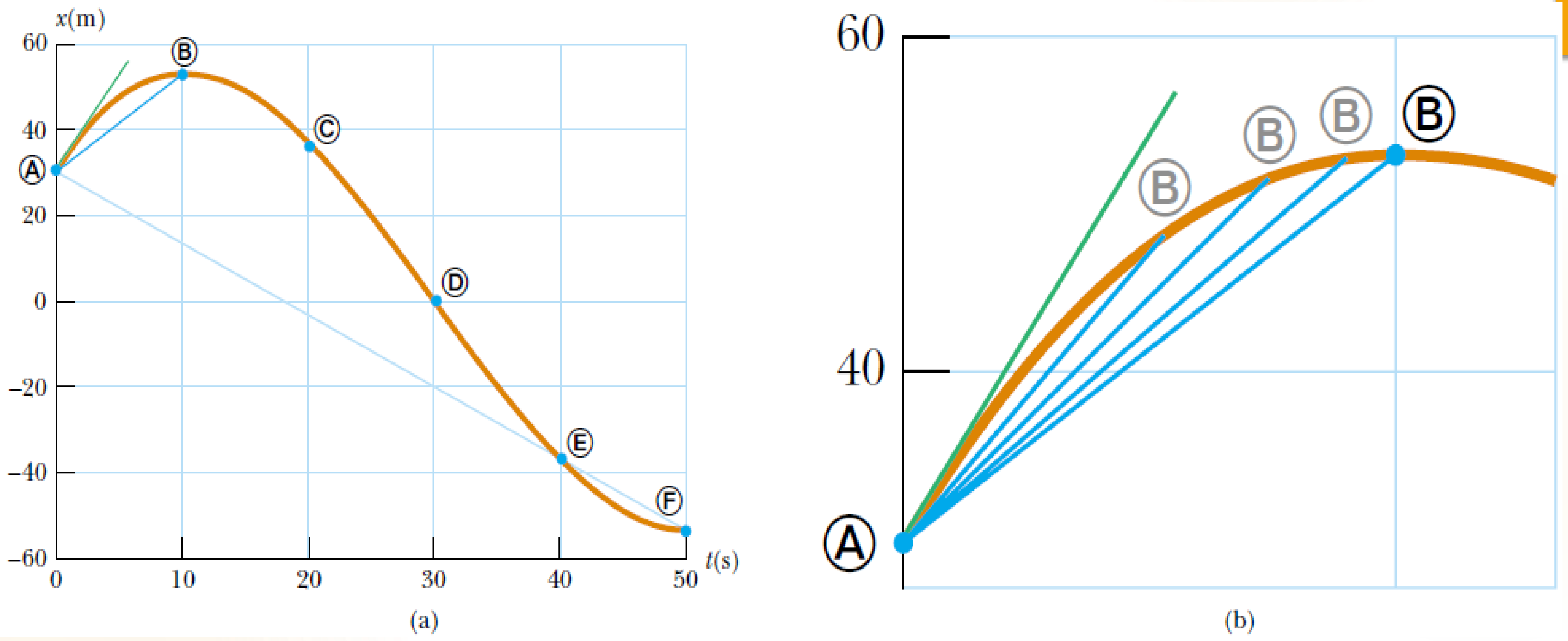
عندما يتحرك الجسم باتجاه +x ثم يعود تصبح الإزاحة أقل من المسافة.

وكذلك الحال لو تحرك ،، -x ثم عاد.

2.2 Instantaneous Velocity and Speed

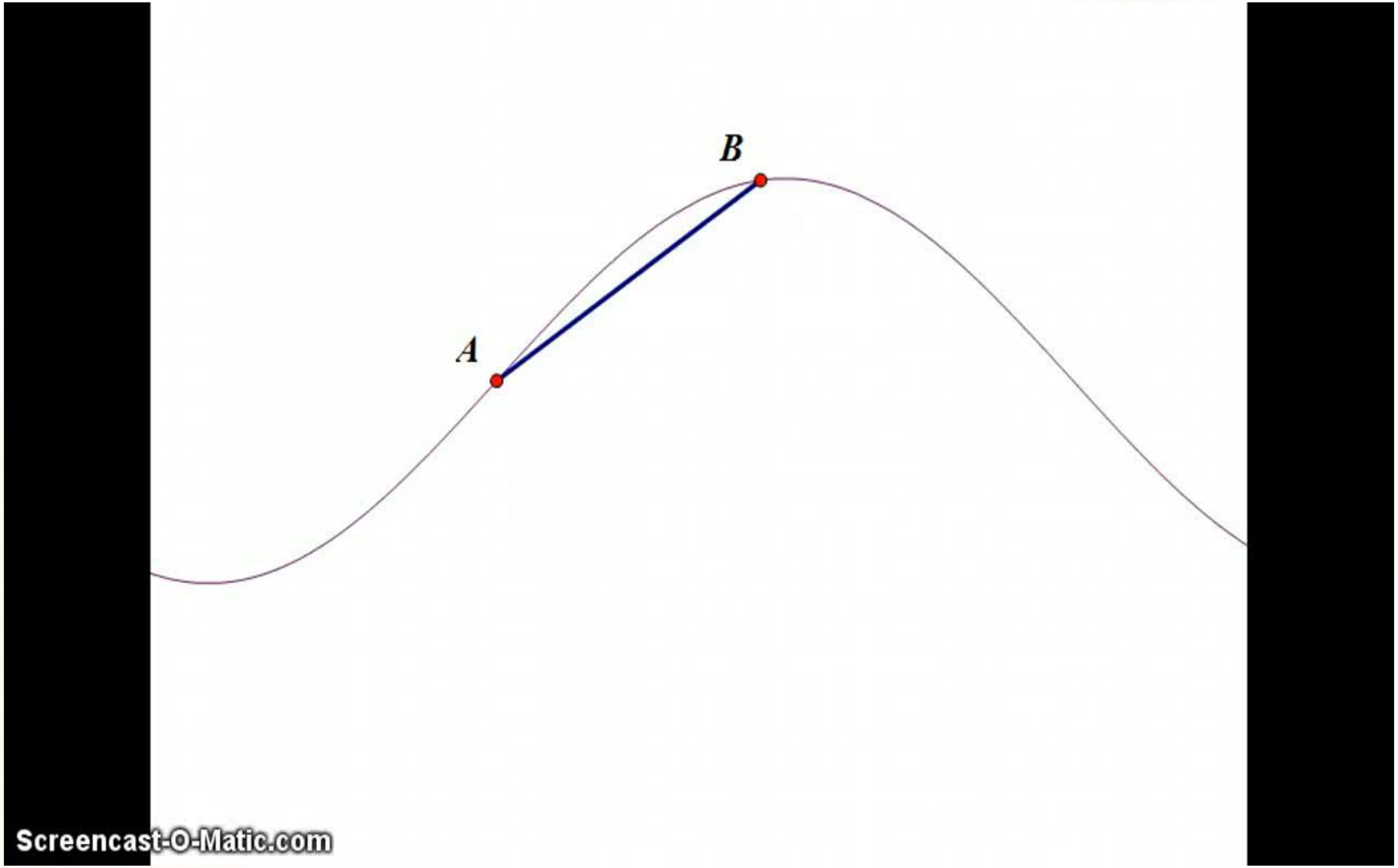
- Often, we need to know the velocity of a particle at a particular instant in time, rather than the average velocity over a finite time interval.
- For example, even though you might want to calculate your average velocity during a long automobile trip, you would be especially interested in knowing your velocity at the *instant you noticed the police* car parked alongside the road ahead of you
- With the invention of calculus (at the late 1600s), scientists began to understand how to describe an object's motion at any moment in time.
- Hence, in this lecture we shall learn how to find instantaneous quantities. In particular; we will find: v(t) and a(t) which are the instantaneous velocity and acceleration respectively.

2.2 Instantaneous Velocity and Speed



- **Figure 2.3**: (a) Representing the motion a car. (b) An enlargement of the upper-left-hand corner of the graph shows how the blue line between positions A and B approaches the green tangent line as point B is moved closer to point A.

2.2 Instantaneous Velocity and Speed



2.2 Instantaneous Velocity and Speed

- We define the instantaneous velocity for a particle moves on x-axis as:

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \qquad (2.5)$$

- There are 3 cases of v_x as:

$v_x > 0$ (Solpe is +): motion is to the right $\rightarrow$

$v_x < 0$ (Slope is -) : motion is to the left $\leftarrow$

$v_x = 0$ (at maxima or minima) particle is momentarily at rest

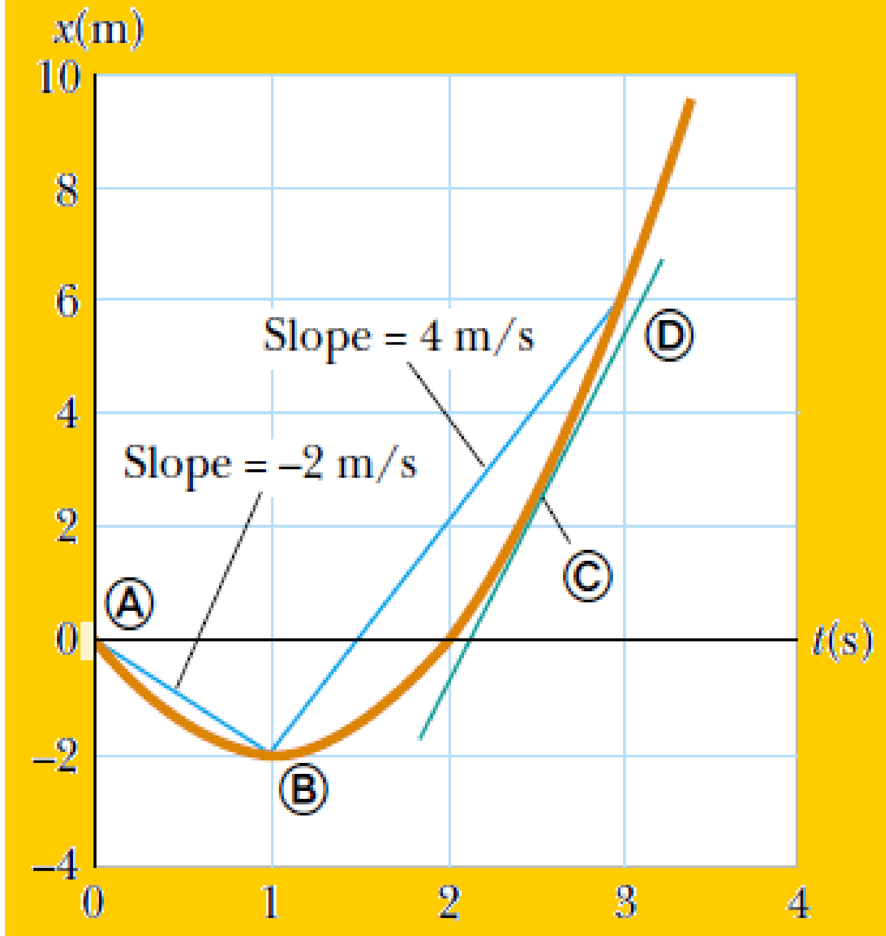
- From here on, we use the word *velocity* to designate instantaneous velocity. When it is average velocity we are interested in, we shall always use the adjective *average*.

- The instantaneous speed is defined as the magnitude of its inst. velocity. It has no direction associated with it and hence carries *no algebraic sign*

2.2 Instantaneous Velocity and Speed

Example: 2.3:

A particle moves along the x axis. Its position varies with time according to the expression $x = -4t + 2t^2$ where x is in meters and t is in seconds. The position–time graph for this motion is shown in the figure. Note that the particle moves in the negative x direction for the first second of motion, is momentarily at rest at the moment t = 1 s, and moves in the positive x direction at times t > 1 s.



Position–time graph for a particle

2.2 Instantaneous Velocity and Speed

Example: 2.3: (continues)

(a) Determine the displacement of the particle in the time intervals t = 0 to t = 1 s and t = 1 s to t = 3 s

1st interval (0 to 1 s):

$$\because x(0) = -4(0) + 2(0) = 0\,m = x_i$$

$$\because x(1) = -4(1) + 2(1) = -2\,m = x_f$$

$$\therefore \Delta x = x_f - x_i = -2 - 0 = -2m$$

2nd interval (1 to 3 s):

$$\because x(1) = -4(1) + 2(1) = -2\,m = x_i$$

$$\because x(3) = -4(3) + 2(9) = 6\,m = x_f$$

$$\therefore \Delta x = x_f - x_i = 6 - (-2) = 8m$$

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2.2 Instantaneous Velocity and Speed

Example: 2.3: (continues)

(b) Calculate the average velocity during these two time intervals.

1st interval (0 to 1 s):

$$\because \bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t} = \frac{-2}{1} = -2m/s$$

2nd interval (1 to 3 s):

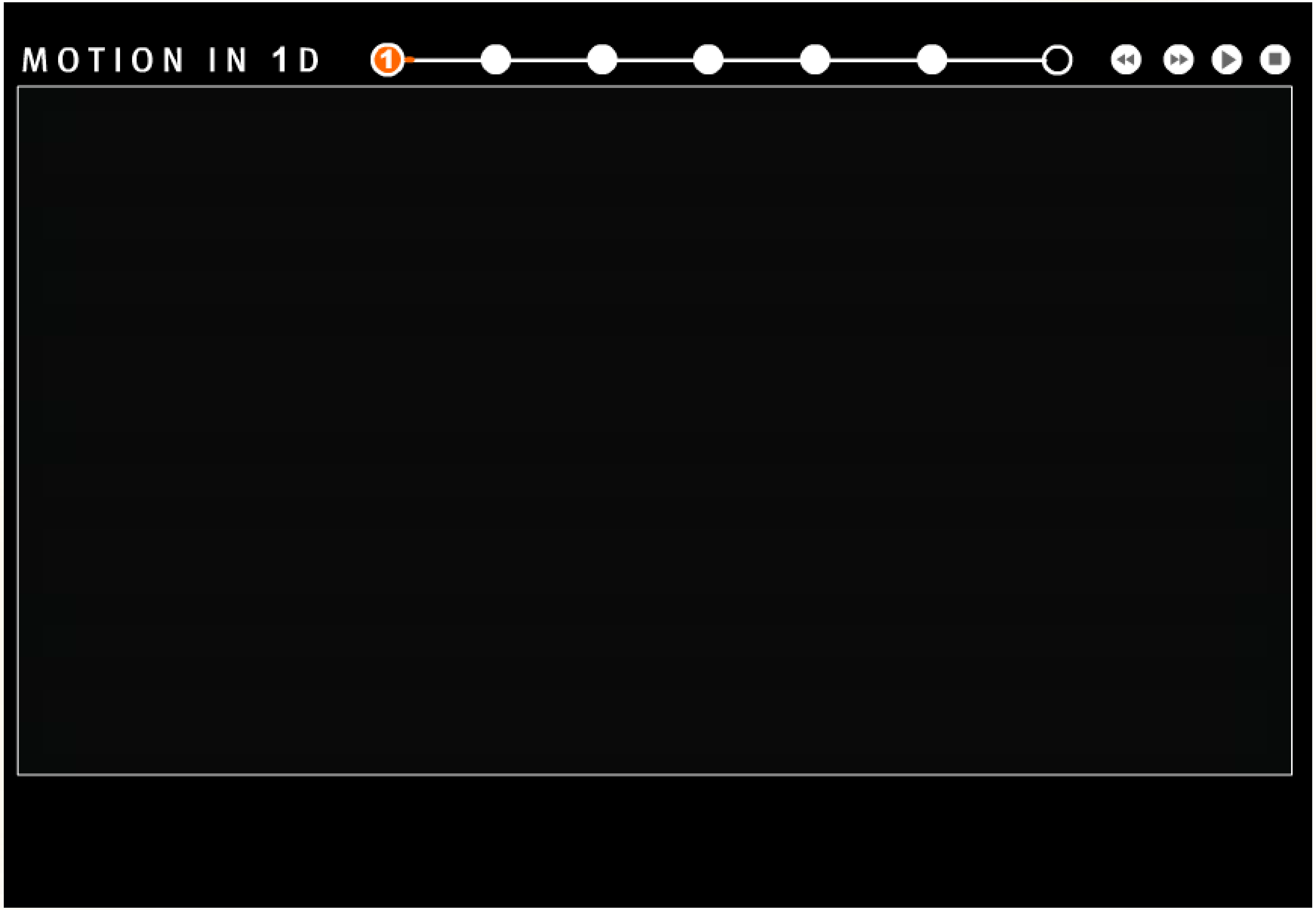
$$\because \bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t} = \frac{8}{2} = 4m/s$$

(c) Find the instantaneous velocity of the particle at t = 2.5 s.

$$\because v_x(t) = -4t + 2t^2$$

$$\therefore v_x(2.5) = -4(2.5) + 2(2.5)^2 = 6m/s$$

Velocity and Speed interactive



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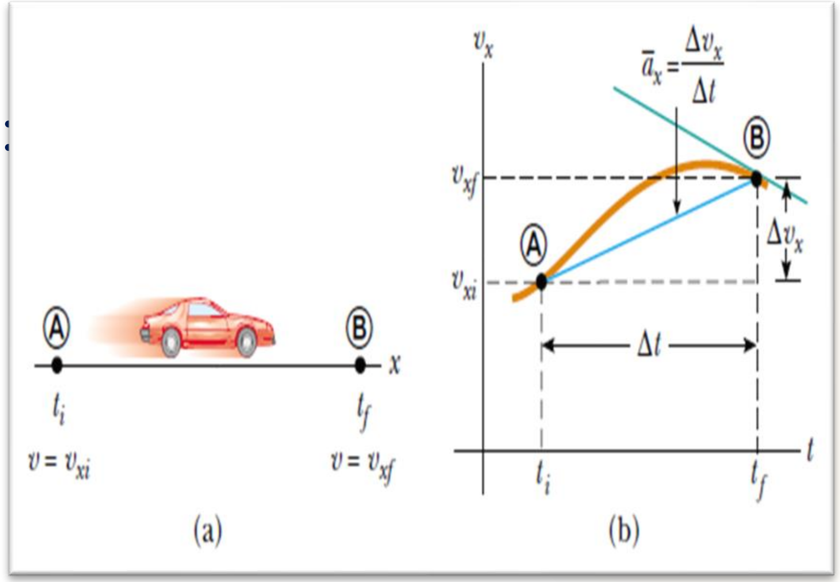
2.3 Acceleration

- Just like with the case of velocity; Acceleration (a) can be and average or instantaneous.
- Average Acceleration is defined as:



$$\bar{a}_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i} \quad (2.6)$$

- instantaneous acceleration is :

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} \quad (2.7)$$

- When the object's velocity and acceleration are in the *same direction*, the object is speeding up. On the other hand, when the object's velocity and acceleration are in *opposite directions*, the object is slowing down.

2.3 Acceleration

Example: 2.5:

The velocity of a particle moving along the x axis varies in time according to the expression $v_x = (40 - 5t^2)$ m/s, where t is in seconds.

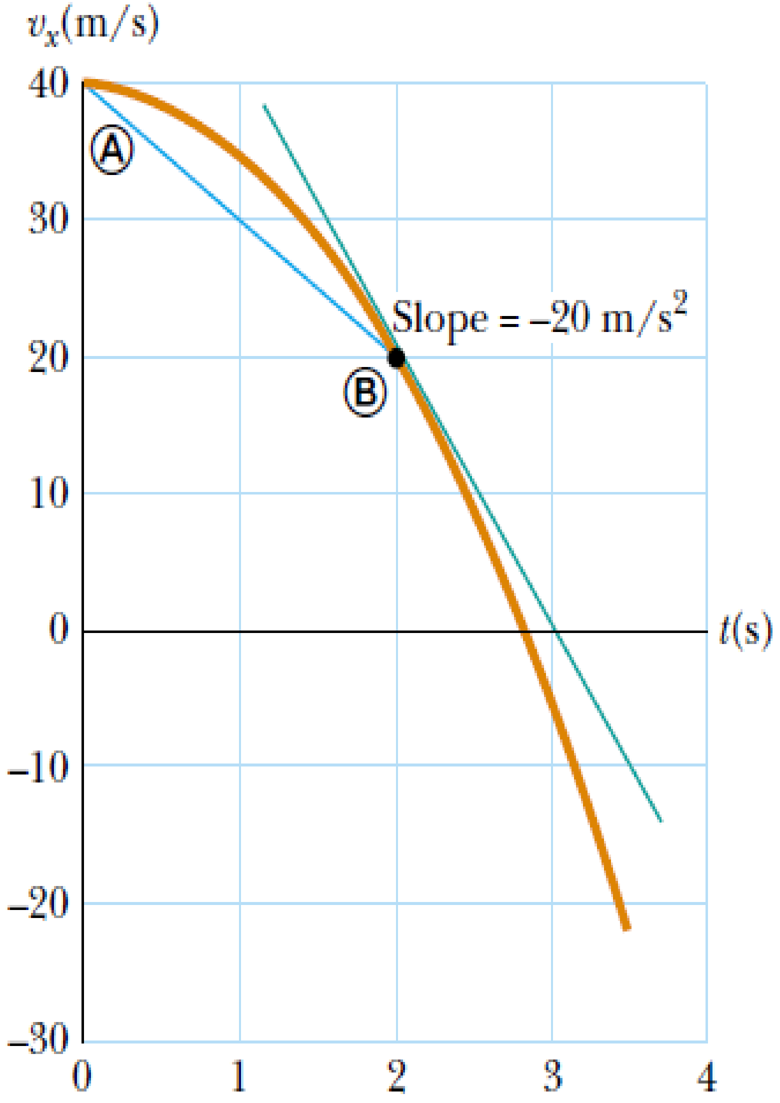
(a) Find the average acceleration in the time interval t = 0 to t = 2.0 s.

$$\because v_x = 40 - 5t^2$$

$$\therefore v_{xi} = v_x(0) = 40 - 5(0) = 40\,\text{m/s}$$

$$\therefore v_{xf} = v_x(2) = 40 - 5(2)^2 = 20\,\text{m/s}$$

$$\rightarrow a_x = \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{\Delta t} = \frac{20 - 40}{2} = -10\,m/s^2$$



2.3 Acceleration

Example: 2.5: (continues)

(b) Determine the acceleration at t = 2.0 s

Solution:

We want to find the instantaneous acceleration at t = 2 s

$$\because v_x = 40 - 5t^2$$

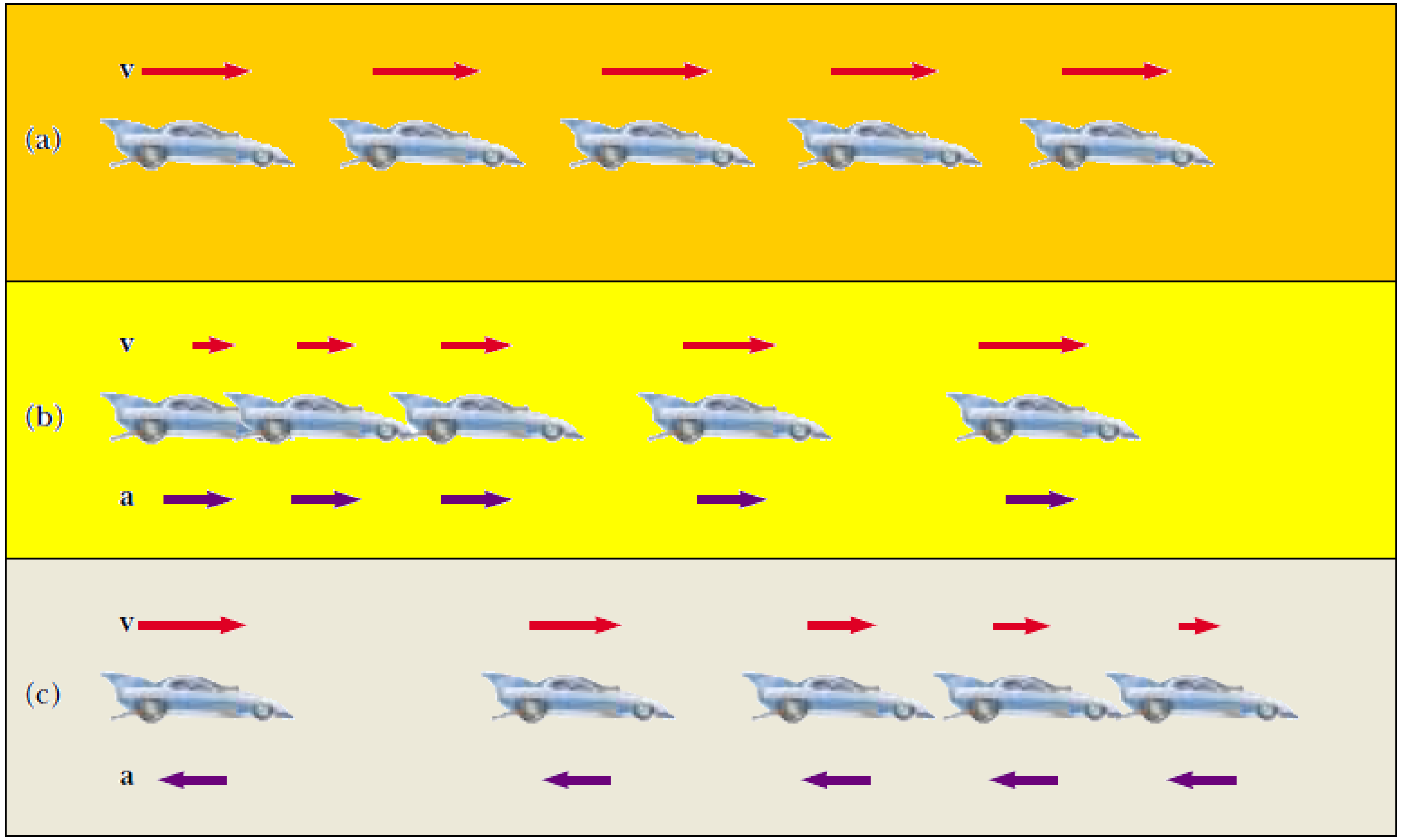
$$\because a_x = \frac{dv_x}{dt}$$

$$\therefore a_x = 0 - 5 \times 2t = -10t$$

$$\therefore a_x(2) = -10(2) = -20 \ \text{m/s}^2$$

Because the velocity of the particle is positive and the acceleration is negative, the particle is slowing down.

2.4 Motion Diagrams



Motion diagram for a car moving at: (a) constant velocity (b) constant acceleration in the direction of its velocity. (c) constant acceleration in the direction opposite the velocity

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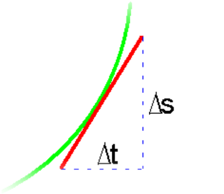
2.3 1-D Motion with Constant Acceleration

If the acceleration of a particle varies in time, its motion can be complex and difficult to analyze. However, a very common and simple type of one-dimensional motion is that in which the *acceleration is constant*. When this is the case, the average acceleration over any time interval is numerically equal to the instantaneous acceleration a_x at any instant within the interval, and the velocity changes at the same rate throughout the motion.

In this case we can use a set of equations shown below:

Kinematic Equations for Motion of a Particle Under Constant Acceleration

Equation	Information Given by Equation
$v_{xf} = v_{xi} + a_x t$	Velocity as a function of time
$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$	Position as a function of velocity and time
$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$	Position as a function of time
$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$	Velocity as a function of position



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2.7 Example

Example: 2.7:

A jet lands on an aircraft carrier at 63 m/s.

(a) What is its acceleration if it stops in 2.0 s due to an arresting cable that snags the airplane and brings it to a stop?

$$v_{xf} = 0, v_{xi} = 63\, m/s, t = 2\, s$$

$$\because v_{xf} = v_{xi} + a_x t$$

$$\therefore 0 = 63 + a_x (2) \rightarrow a_x = -\frac{63}{2} = -31\,\text{m/s}^2$$

(b) If the plane touches down at position $x_i=0$, what is the final position of the plane?

$$\because x_f = x_i + v_{xi} t + \frac{1}{2} a_x t^2$$

$$\therefore x_f = 0 + 63(2) + \frac{1}{2}(-31)(4) = 64\, m$$

Quiz

Quick Quiz 2.2 If a car is traveling eastward and slowing down, what is the direction of the force on the car that causes it to slow down? (a) eastward (b) westward (c) neither of these.

Quick Quiz 2.4 Which of the following is true? (a) If a car is traveling eastward, its acceleration is eastward. (b) If a car is slowing down, its acceleration must be negative. (c) A particle with constant acceleration can never stop and stay stopped.

2.6 Freely Falling Objects

- In this section we will be using same set of equations in Section 2.3 but with the following notes:
  - #1: a → $-9.8\ m/s^2$
  - #2: velocity (v_{xi} and v_{xf}) takes (+) if the object is moving UP and (-) if it is moving DOWN
  - #3: x → y
  - #4: y takes (+) if the object is above its firing level and (-) if it is below its firing level.
- With these notes in minde, just select the right equation and use it directly. If you solved it and find (-) then use the above to explain the results.

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2.6 Freely Falling Objects

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- With these notes in minde, just select the right equation and use it directly. If you solved it and find (-) then use the above to explain the results.

2.6 Freely Falling Objects

- **Quick Quiz 2.6** A ball is thrown upward. While the ball is in free fall, does its acceleration (a) increase (b) decrease (c) increase and then decrease (d) decrease and then increase (e) remain constant?

- **Quick Quiz 2.7** After a ball is thrown upward and is in the air, its speed (a) increases (b) decreases (c) increases and then decreases (d) decreases and then increases (e) remains the same

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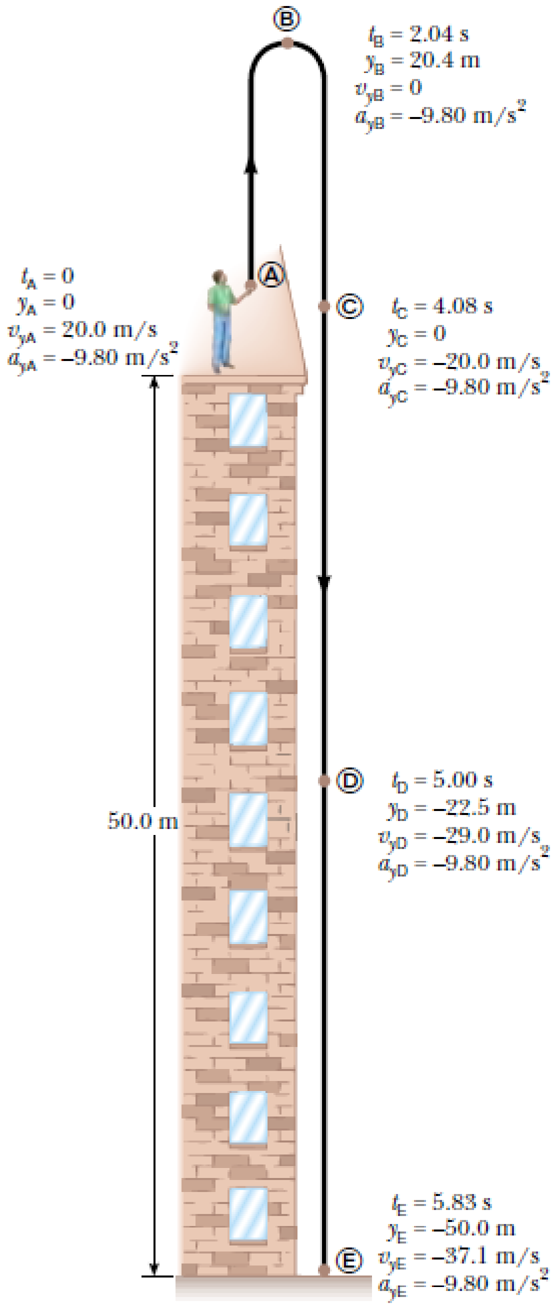
2.6 Freely Falling Objects

Example: 2.12:

A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The building is 50.0 m high, and the stone just misses the edge of the roof on its way down, as shown in Figure 2.14. Using $t_A = 0$ as the time the stone leaves the thrower's hand at position A, determine:

(A) the time at which the stone reaches its maximum height.



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2.6 Freely Falling Objects

Example: 2.12:

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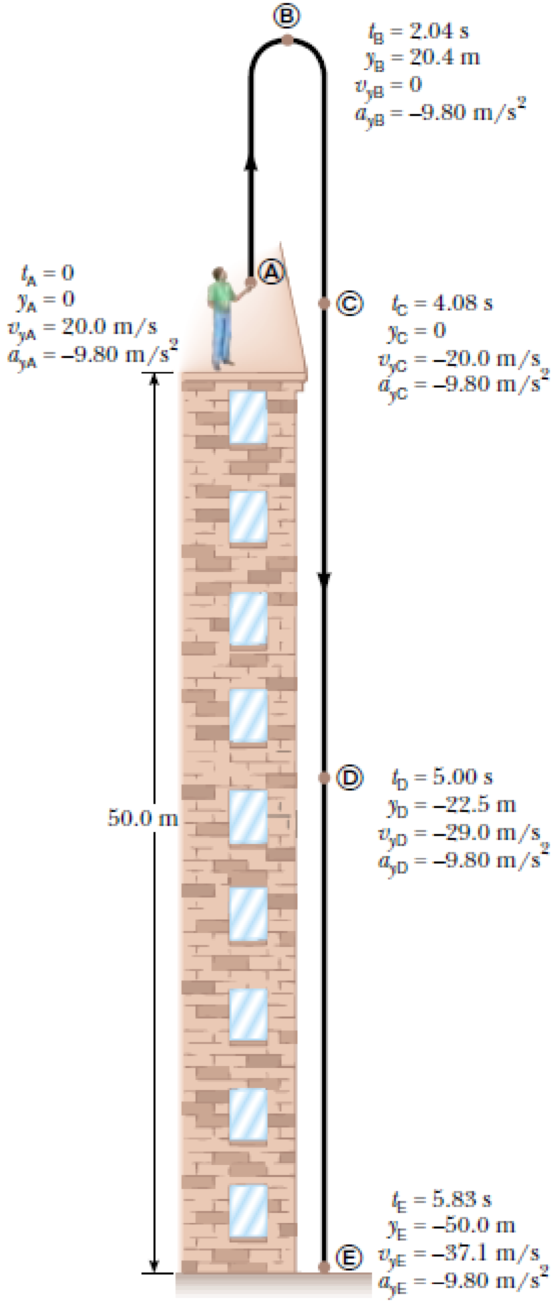
(B) *the maximum height*.

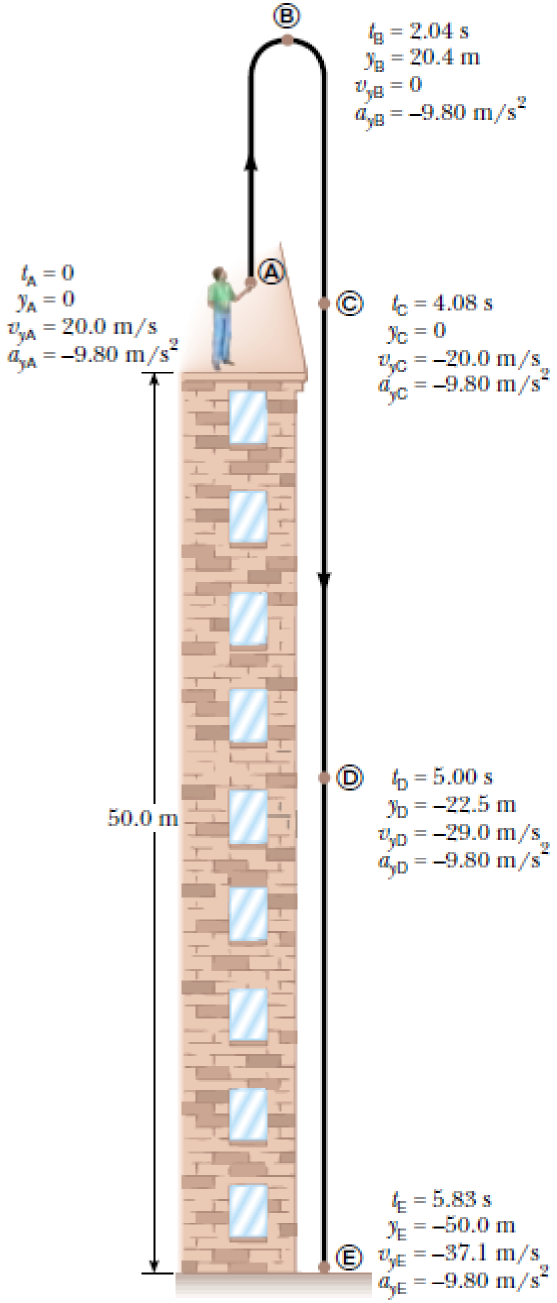


2.6 Freely Falling Objects

Example: 2.12:

A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The building is 50.0 m high, and the stone just misses the edge of the roof on its way down, as shown in Figure 2.14. Using $t_A = 0$ as the time the stone leaves the thrower's hand at position A, determine:

(C) *the time at which the stone returns to the height from which it was thrown,*



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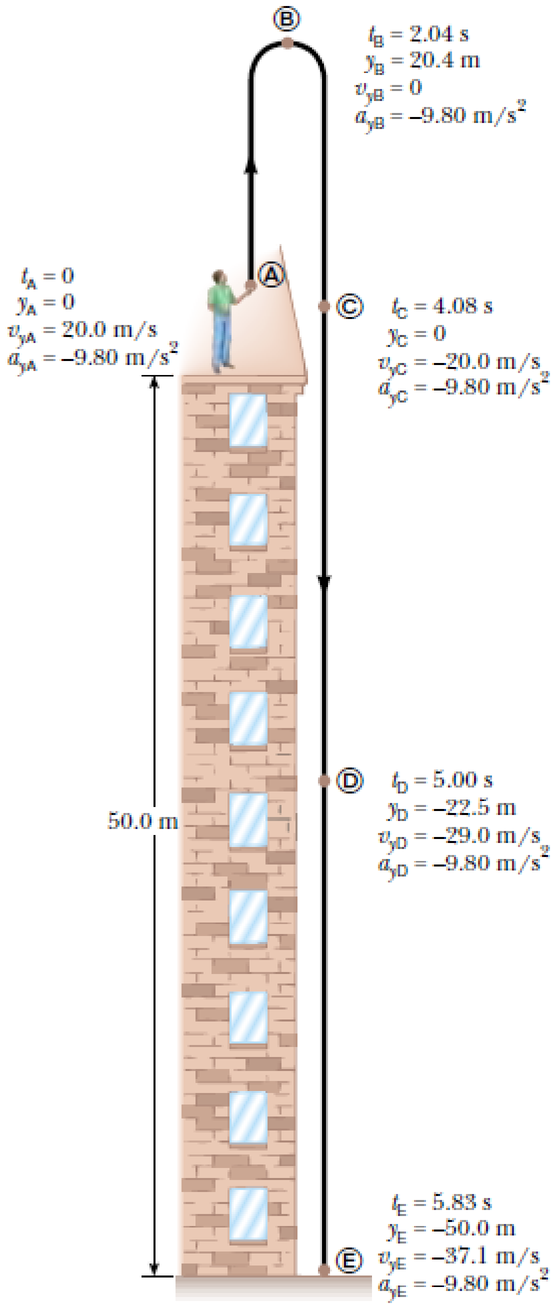
2.6 Freely Falling Objects

Example: 2.12:

A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The building is 50.0 m high, and the stone just misses the edge of the roof on its way down, as shown in Figure 2.14. Using $t_A = 0$ as the time the stone leaves the thrower's hand at position A, determine:

(D) *the velocity of the stone at this instant*



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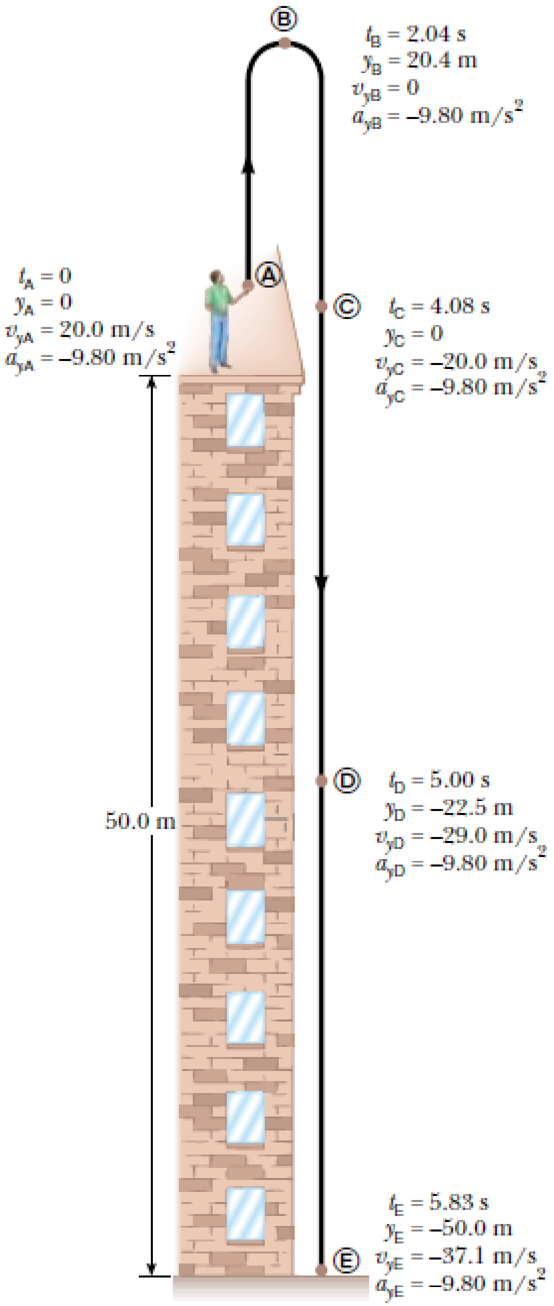
2.6 Freely Falling Objects

Example: 2.12:

A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The building is 50.0 m high, and the stone just misses the edge of the roof on its way down, as shown in Figure 2.14. Using $t_A = 0$ as the time the stone leaves the thrower's hand at position A, determine:

(E) *the velocity and position of the stone at t = 5.00* s.



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Example: 2.12

Solution to Ex. 2.12:

$u_i = +20\ m/s \qquad y_i = -50\ m$

a) find t to max. height?

$\therefore u_{yf} = u_{yi} - 9.8t \rightarrow 0 = +20 - 9.8t$

$\rightarrow t = 2.04\ s$

b) find max. height?

$\therefore y_f = u_{yi}t - \frac{1}{2} \times 9.8 \times t^2 = +20 \times 2.04 - \frac{1}{2} \times 9.8 \times (2.04)^2$

$= 20.4\ m$ (w.r.t top of building)

$= 20.4 + 50 = 70.4$ (w.r.t. ground)

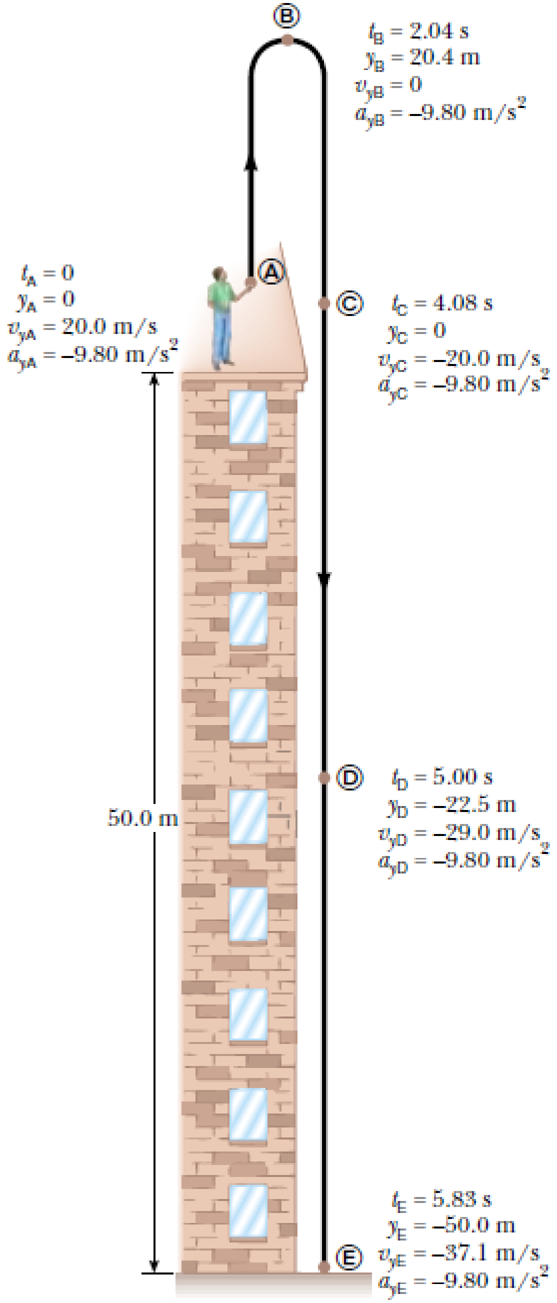
c) $\because$ time↑ = time↓ $\rightarrow t = 2 \times 2.04 = 4.08\ s$

d) $\because u\uparrow = -u\downarrow \rightarrow u = -20\ m/s$

e) find u_y and y at $t = 5\ s$

$\therefore u_{yf} = u_{yi} - 9.8t = 20 - 9.8 \times 5 = -29\ m/s$ ↓ falling down

$\therefore y = u_{yi}t - \frac{1}{2} \times 9.8 \times t^2 = 20 \times 5 - \frac{1}{2} \times 9.8 \times 25 = -25.5\ m$ تحت البناية



Chapter Summary

- After a particle moves along the x axis from some initial position x_i to some final position x_f, its displacement is

$$\Delta x = x_f - x_i \qquad (2.1)$$

- The average velocity of a particle during some time interval is the displacement Δx divided by the time interval Δt during which that displacement occurs:

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t} \qquad (2.2)$$

Chapter Summary

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$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t} \qquad (2.2)$$

- The average speed of a particle is equal to the ratio of the total distance it travels to the total time interval during which it travels that distance:

$$\text{Average Speed} = \frac{\text{Total Distance}}{\text{Total Time}} \qquad (2.3)$$

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Chapter Summary

- The instantaneous velocity of a particle is defined as:

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \qquad (2.5)$$

- The instantaneous speed of a particle is equal to the magnitude of its instantaneous velocity.
- The average acceleration of a particle is defined as:

$$\bar{a}_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i} \qquad (2.6)$$

- The instantaneous acceleration is defined as:

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} \qquad (2.7)$$

Chapter Summary (continued)

- When the object's velocity and acceleration are in the same direction, the object is speeding up. On the other hand, when the object's velocity and acceleration are in opposite directions, the object is slowing down.
- The equations of kinematics for a particle moving along the x axis with uniform acceleration a_x are:

Kinematic Equations for Motion of a Particle Under Constant Acceleration

Equation	Information Given by Equation
$v_{xf} = v_{xi} + a_x t$	Velocity as a function of time
$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$	Position as a function of velocity and time
$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$	Position as a function of time
$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$	Velocity as a function of position

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PHYS 103 (GENERAL PHYSICS)

CHAPTER 3: VECTORS

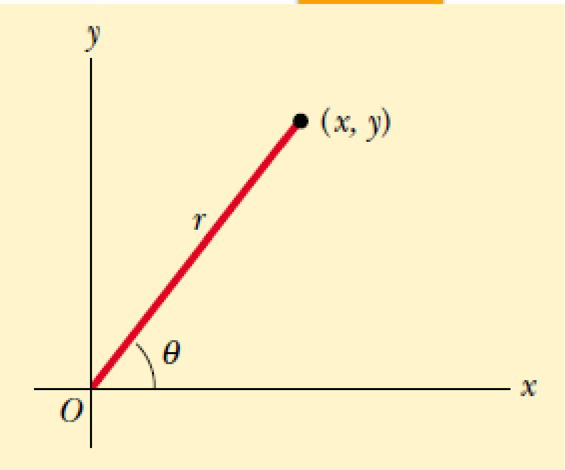
THIS PRESENTATION HAS BEEN PREPARED BY: ***DR. NASSR S. ALZAYED***

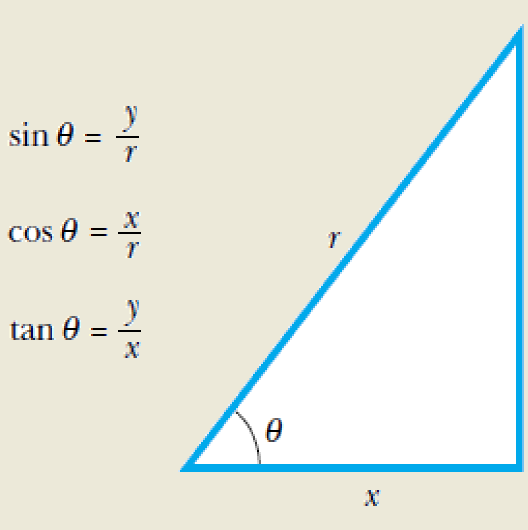
Lecture Outline

- Here is a quick list of the subjects that we will cover in this presentation. It is based on Serway, Ed. 6
- *3.1 Coordinate Systems (Cartezian & Polar)*
- *3.2 Vector and Scalar Quantities*
- *3.3 Some Properties of Vectors (Addition, subtraction, …)*
- *3.4 Components of a Vector and Unit Vectors*
- *Examples*
- *Lecture Summary*
- *Activities (Interactive Flashes)*
- *Quizzes*

3.1 Coordinate Systems

- Many aspects of physics involve a description of a location in space. This usually is implemented using *coordinate systems*
- *Cartesian coordinate system* is one simple system in which horizontal and vertical axes intersect at a point defined as the origin.
- Sometimes it is more convenient to represent a point in a plane by its plane polar coordinates (r, θ), In this *polar coordinate system*, r is the distance from the origin to the point having Cartesian coordinates (x, y), and θ is the angle between a line drawn from the origin to the point and a fixed axis





3.1 Conversion Between Coordinate Systems

- we can obtain the Cartesian coordinates from Polar coordinates by using the equations:

$$x = r\cos\theta \qquad (3.1)$$

$$y = r\sin\theta \qquad (3.2)$$

$$\tan\theta = \frac{y}{x} \qquad (3.3)$$

$$r = \sqrt{x^2 + y^2} \qquad (3.4)$$

- These four expressions relating the coordinates (x, y) to the coordinates (r, θ) apply only when positive θ is an angle measured counterclockwise from the positive x axis.
- If the reference axis for the polar angle θ is chosen to be one other than the positive x axis or if the sense of increasing θ is chosen differently, then the expressions relating the two sets of coordinates will change.

Example 3.1 Polar Coordinates

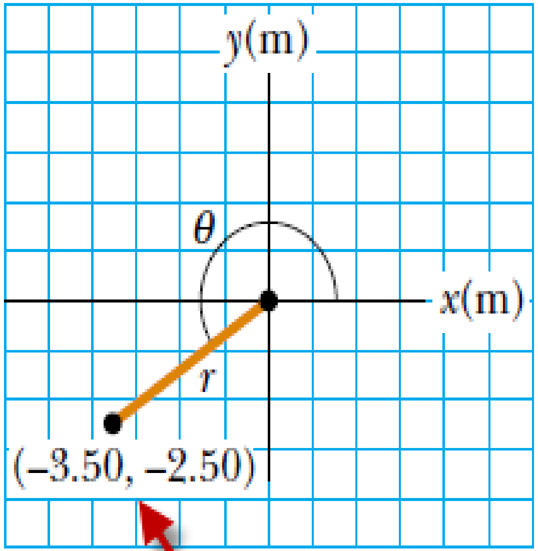
- The Cartesian coordinates of a point in the xy plane are (x, y) = (-3.50, -2.50) m, as shown in the figure. Find the polar, (r, θ), coordinates of this point.
- Solution:

$$\because r = \sqrt{x^2 + y^2} = \sqrt{(-3.5)^2 + (-2.5)^2} = 4.30\,m$$

$$\because \tan\theta = \frac{y}{x} = \frac{-2.5}{-3.5} = 0.714$$

$$\therefore \theta = 216^o$$

- Note that you must use the signs of x and y to find that the point lies in the *third quadrant* of the coordinate system. That is, θ = 216° and not 35.5°.



x is (-) and y is (-) Hence, the point must be in the ***third quadrant***

3.2 Vector and Scalar Quantities

- ***A scalar quantity*** is completely specified by a single value with an appropriate unit and *has no direction*.
- ***A vector quantity*** is completely specified by a number and appropriate units *plus a direction*.
- Below examples of scalar and vector quantities

scalar quantities	**vector quantities**
Temperature	*Velocity*
Density	*Acceleration*
Distance	*Force*
Mass	*Displacement*
Speed	*Torque*
Volume	*Weight*

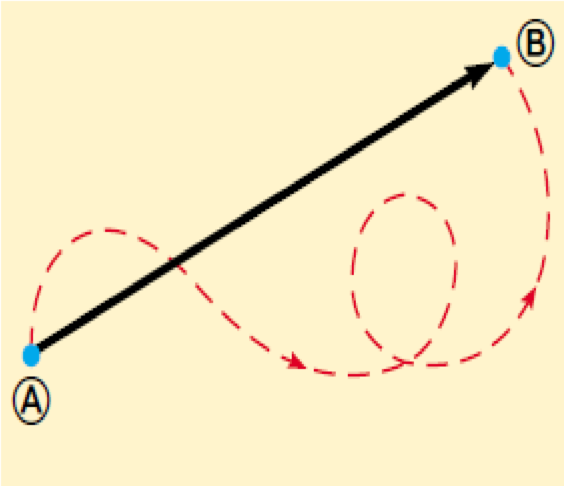


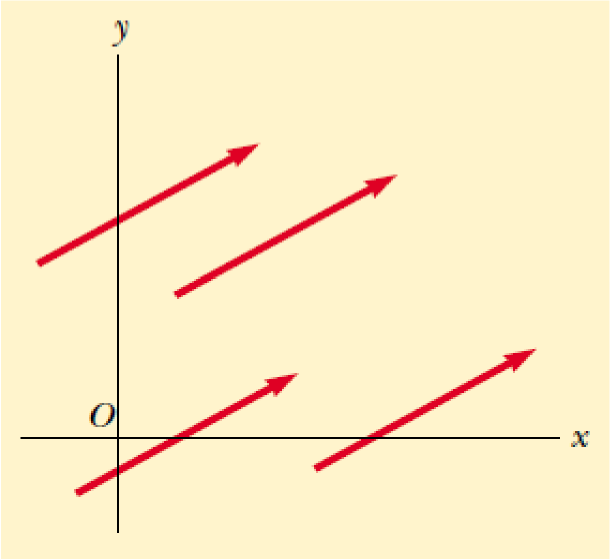
Figure 3.4 As a particle moves from Ⓐ to Ⓑ along an arbitrary path represented by the broken line, its displacement is a vector quantity shown by the arrow drawn from Ⓐ to Ⓑ.

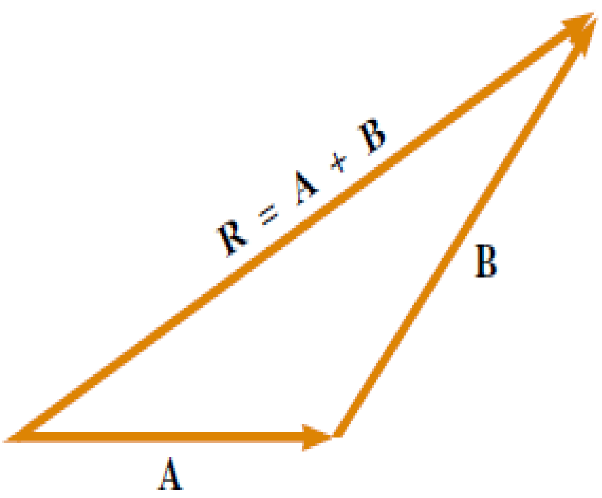
3.2 Vector and Scalar Quantities

Quick Quiz 3.1 Which of the following are vector quantities and which are scalar quantities? (a) your age (b) acceleration (c) velocity (d) speed (e) mass

3.3 Some Properties of Vectors (1)

- *Equality of Two Vectors*: **A = B if** $A = B$ and if **A** and **B** point in the same direction along parallel lines
- *Adding Vectors*: To add vector **B** to vector **A**, first draw vector **A** on graph paper, and then draw vector **B** to the same scale with its tail starting from the tip of **A**, as shown in Figure. The resultant vector **R** = **A** + **B** is the vector drawn from the tail of **A** to the tip of **B**.
- It is also possible to add vectors using Unit vectors. We shall discuss this feature later in this lecturer.





3.3 Some Properties of Vectors (2)

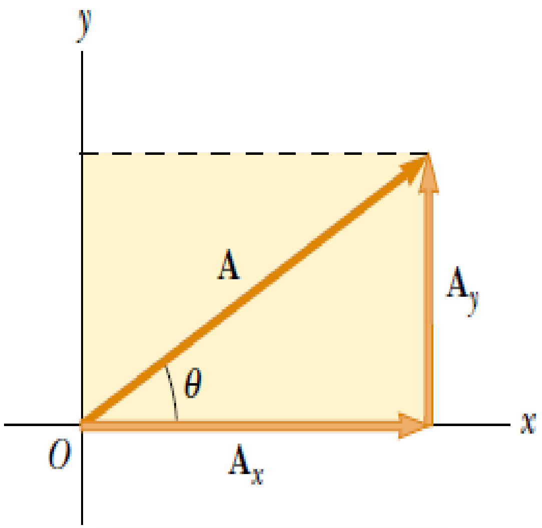
- *commutative law of addition* : **A + B = B + A**
- *associative law of addition:* **A + (B + C) = (A + B) + C**
- *Negative of a Vector***:** The negative of the vector **A** is defined as the vector that when added to **A** gives zero for the vector sum. That is:

A + (-**A**) = 0. The vectors **A** and -**A** have the same magnitude but point in *opposite directions*

- *Subtracting Vectors:* We define the operation **A - B** as vector -**B** added to vector **A**: **A** - **B** = **A** + (-**B**)
- *Multiplying a Vector by a Scalar:* m**A** has *same* direction of **A** with mA in magnitude. -m**A** has opposite direction of **A** with mA in magnitude**.**
- *For example*, the vector 5**A** is five times as long as **A** and points in the same direction as **A**

3.4 Components of a Vector and Unit Vectors

▶ Consider a vector **A** lying in the xy plane and making an arbitrary angle θ with the positive x axis, as shown in Figure. This vector can be expressed as the sum of two other vectors $\mathbf{A}_x$ and $\mathbf{A}_y$.
$\mathbf{A}_x$ and $\mathbf{A}_y$ are the *VECTOR COMPONENTS* of The vector **A**
A_x and A_y are the *COMPONENTS* of The vector **A**



$$A_x = A\cos\theta \qquad (3.8)$$

$$A_y = A\sin\theta \qquad (3.9)$$

$$\tan\theta = \frac{A_y}{A_x} \qquad (3.10)$$

$$A = \sqrt{{A_x}^2 + {A_y}^2} \qquad (3.11)$$

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3.2 Vector and Scalar Quantities

Quick Quiz 3.2 The magnitudes of two vectors **A** and **B** are $A = 12$ units and $B = 8$ units. Which of the following pairs of numbers represents the *largest* and *smallest* possible values for the magnitude of the resultant vector $\mathbf{R} = \mathbf{A} + \mathbf{B}$? (a) 14.4 units, 4 units (b) 12 units, 8 units (c) 20 units, 4 units (d) none of these answers.

Quick Quiz 3.3 If vector **B** is added to vector **A**, under what condition does the resultant vector $\mathbf{A} + \mathbf{B}$ have magnitude $A + B$? (a) **A** and **B** are parallel and in the same direction. (b) **A** and **B** are parallel and in opposite directions. (c) **A** and **B** are perpendicular.

Quick Quiz 3.4 If vector **B** is added to vector **A**, which *two* of the following choices must be true in order for the resultant vector to be equal to zero? (a) **A** and **B** are parallel and in the same direction. (b) **A** and **B** are parallel and in opposite directions. (c) **A** and **B** have the same magnitude. (d) **A** and **B** are perpendicular.

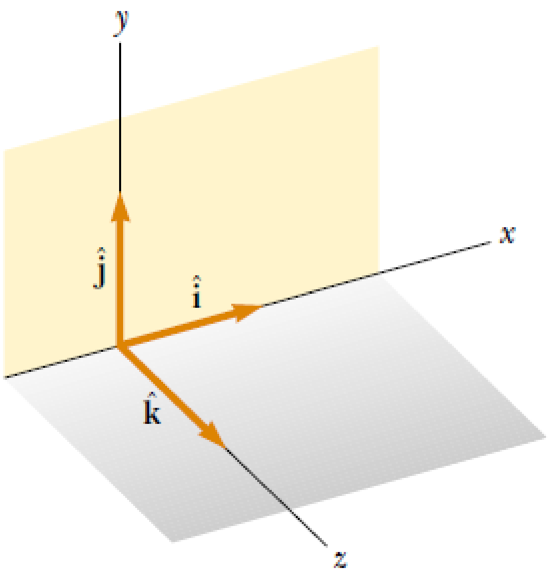
3.4 Unit Vectors

- A unit vector is a dimensionless vector having a magnitude of exactly 1.

  on x: we use: $\hat{\imath}$
  on y: we use: $\hat{\jmath}$
  on z: we use: $\hat{k}$
  $|\hat{\imath}| = |\hat{\jmath}| = \left|\hat{k}\right| = 1$
  We express a vector using unit vetors as:
  $A = A_x\hat{\imath} + A_y\hat{\jmath}$

Vector addition is easier with unit vectors

$$R = A + B = R_x\hat{\imath} + R_y\hat{\jmath}$$
$$\therefore R = (A_x\hat{\imath} + A_y\hat{\jmath}) + (B_x\hat{\imath} + B_y\hat{\jmath})$$
$$\Rightarrow R = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath}$$
$$\therefore R_x = A_x + B_x R_y = A_y + B_y$$



3.4 Unit Vectors

- To find magnitude of the Resultant vector R and the angle it makes with +tive x-axis; we just do same as we did before:

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2} \qquad (3.16)$$

$$\tan\theta = \frac{R_y}{R_x} = \frac{A_x + B_x}{A_y + B_y} \qquad (3.17)$$

in 3D:

$$A = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k} \qquad (3.18)$$

$$B = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k} \qquad (3.19)$$

The sum of A and B is:

$$R = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath} + (A_z + B_z)\hat{k} \qquad (3.20)$$

magnetude of the R:

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2 + (A_z + B_z)^2}$$

Example 3.3 The Sum of Two Vectors

- To Find the sum of two vectors A and B lying in the xy plane and given by:

$$A = (2.0\hat{i} + 2.0\hat{j})\,m \quad \text{and} \quad B = (2.0\,\hat{\text{i}} - 4.0\,\hat{\text{j}})\,\text{m}$$

Solution:

$$\because R = A + B = (2.0\hat{i} + 2.0\hat{j}) + (2.0\,\hat{\text{i}} - 4.0\,\hat{\text{j}}) = (4.0\,\hat{\text{i}} - 2.0\,\hat{\text{j}})\,\text{m}$$

$$\rightarrow |\text{R}| = \sqrt{R_x^{\,2} + R_y^{\,2}} = \sqrt{(4)^2 + (-2)^2} = 4.5\,m$$

$$\theta = \tan^{-1}\frac{-2}{4} = \tan^{-1}(-0.5) = -27^o$$

- But because R is in the $4^{th.}$ quadrant; this angle is not the actual angle based on our convention. We need to add 360^o to this angle:

$$\therefore \theta = -27^o + 360^o = 333^o$$

Example 3.3 The Resultant Displacement

A particle undergoes three consecutive displacements:

$d_1 = (15\hat{i} + 30\hat{j} + 12\hat{k})$ cm , $d_2 = (23\hat{i} - 14\hat{j} - 5\hat{k})$ cm

and $d_3 = (-13\hat{i} + 15\hat{j})$ cm

Find the components of the resultant displacement and its magnitude.

Solution:

$\because R = d_1 + d_2 + d_3$

$\therefore R = (15\hat{i} + 30\hat{j} + 12\hat{k}) + (23\hat{i} - 14\hat{j} - 5\hat{k}) + (-13\hat{i} + 15\hat{j})$

$\Rightarrow R = (25\hat{i} + 31\hat{j} + 7\hat{k})$ cm

$|\mathrm{R}| = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{(25)^2 + (31)^2 + (7)^2} = 40cm$

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Example 3.5 Taking a hike (a)

- A hiker begins a trip by first walking 25.0 km southeast from her car. She stops and sets up her tent for the night. On the second day, she walks 40.0 km in a direction 60.0° north of east, at which point she discovers a forest ranger's tower.

(a) *Determine the components of the hiker's displacement for each day.*

Components of her first day:

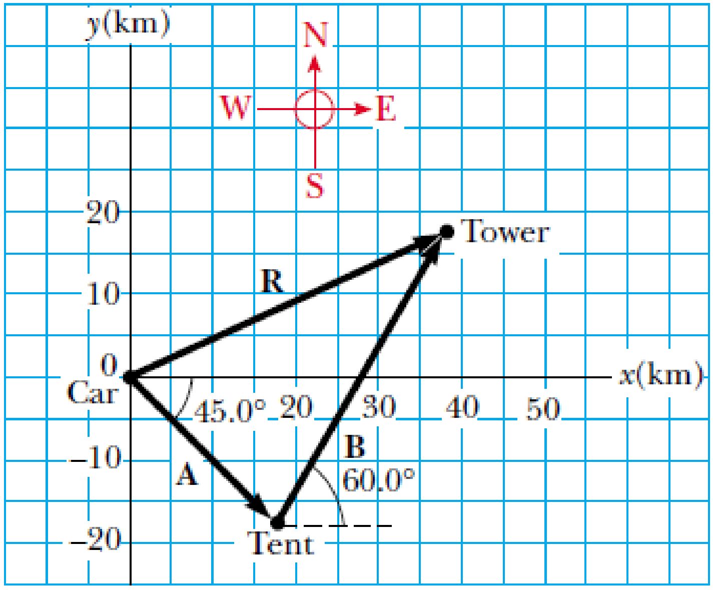
$$A_x = A\cos(-45^o) = 25\cos(-45^o) = 17.7 \text{ km}$$

$$A_y = A\sin(-45^o) = 25\sin(-45^o) = -17.7 \text{ km}$$

Components of her 2nd day:

$$B_x = B\cos(60^o) = 25\cos(60^o) = 20 \text{ km}$$

$$B_y = B\sin(60^o) = 25\sin(60^o) = 34.6 \text{ km}$$



Please note that: cos(315°) = cos(-45°) and sin (315°) = sin (-45°)
315° is the angle between +x and the vector A (counterclockwise)

Example 3.5 Taking a hike (b)

- (b) *Determine the components of the hiker's resultant displacement R for the trip. Find an expression for R in terms of unit vectors..*

Solution:

$\because$ R=A+B

$$\therefore R_x = A_x + B_x = 17.7 + 20 = 37.7\,km$$

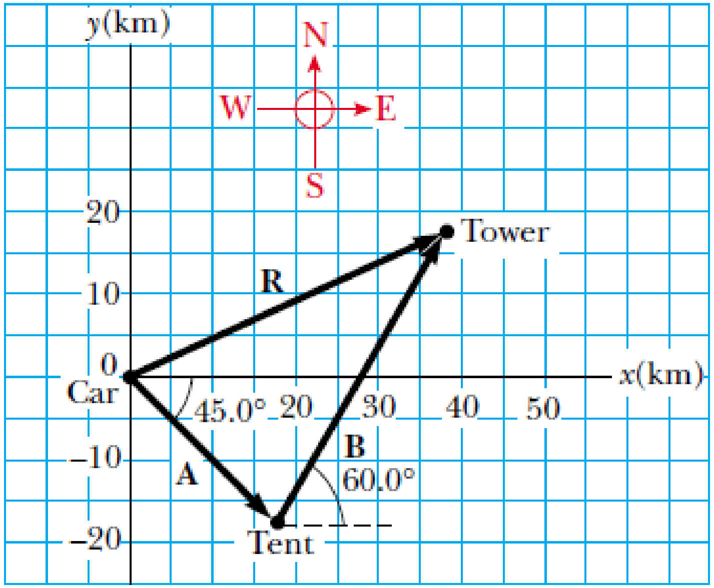
$$R_y = A_y + B_y = -17.7 + 34.6 = 16.9\,km$$

using unit vector notation:

$$R = (37.7\hat{i} + 16.9\hat{j})\,km$$

$$\therefore |R| = \sqrt{(37.7)^2 + (169)^2} = 41.3\,km$$

$$\theta = \tan^{-1}\frac{16.9}{37.7} = 24.1^o$$



Example 3.5 Taking a hike (b)

Quick Quiz 3.6 If at least one component of a vector is a positive number, the vector cannot (a) have any component that is negative (b) be zero (c) have three dimensions.

Quick Quiz 3.7 If $\mathbf{A} + \mathbf{B} = 0$, the corresponding components of the two vectors $\mathbf{A}$ and $\mathbf{B}$ must be (a) equal (b) positive (c) negative (d) of opposite sign.

Quick Quiz 3.8 For which of the following vectors is the magnitude of the vector equal to one of the components of the vector? (a) $\mathbf{A} = 2\hat{\mathbf{i}} + 5\hat{\mathbf{j}}$ (b) $\mathbf{B} = -3\hat{\mathbf{j}}$ (c) $\mathbf{C} = +5\hat{\mathbf{k}}$

Lecture Summary

- ***Scalar quantities***: are those that have only a numerical value and no associated direction.
- ***Vector quantities***: have both magnitude and direction and obey the laws of vector addition.
- ***The magnitude of a vector*** is always a positive number.
- When two or more vectors are added together, all of them must have the same units and all of them must be the same type of quantity.
- We can add two vectors **A** and **B** graphically. In this method, the resultant vector **R = A + B** runs from the tail of **A** to the tip of **B**.
- A 2nd method involves components of the vectors. $\mathbf{A}_x$ of the vector **A** is equal to the projection of **A** along the x axis of a coordinate system, where $\mathbf{A}_x = \mathbf{A} \cos\theta$. $\mathbf{A}_y$ of **A** is the projection of **A** along the y axis, where $\mathbf{A}_y = \mathbf{A} \sin\theta$

Lecture Summary (continued)

- Be sure you can determine which trigonometric functions you should use in all situations, especially when θ is defined as something other than the counterclockwise angle from the positive x axis.
- If a vector **A** has an x component $\mathbf{A}_x$ and a y component $\mathbf{A}_y$, the vector can be expressed in unit–vector form as

$$A = A_x \hat{i} + A_y$$

- We can find the resultant of two or more vectors by resolving all vectors into their x and y components, adding their resultant x and y components, and then using the Pythagorean theorem to find the magnitude of the resultant vector. We can find the angle that the resultant vector makes with respect to the x axis by using a suitable trigonometric function.

Thanks

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PHYS 103 (GENERAL PHYSICS)

CHAPTER 4: 2-D MOTION

THIS PRESENTATION HAS BEEN PREPARED BY: ***DR. NASSR S. ALZAYED***

Chapter Outline

- Here is a quick list of the subjects that we will cover in this presentation. It is based on Serway, Ed. 6

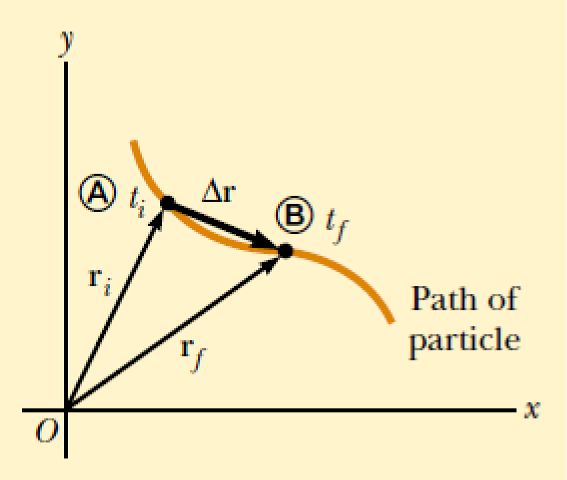
- *4.1 The Position, Velocity, and Acceleration Vectors*
- *4.2 Two-Dimensional Motion with Constant Acceleration*
- *4.3 Projectile Motion*
- *4.4 Uniform Circular Motion*
- *4.5 Tangential and Radial Acceleration*
- *Examples*
- *Quizzes*
- *Chapter Summary*

4.1 Position, Velocity, and Acceleration Vectors

- In the figure: when the particle moves from position *A* to position *B*, we can say that particle moved from position : $\mathbf{r}_i$ to position $\mathbf{r}_f$.
- $\mathbf{r}_i$ and $\mathbf{r}_f$ are initial and final position vectors respectively.
- We can express the *DISPLACEMENT* that was made by the particle as:

$$\Delta r = r_f - r_i \qquad (4.1)$$

- We have already discussed the displacement in lecture No. 3. Please see the difference between the path (distance) of the particle (orange line) and displacement. Distance is not a vector while displacement is a vector.



4.1 The Average Velocity

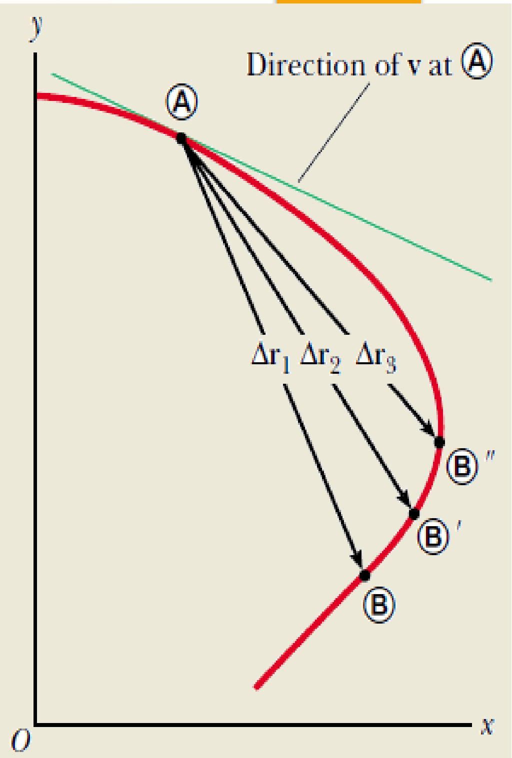
- The average velocity is defined as:

$$\overline{\mathrm{v}} = \frac{\Delta r}{\Delta t} = \frac{r_f - r_i}{t_f - t_i} \qquad (4.2)$$

- We can get the instantaneous velocity (velocity as a function of time) as follows:

$$v = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t} = \frac{dr}{dt} \qquad (4.3)$$

- the instantaneous velocity equals the derivative of the position vector with respect to time



- The direction of the instantaneous velocity vector is along a line tangent to the path at that point and in the direction of motion.
- The magnitude of the instantaneous velocity vector $\upsilon = |\overline{\mathrm{v}}|$ is called the speed, which is a scalar quantity.

4.1 The Average acceleration

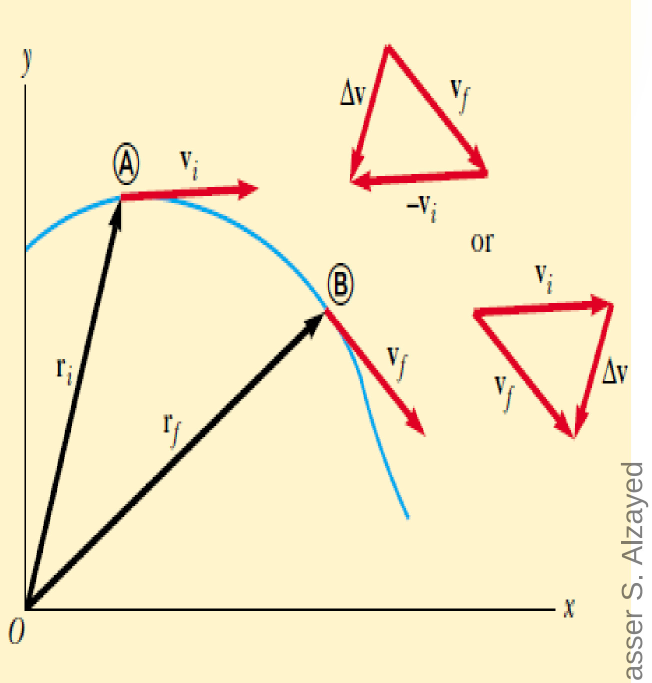
- The average acceleration is defined as:

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} \qquad (4.4)$$

- We can get the instantaneous acceleration (acceleration as a function of time) as follows:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \qquad (4.5)$$

- Note: the magnitude of the velocity vector (the speed) may change with time as in straight-line (one-dimensional) motion.
- the direction of the velocity vector may change with time even if its magnitude (speed) remains constant, as in curved-path (2-d) motion.



4.2 2-D Motion with Cons. Acceleration

- The Let us consider 2-dimensional motion during which the acceleration remains constant in both magnitude and direction.
- The position vector for a particle moving in the xy plane can be written:

$$r = x\hat{i} + y\hat{j} \qquad (4.6)$$

- Please note that: *r*, *x* and *y* are time-dependant. They change with time as the particle moves.
- the velocity of the particle can be derived as:

$$v = \frac{dr}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = v_x\hat{i} + v_y\hat{j} \qquad (4.7)$$

- Because a is assumed constant, its components a_x and a_y also are also constants.
- Hence, for every component; we can use Table 2.2 (Chapter 2)

4.2 2-D Motion: x and y equations

- We will have 2 sets of Equations; one for each direction.
- For x-direction; we have:

$$v_{xf} = v_{xi} + a_x t$$

$$x_f = v_{xi} t + \frac{1}{2} a_x t^2$$

$$v_{xf}^2 = v_{xi}^2 + 2 a_x x_f$$

- For y-direction; we have:

$$v_{yf} = v_{yi} + a_y t$$

$$y_f = v_{yi} t + \frac{1}{2} a_y t^2$$

$$v_{yf}^2 = v_{yi}^2 + 2 a_y y_f$$

4.2 2-D Motion velocity and position vectors

- Hence, vectors of velocity ***v*** and position ***r*** are.

$$\because v = v_x \hat{i} + v_y \hat{j}$$

$$\therefore v_f = (v_{xi} + a_x t)\hat{i} + (v_{yi} + a_y t)\hat{j}$$

$$= (v_{xi}\hat{i} + v_{yi}\hat{j}) + (a_x \hat{i} + a_y \hat{j})\mathrm{t}$$

$$= v_i + at \qquad (4.8)$$

$$r_f = (v_{xi} t + \frac{1}{2} a_x t^2)\hat{i} + (v_{yi} t + \frac{1}{2} a_y t^2)\hat{j}$$

$$= (v_{xi}\hat{i} + v_{yi}\hat{j})t + \frac{1}{2}(a_x \hat{i} + a_y \hat{j})t^2$$

$$= v_i t + \frac{1}{2} a t^2 \qquad (4.9)$$

Example 4.1 Motion in a Plane

- A particle starts from the origin at t = 0 with an initial velocity having an *x* component of 20 m/s and a *y* component of -15 m/s. The particle moves in the *xy* plane with an *x* component of acceleration only, given by $a_x = 4.0\ \text{m/s}^2$.

(A) Determine the components of the velocity vector at any time and the total velocity vector at any time.

Solution:

We want to find v_{xf} and v_{yf} as functions of t then find **v**(t).

$$v_{xf} = v_{xi} + a_x t = (20 + 4t)\ \text{m/s}$$

$$v_{yf} = v_{yi} + a_y t = (-15 + 0t) = -15\ \text{m/s}$$

$$\Rightarrow v_f = v_x \hat{i} + v_y \hat{j} = [(20 + 4t)\hat{i} - 15\hat{j}]\ \text{m/s}$$

Example 4.1 Motion in a Plane

(B) Calculate the velocity and speed of the particle at t = 5.0 s.

Solution: We want to get **v** as (vector), its direction, and find the Speed:

$$v_f = v_x \hat{i} + v_y \hat{j} = [(20 + 4t)\hat{i} - 15\hat{j}] \text{ m/s}$$

$$\therefore v_f = [(20 + 4 \times 5)\hat{i} - 15\hat{j}] \text{ m/s}$$

$$\therefore v_f = (40\hat{i} - 15\hat{j}) \text{ m/s}$$

for direction:

$$\because \theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \tan^{-1}\left(\frac{-15}{40}\right) = -21^o$$

for speed: we find magnetude of v:

$$|v_f| = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{(40)^2 + (-15)^2} = 43\, m/s$$

Example 4.1 Motion in a Plane (continued)

(C) Determine the x and y coordinates of the particle at any time t and the position vector at this time.

Solution: We want to get x and y then **r** as (vector):

$$\because x_f = v_{xi}t + \frac{1}{2}a_x t^2 \Rightarrow x_f = 20t + \frac{1}{2}(4)t^2 = (20t + 2t^2)\,m$$

$$\because y_f = v_{yi}t + \frac{1}{2}a_y t^2 \Rightarrow y_f = -15t + \frac{1}{2}(0)t^2 = (-15t)\,m$$

$$\because r_f = x_f\hat{i} + y_f\hat{j} \Rightarrow r_f = [(20t + 2t^2)\hat{i} + -15t\hat{j}]\,m$$

to find magnetude of r_f at t=5 s:

$$|r_f| = \sqrt{x_{fx}^2 + y_{fy}^2} = \sqrt{150^2 + (-75)^2} = 170\,m$$

- *Please note: the last value is NOT the DISTANCE.*

Practice Quiz 4.1 & 4.2

Quick Quiz 4.1 Which of the following cannot *possibly* be accelerating? (a) An object moving with a constant speed (b) An object moving with a constant velocity (c) An object moving along a curve.

Quick Quiz 4.2 Consider the following controls in an automobile: gas pedal, brake, steering wheel. The controls in this list that cause an acceleration of the car are (a) all three controls (b) the gas pedal and the brake (c) only the brake (d) only the gas pedal.

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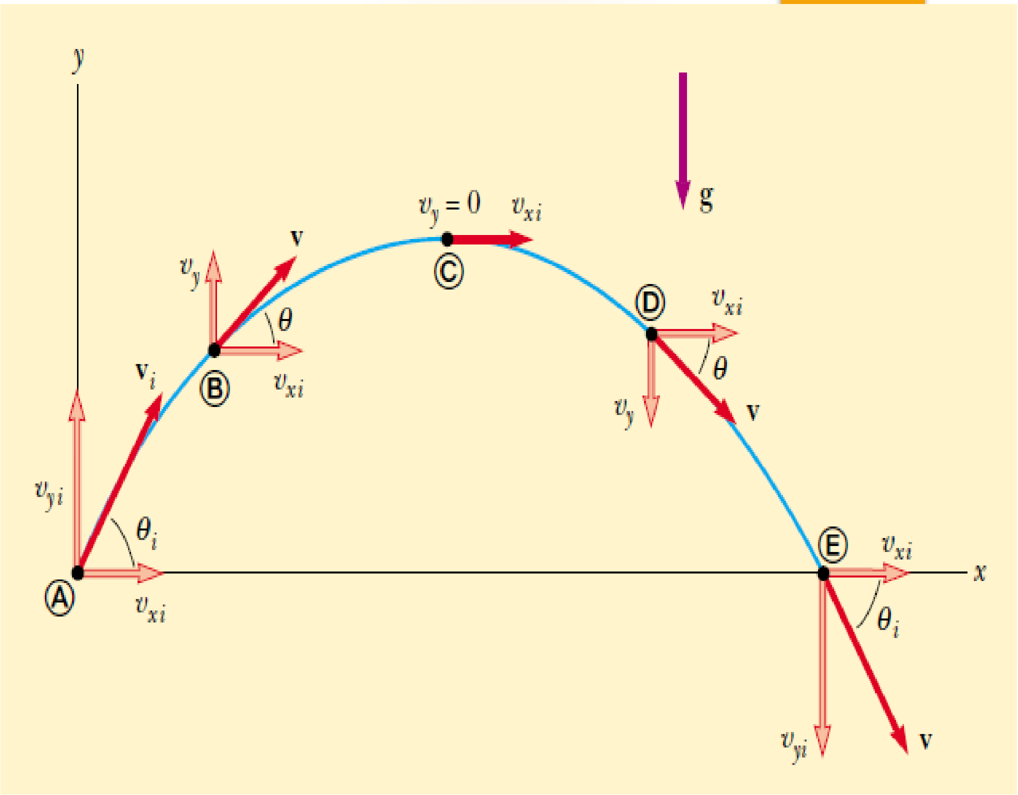
4.3 Projectile Motion

Anyone who has observed a baseball in motion has observed projectile motion. The ball moves in a curved path, and its motion is simple to analyze if we make two assumptions: (1) the free-fall acceleration g is constant over the range of motion and is directed downward, and (2) the effect of air resistance is negligible.

We find that the path of a projectile, which we call its trajectory, is always a *parabola*



The parabolic path of a projectile that leaves the origin with a velocity v_i. The x component of v remains constant in time. The y component of velocity is zero at the peak of the path.

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4.3 Projectile Motion (x & y equations)

- We will be having 2 sets of equations: 1 for x and 1 for y directions:

Fro x directon:

$$v_{xi} = v_i \cos\theta_i \qquad (4.10a)$$

$$x_f = v_{xi} t = (v_i \cos\theta_i) t \qquad (4.11a)$$

Fro y directon:

$$v_{yi} = v_i \sin\theta_i \qquad (4.10b)$$

$$y_f = v_{yi} t + \frac{1}{2} a_y t^2 = (v_i \sin\theta_i) t - \frac{1}{2} g t^2 \qquad (4.12)$$

- Please note that you can solve for x or y independently.

4.3 Projectile Motion (trajectory equation)

- We will be having 2 sets of equations: 1 for x and 1 for y directions:

$$(4.11a) \rightarrow t = \frac{x_f}{v_i \cos\theta_i}$$

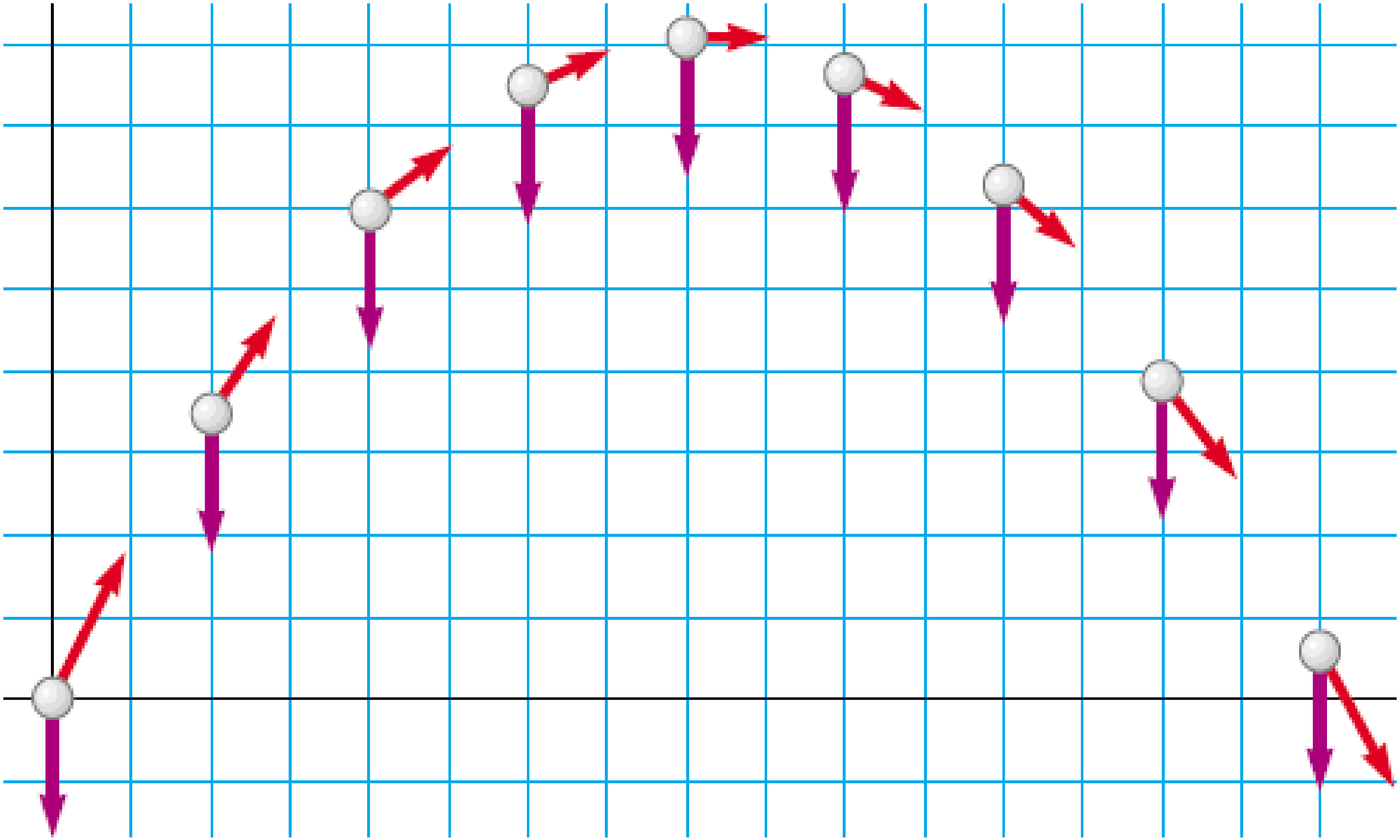
$$\therefore \mathrm{y_f} = (v_i \sin\theta_i)\frac{x_f}{v_i \cos\theta_i} - \frac{1}{2} g \left[\frac{x_f}{v_i \cos\theta_i}\right]^2$$

$$\Rightarrow \mathrm{y_f} = (\tan\theta_i) x_f - \frac{g}{2v_i^{\,2} \cos\theta_i^{\,2}} x_f^{\,2}$$

$$OR : y = ax - bx^2$$

- This is the equation of a *parabola* that passes through the origin.

4.3 Projectile Motion (motion diagram)



Time of Flight of a Projectile

- We will consider the maximum height reached by a projectile:

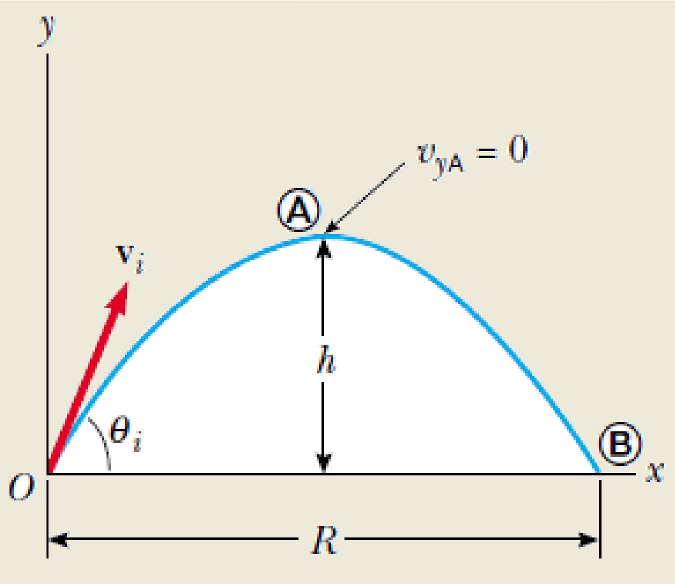
1st: time of fleight: at maxi. height $v_{yf} = 0$

$$\because v_{yf} = v_{yi} + a_y t = 0$$

$$\Rightarrow 0 = v_i \sin\theta_i - g t_{max}$$

$$\Rightarrow t_{max} = \frac{v_i \sin\theta_i}{g}$$

$$\therefore t_{fleight} = \frac{2 v_i \sin\theta_i}{g}$$



- Time of flight is twice the time required to reach to the max. point. We call this Time-Of-flight and is true only if the projectile final destination is on the same level as its starting point.

Maximum Height of a Projectile

- Maximum height of a projectile can be calculated using last equation:

at max. point, t is t_{max}

$$\because y_{max} = (v_i \sin\theta_i) t_{max} - \frac{1}{2} g \left[t_{max}\right]^2$$

$$\therefore y_{max} = (v_i \sin\theta_i) \frac{v_i \sin\theta_i}{g} - \frac{1}{2} g \left[\frac{v_i \sin\theta_i}{g}\right]^2$$

$$or: h = (v_i \sin\theta_i) \frac{v_i \sin\theta_i}{g} - \frac{1}{2} g \left[\frac{v_i \sin\theta_i}{g}\right]^2$$

$$\therefore h = \frac{v_i^{\,2} \sin\theta_i^{\,2}}{2g} \qquad (4.13)$$

Horizontal Range of a Projectile

- Horizontal Range of a projectile can be calculated using last equation:

$$\because t_{fleight} = \frac{2v_i \sin\theta_i}{g}$$

$$\because R = v_{xi} t_{fleight}$$

$$\therefore R = (v_i \cos\theta_i)\frac{2v_i \sin\theta_i}{g} = \frac{2v_i^{\,2} \sin\theta_i \cos\theta_i}{g}$$

$$\because \sin(2\theta) = 2\sin\theta\cos\theta$$

$$\therefore R = \frac{v_i^{\,2}\sin 2\theta_i}{g} \qquad (4.14)$$

- Not that R is Max. when θ = 45°. $R_{\max} = \frac{v_i^{2}}{g}$

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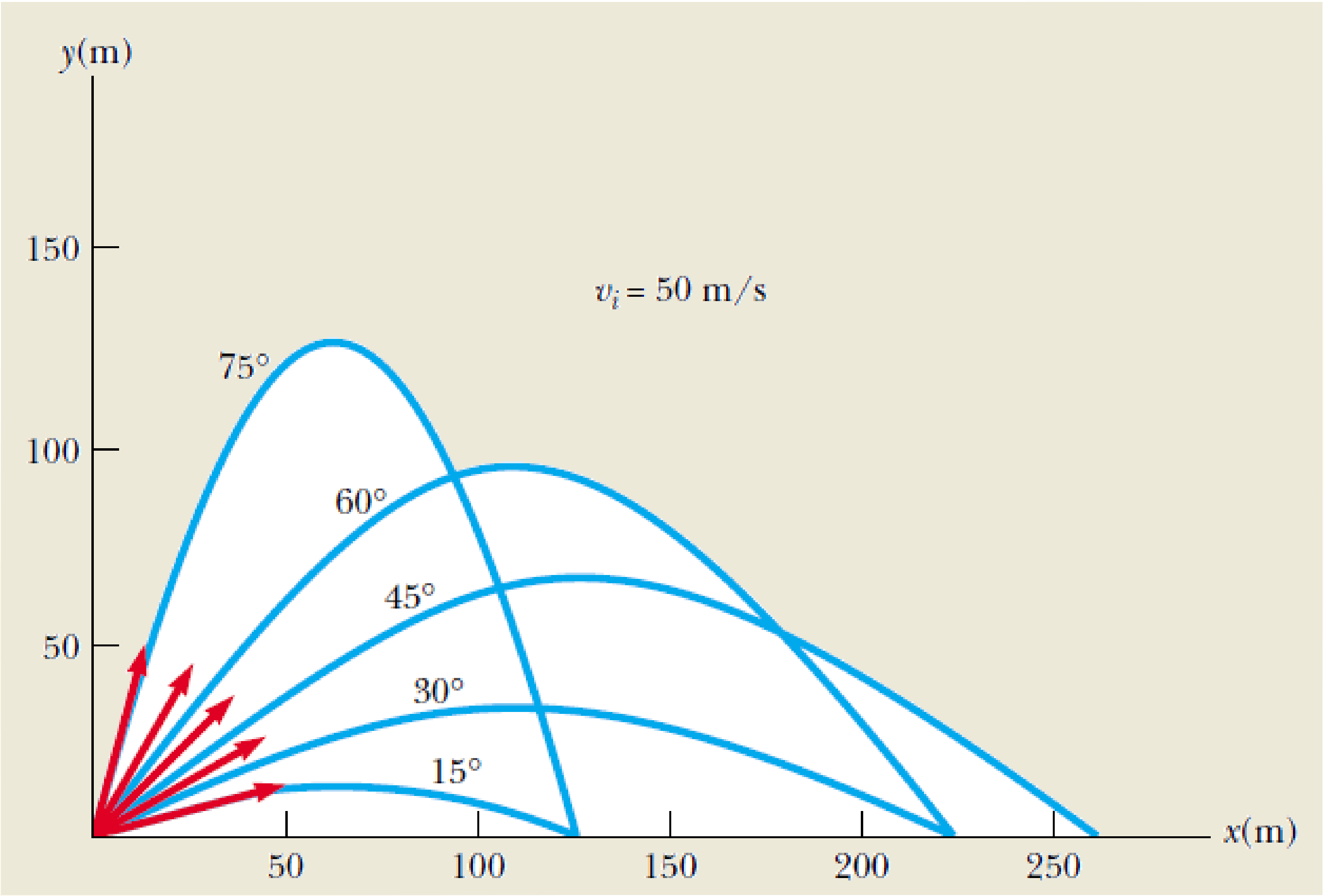
Effect of starting angle on a Projectile



A projectile launched from the origin with an initial speed of 50 m/s at various angles of projection.

Example 4.3 The Long Jump

- A long-jumper leaves the ground at an angle of 20.0° above the horizontal and at a speed of 11.0 m/s.
- (A) How far does he jump in the horizontal direction?

We can find the distance from Range (R):

$$\therefore R = \frac{v_i^{\,2} \sin 2\theta_i}{g} = \frac{11^2 \times \sin 40}{9.8} = 7.94\,m$$

- (B) What is the maximum height reached?

We can use the max. height equation directly:

$$\because h = \frac{v_i^{\,2} \sin \theta_i^{\,2}}{2g} = \frac{11^2 \sin 20^2}{2 \times 9.8} = 0.722\,m$$

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Example 4.5 That's Quite an Arm!

- A stone is thrown from the top of a building upward at an angle of 30.0° to the horizontal with an initial speed of 20.0 m/s, as shown in the Figure. If the height of the building is 45.0 m.
- (A) how long does it take the stone to reach the ground?

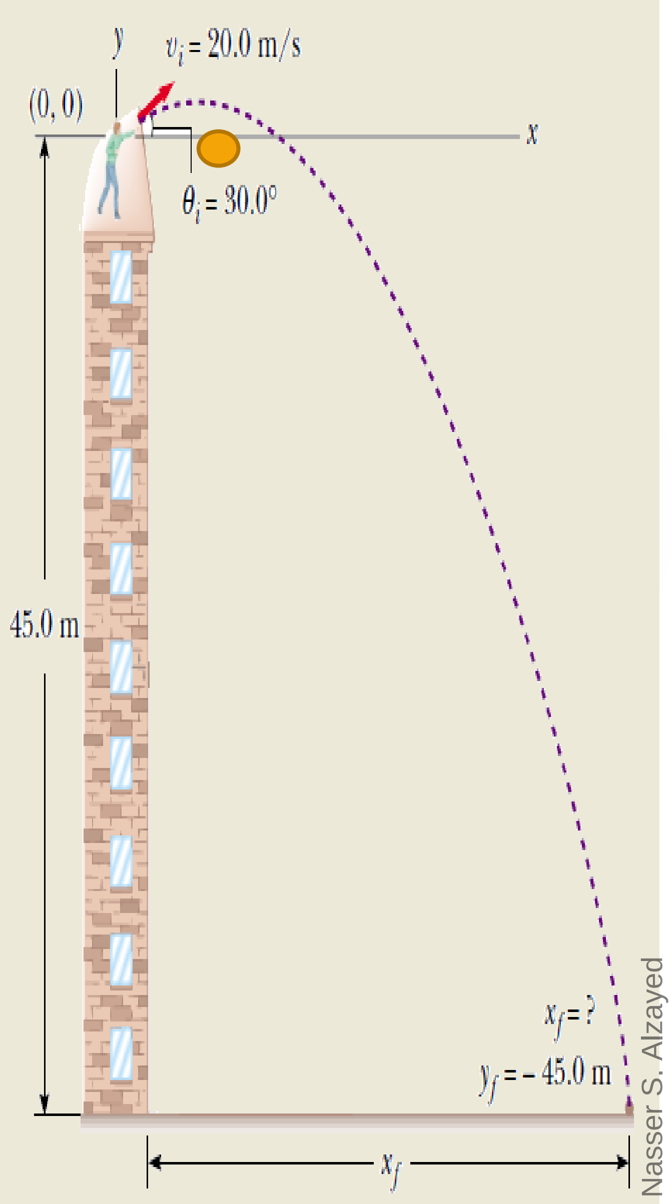
$$\because y_f = (v_i \sin\theta_i)t - \frac{1}{2}gt^2$$

$$\because v_i = 20\ m/s,\quad \theta_i = 30^o,\ y_f = -45\ m$$

$$\Rightarrow -45 = (20\sin 30)t - \frac{1}{2}\times 9.8t^2$$

$$\Rightarrow 4.9t^2 - 10t - 45 = 0$$

$$\text{Solving}: t = \frac{10 \pm \sqrt{100+882}}{9.8} = 4.22s$$



Example 4.5 (Continued)

- (B) What is the speed of the stone just before it strikes the ground?

To solve: we must find components of velocity (v_{xf} and v_{yf}) just at the ground level. Then we calculate the magnitude = speed

$$v_{xf} = v_{xi} = 20\cos 30 = 17.32\ m/s$$

$$\because v_{yf} = v_{yi} - gt = v_i \sin 30 - 9.8t$$

$$\therefore v_{yf} = 20\sin 30 - 9.8(4.22) = -31.36 m/s$$

$$\Rightarrow speed = v_f = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{17.32^2 + (-31.36)^2} = 35.9 m/s$$

- (C) What is the distance between the building and the striking point? $\because x_f = v_{xi}t = 20\cos 30(4.22) = 73.1m$

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Example 4.3 The Long Jump

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- (A) How far does he jump in the horizontal direction?

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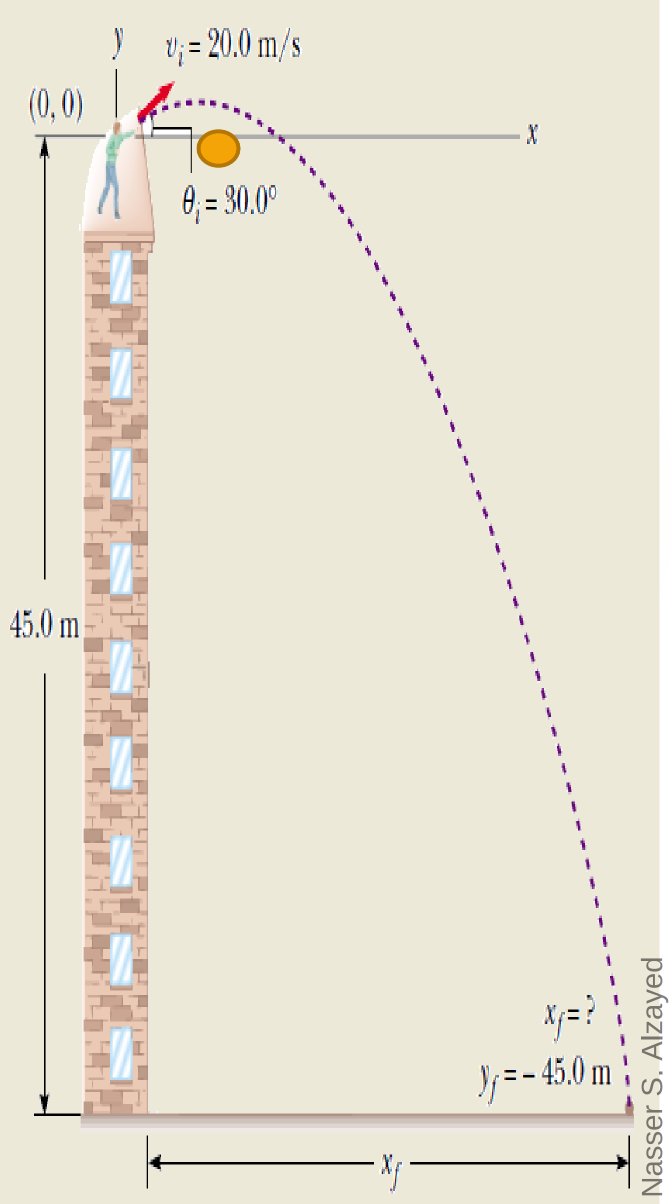
$$\text{Solving}: t = \frac{10 \pm \sqrt{100+882}}{9.8} = 4.22s$$



Example 4.5 (Continued)

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To solve: we must find components of velocity (v_{xf} and v_{yf}) just at the ground level. Then we calculate the magnitude = speed

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$$\Rightarrow speed = v_f = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{17.32^2 + (-31.36)^2} = 35.9 m/s$$

- (C) What is the distance between the building and the striking point? $\because x_f = v_{xi}t = 20\cos 30(4.22) = 73.1 m$

Example 4.6 a package dropped by airplane

A plane drops a package of supplies to a party of explorers. If the plane is traveling horizontally at 40.0 m/s and is 100 m above the ground, *where does the package strike the ground relative to the point at which it is released?*

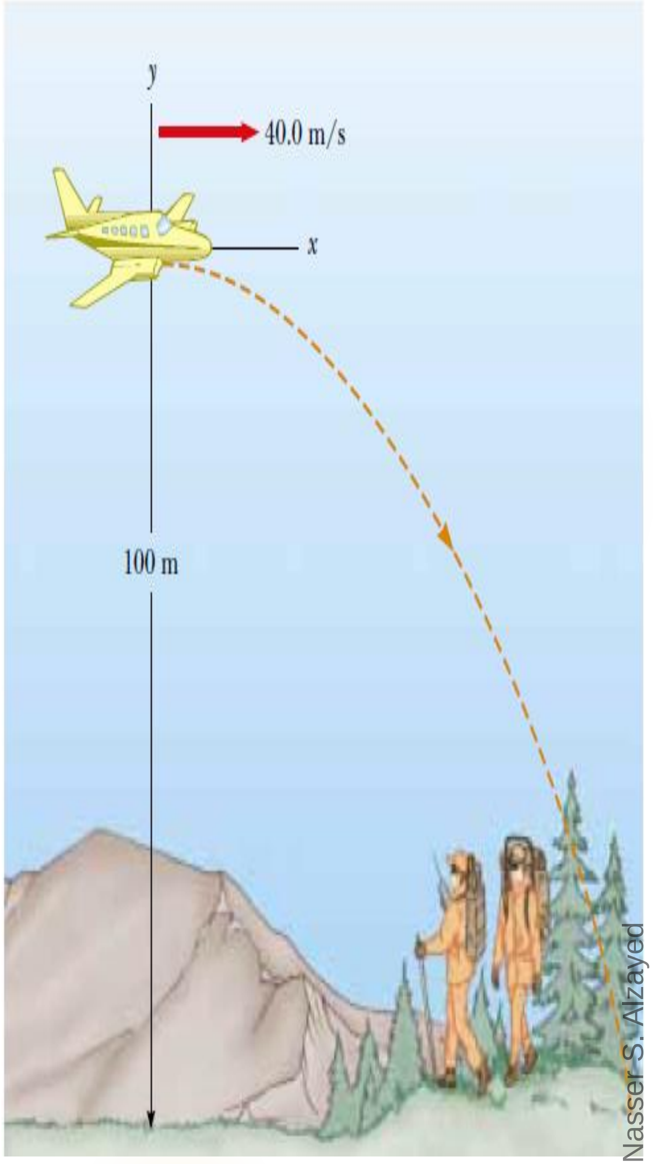
$$\because x_f = v_{xi} t = 40t \quad \text{(we need t)}$$

$$\because y_f = v_{yi} t - \frac{1}{2} g t^2$$

$$\therefore -100 = 0 - \frac{1}{2} \times 9.8 t^2$$

$$\Rightarrow t = \sqrt{\frac{2 \times 100}{9.8}} = 4.52 s$$

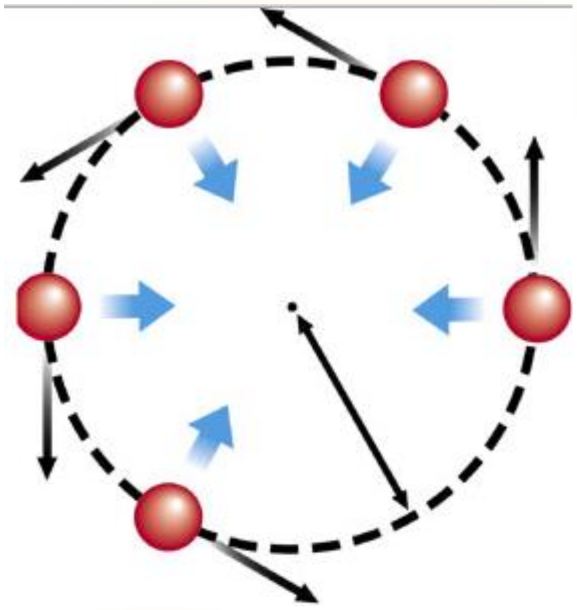
$$\therefore x_f = 40 \times 4.52 = 181 m$$



4.4 Uniform Circular Motion

- A *uniform circular motion* is of an object (e.g. a car) moving in a circular path with constant speed v.
- *Note: even though speed is constant: velocity is not.*
- We consider velocity not constant in three cases:
  1. Magnitude of v is changing
  2. Direction of v is changing
  3. Both of magnitude and direction are changing
- centripetal acceleration is given as follows:

$$a_c = \frac{v^2}{r} \qquad (4.15)$$



- where: *v* is the speed, *r* is the radius of circulation.
- Period of circulation:

$$T = \frac{2\pi r}{v} \qquad (4.16)$$

4.5 Tangential and Radial Acceleration

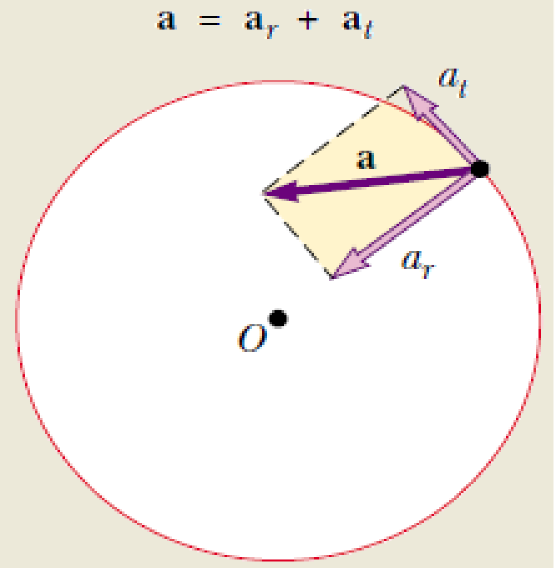
- In circular motion; there are 2 different accelerations: Radial: a_r and Tangential: a_t.
- Total Acceleration is the Vector sum of both of these 2 components.

$$a = a_r + a_t \qquad (4.17)$$

where:

$$a_t = \frac{dv}{dt} \qquad \text{and} \qquad a_r = -a_c = -\frac{v^2}{r}$$

$$\therefore a = \sqrt{a_r^{\,2} + a_t^{\,2}}$$



Quizzes 4.3-4.5

Quick Quiz 4.3 Suppose you are running at constant velocity and you wish to throw a ball such that you will catch it as it comes back down. In what direction should you throw the ball relative to you? (a) straight up (b) at an angle to the ground that depends on your running speed (c) in the forward direction.

Quick Quiz 4.4 As a projectile thrown upward moves in its parabolic path (such as in Figure 4.8), at what point along its path are the velocity and acceleration vectors for the projectile perpendicular to each other? (a) nowhere (b) the highest point (c) the launch point.

Quick Quiz 4.5 As the projectile in Quick Quiz 4.4 moves along its path, at what point are the velocity and acceleration vectors for the projectile parallel to each other? (a) nowhere (b) the highest point (c) the launch point.

Quizzes 4.6-4.8

Quick Quiz 4.6 Rank the launch angles for the five paths in Figure 4.11 with respect to time of flight, from the shortest time of flight to the longest.

Quick Quiz 4.7 Which of the following correctly describes the centripetal acceleration vector for a particle moving in a circular path? (a) constant and always perpendicular to the velocity vector for the particle (b) constant and always parallel to the velocity vector for the particle (c) of constant magnitude and always perpendicular to the velocity vector for the particle (d) of constant magnitude and always parallel to the velocity vector for the particle.

Quick Quiz 4.8 A particle moves in a circular path of radius r with speed v. It then increases its speed to $2v$ while traveling along the same circular path. The centripetal acceleration of the particle has changed by a factor of (a) 0.25 (b) 0.5 (c) 2 (d) 4 (e) impossible to determine

Quizzes 4.9-4.11

Quick Quiz 4.9 A particle moves along a path and its speed increases with time. In which of the following cases are its acceleration and velocity vectors parallel? (a) the path is circular (b) the path is straight (c) the path is a parabola (d) never.

Quick Quiz 4.10 A particle moves along a path and its speed increases with time. In which of the following cases are its acceleration and velocity vectors perpendicular everywhere along the path? (a) the path is circular (b) the path is straight (c) the path is a parabola (d) never.

Quick Quiz 4.11 A passenger, observer A, in a car traveling at a constant horizontal velocity of magnitude 60 mi/h pours a cup of coffee for the tired driver. Observer B stands on the side of the road and watches the pouring process through the window of the car as it passes. Which observer(s) sees a parabolic path for the coffee as it moves through the air? (a) A (b) B (c) both A and B (d) neither A nor B.

Chapter Summary

- Displacement of a particle in 2-D is:

$$\Delta r = r_f - r_i \qquad (4.1)$$

- The average velocity is defined as:

$$\overline{\mathrm{v}} = \frac{\Delta r}{\Delta t} = \frac{r_f - r_i}{t_f - t_i} \qquad (4.2)$$

- instantaneous velocity:

$$v = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t} = \frac{dr}{dt} \qquad (4.3)$$

- The average acceleration is defined as:

$$\overline{\mathrm{a}} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} \qquad (4.4)$$

- the instantaneous acceleration:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \qquad (4.5)$$

Chapter Summary (continued)

- Constant Acceleration motion of a particle in 2-D:

$$v_{xf} = v_{xi} + a_x t \qquad v_{yf} = v_{yi} + a_y t$$

$$x_f = v_{xi} t + \frac{1}{2} a_x t^2 \qquad y_f = v_{yi} t + \frac{1}{2} a_y t^2$$

$$v_{xf}^2 = v_{xi}^2 + 2a_x x_f \qquad v_{yf}^2 = v_{yi}^2 + 2a_y y_f$$

- Velocity and position in Vector form in 2-D motion:

$$\because v = v_x \hat{i} + v_y \hat{j}$$

$$\therefore v_f = (v_{xi} + a_x t)\hat{i} + (v_{yi} + a_y t)\hat{j}$$

$$= (v_{xi}\hat{i} + v_{yi}\hat{j}) + (a_x \hat{i} + a_y \hat{j})\mathrm{t}$$

$$= v_i + at$$

$$r_f = (v_{xi} t + \frac{1}{2} a_x t^2)\hat{i} + (v_{yi} t + \frac{1}{2} a_y t^2)\hat{j}$$

$$= (v_{xi}\hat{i} + v_{yi}\hat{j})t + \frac{1}{2}(a_x \hat{i} + a_y \hat{j})t^2$$

$$= v_i t + \frac{1}{2} a t^2$$

Lecture Summary

- Projectile motion is one type of two-dimensional motion under constant acceleration, where $a_x = 0$ and $a_y = -g$.
- It is useful to think of projectile motion as the superposition of two motions:
  - (1) constant-velocity motion in the x direction
  - (2) free-fall motion in the vertical direction subject to a constant downward acceleration of magnitude $g = 9.80\ m/s^2$.
- A particle moving in a circle of radius r with constant speed v is in uniform circular motion.
- If a particle moves along a curved path in such a way that both the magnitude and the direction of v change in time, then the particle has an acceleration vector that can be described by two component vectors: (1) a radial component vector a_r that causes the change in direction of v and (2) a tangential component vector a_t that causes the change in magnitude of v.

Thanks

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King Saud University
College of Science
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PHYS 103 (GENERAL PHYSICS)

CHAPTER 5: THE LAWS OF MOTION (PART 1)

THIS PRESENTATION HAS BEEN PREPARED BY: *DR. NASSR S. ALZAYED*

Chapter Outline

- Here is a quick list of the subjects that we will cover in this presentation. It is based on Serway, Ed. 6
- *5.1 The Concept of Force*
- *5.2 Newton's First Law and Inertial Frames*
- *5.3 Mass*
- *5.4 Newton's Second Law*
- *5.5 The Gravitational Force and Weight*
- *5.6 Newton's Third Law*
- *5.7 Applications on Newton's Laws*
  - *Objects in Equilibrium*
  - *Traffic Light at Rest*
  - *Weighing a Fish in an Elevator*
  - *The Atwood Machine*
  - *Acceleration of Two Objects Connected by a Cord*
- *5.8 Forces of Friction*
- *Examples and Quizzes*

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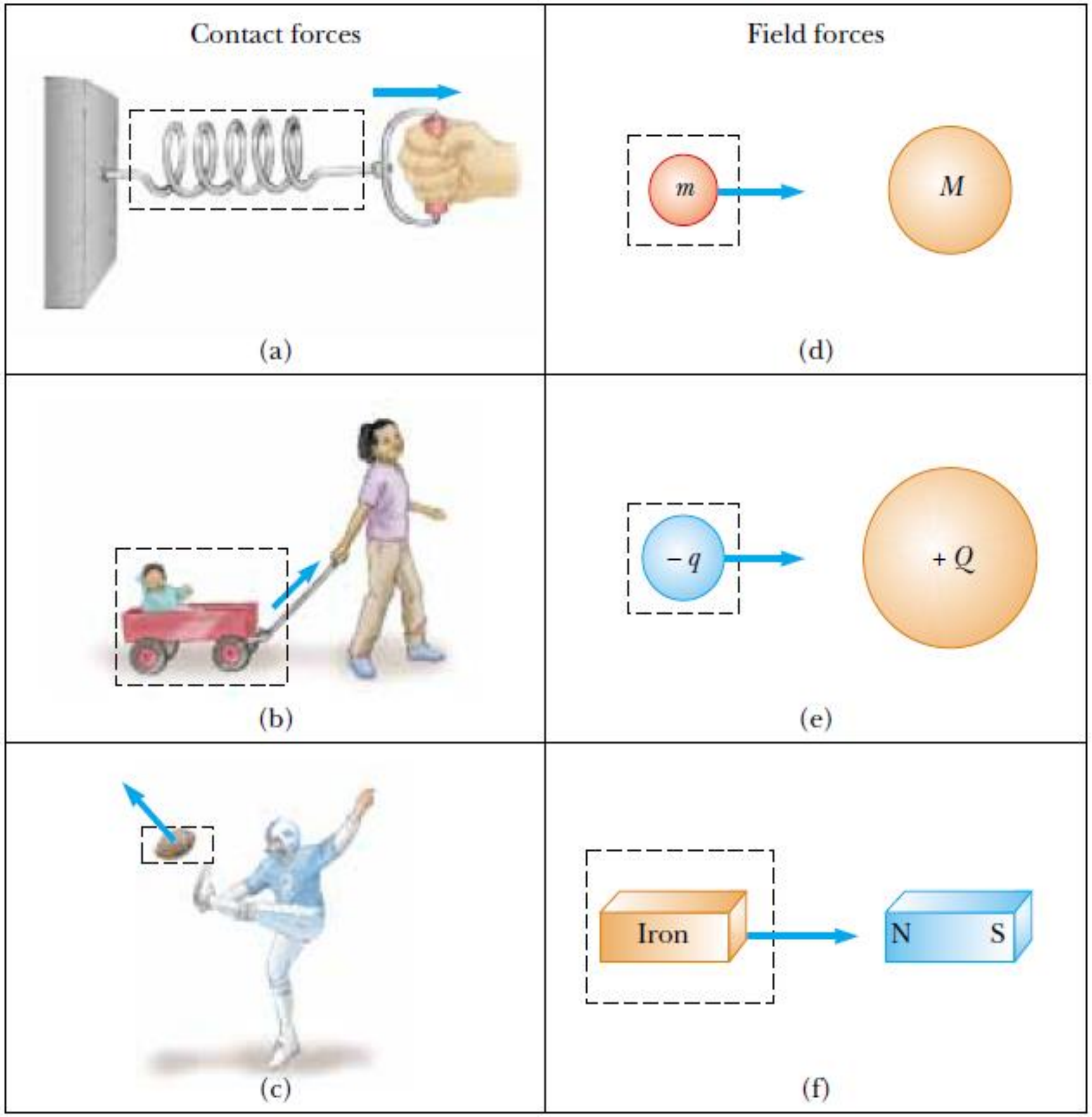
5.1 The Concept of Force

- An object accelerates due to an external force.
- If the net force exerted on an object is zero, the acceleration of the object is zero and its velocity remains constant.
- When the velocity of an object is constant (*including when the object is at rest*), the object is said to be in equilibrium.
- There are 2 types of forces:
  - Contact forces (e.g. when you pull a spring or press it)
  - Field forces (e.g. the force between earth and the moon)
- The only known *fundamental* forces in nature are all field forces: (1) *gravitational forces* between objects, (2) *electromagnetic forces* between electric charges, (3) *nuclear forces* between subatomic particles, and (4) *weak forces* that arise in certain radioactive decay processes.

5.1 Examples of Contact and Field forces



5.2 Newton's First Law

- *In the absence of external forces, when viewed from an inertial reference frame, an object at rest remains at rest and an object in motion continues in motion with a constant velocity.*
- In simpler terms, we can say that when no force acts on an object, the acceleration of the object is zero.
- If nothing acts to change the object's motion, then its velocity does not change.
- From the *first law*, we conclude that any isolated object (one that does not interact with its environment) is either at rest or moving with constant velocity.
- The tendency of an object to resist any attempt to change its velocity is called *inertia*.

Quiz 5.1

Quick Quiz 5.1 Which of the following statements is most correct? (a) It is possible for an object to have motion in the absence of forces on the object. (b) It is possible to have forces on an object in the absence of motion of the object. (c) Neither (a) nor (b) is correct. (d) Both (a) and (b) are correct.

5.3 Concept of Mass

- **Mass** is that property of an object that specifies how much resistance an object exhibits to changes in its velocity, the SI unit of mass is the kilogram.
- The greater the mass of an object, the less that object accelerates under the action of a given applied force.
- Mass is an inherent property of an object and is independent of the object's surroundings.
- Mass should not be confused with **weight**. Mass and weight are two different quantities. The weight of an object is equal to the magnitude of the gravitational force exerted on the object and varies with location.
- On the other hand, the mass of an object is the same everywhere: an object having a mass of 2 kg on the Earth also has a mass of 2 kg on the Moon

5.4 Newton's Second Law

- *Acceleration of an object is directly proportional to the force acting on it.*
- In mathematical form: we can write this law as:

$$\sum \mathbf{F} = m\mathbf{a} \qquad (5.2)$$

$$\Rightarrow$$

$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum F_z = ma_z \qquad (5.3)$$

- **Units of Mass, Acceleration, and Force[a]**

System of Units	Mass	Acceleration	Force
SI	kg	m/s^2	$N = kg \cdot m/s^2$
U.S. customary	slug	ft/s^2	$lb = slug \cdot ft/s^2$

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Quizzes 5.2, 5.3 and 5.4

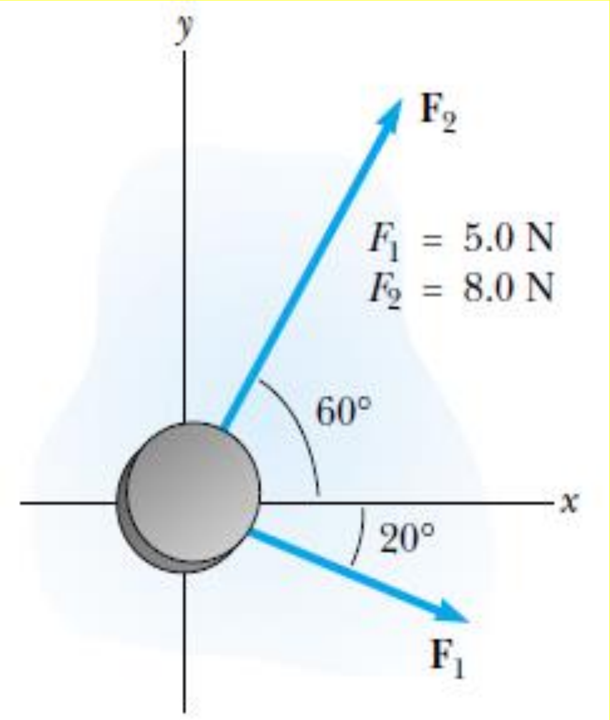
Quick Quiz 5.2 An object experiences no acceleration. Which of the following *cannot* be true for the object? (a) A single force acts on the object. (b) No forces act on the object. (c) Forces act on the object, but the forces cancel.

Quick Quiz 5.3 An object experiences a net force and exhibits an acceleration in response. Which of the following statements is *always* true? (a) The object moves in the direction of the force. (b) The acceleration is in the same direction as the velocity. (c) The acceleration is in the same direction as the force. (d) The velocity of the object increases.

Quick Quiz 5.4 You push an object, initially at rest, across a frictionless floor with a constant force for a time interval Δt, resulting in a final speed of v for the object. You repeat the experiment, but with a force that is twice as large. What time interval is now required to reach the same final speed v? (a) $4\Delta t$ (b) $2\Delta t$ (c) Δt (d) $\Delta t/2$ (e) $\Delta t/4$.

Example 5.1 An Accelerating Hockey Puck

A hockey puck having a mass of 0.30 kg slides on the horizontal, frictionless surface. Two hockey sticks strike the puck simultaneously, exerting the forces on the puck shown in the figure. The force $\boldsymbol{F}_1$ has a magnitude of 5.0 N, and the force $\boldsymbol{F}_2$ has a magnitude of 8.0 N. *Determine both the magnitude and the direction of the puck's acceleration*



Solution:

To analyze the problem, we resolve the force vectors into components.

We should find first x-component of the net force, then, y-component. We then find x and y comp. of the acceleration a and find its mag. And direction

Example 5.1 (Continued)

$$\sum F_x = F_{1x} + F_{2x}$$

$$= F_1 \cos(-20) + F_2 \cos(60) = 5 \times 0.94 + 8 \times 0.5 = 8.7\,N$$

$$\sum F_y = F_{1y} + F_{2y}$$

$$= F_1 \sin(-20) + F_2 \sin(60) = 5 \times -0.34 + 8 \times 0.87 = 5.2\,N$$

$$\therefore a_x = \frac{\sum F_x}{m} = \frac{8.7}{0.3} = 29\,m/s^2 \;,\; a_y = \frac{\sum F_y}{m} = \frac{5.2}{0.3} = 17\,m/s^2$$

$$\Rightarrow a = \sqrt{a_x{}^2 + a_y{}^2} = \sqrt{29^2 + 17^2} = 34\,m/s^2$$

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{17}{29}\right) = 30^o$$

5.5 The Gravitational Force and Weight

- The attractive force exerted by the Earth on an object is called the gravitational force $\mathbf{F}_g$
- This force is directed toward the center of the Earth,3 and its magnitude is called the weight of the object.
- Using equation (5.2) with $\boldsymbol{a} = \boldsymbol{g}$ we have:

$$\boldsymbol{F}_g = m\,\boldsymbol{g} \qquad (5.6)$$

- Thus: the weight of an object = mg
- Kilogram is Not a Unit of Weight: You may have seen the "conversion" 1 kg = 2.2 lb. Despite popular statements of weights expressed in kilograms, the kilogram *is not a unit of weight*, it is a unit of mass. The conversion statement is not an equality; it is an equivalence that is only valid on the surface of the Earth.

Quizzes 5.5, and 5.6

Quick Quiz 5.5 A baseball of mass m is thrown upward with some initial speed. A gravitational force is exerted on the ball (a) at all points in its motion (b) at all points in its motion except at the highest point (c) at no points in its motion.

Quick Quiz 5.6 Suppose you are talking by interplanetary telephone to your friend, who lives on the Moon. He tells you that he has just won a newton of gold in a contest. Excitedly, you tell him that you entered the Earth version of the same contest and also won a newton of gold! Who is richer? (a) You (b) Your friend (c) You are equally rich.

5.6 Newton's Third Law

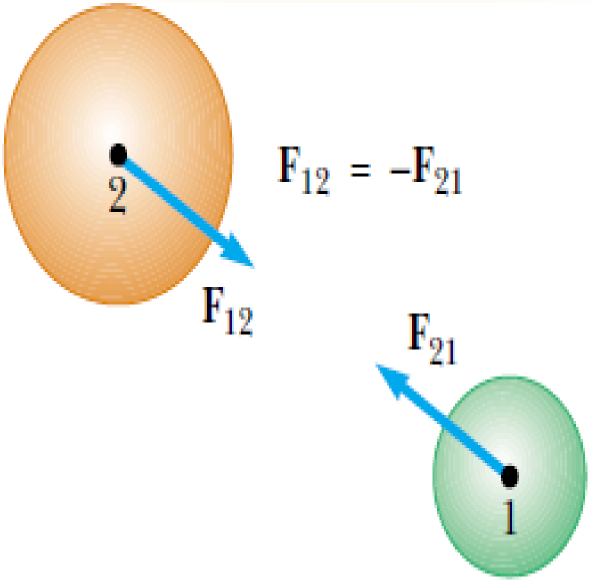
- *If two objects interact, the force F_{12} exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force F_{21} exerted by object 2 on object 1:*

$$\mathbf{F}_{12} = -\mathbf{F}_{21} \qquad (5.7)$$

where $\mathbf{F}_{ab}$ means "the force exerted by a on b." The third law, which is illustrated in the figure is equivalent to stating that forces always occur in pairs, or that a single isolated force cannot exist.

The force that object 1 exerts on object 2 may be called the *action* force and the force of object 2 on object 1 the *reaction* force.

The action force is equal in magnitude to the reaction force and opposite in direction.



5.7 Applications of Newton's Laws

- ***when we apply Newton's laws to an object, we are interested only in external forces that act on the object***
- **Objects in Equilibrium:**

If the acceleration of an object is zero, the particle is in **equilibrium**

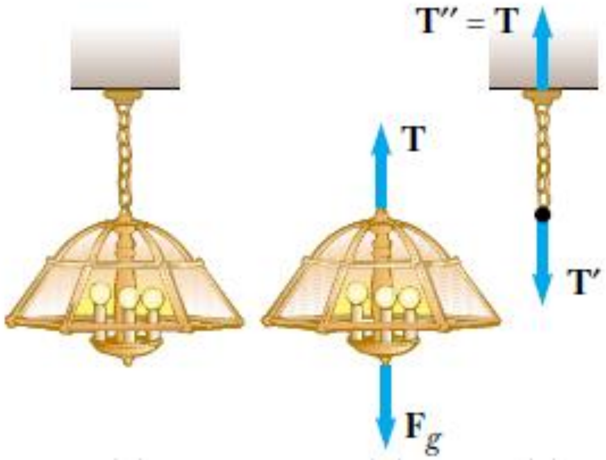
$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

- For example: a lamp hang by a robe from the ceiling, is in equilibrium because:

$$\sum F_y = T - mg = 0$$

$$\Rightarrow ma = 0 \;\; \Rightarrow a = 0$$

- A lamp suspended from a ceiling by a chain of negligible mass balanced Under the effect of two forces **T** and $\mathbf{F}_g$.



Quizzes 5.7, to 5.10

Quick Quiz 5.7 If a fly collides with the windshield of a fast-moving bus, which object experiences an impact force with a larger magnitude? (a) the fly (b) the bus (c) the same force is experienced by both.

Quick Quiz 5.8 If a fly collides with the windshield of a fast-moving bus, which object experiences the greater acceleration: (a) the fly (b) the bus (c) the same acceleration is experienced by both.

Quick Quiz 5.9 Which of the following is the reaction force to the gravitational force acting on your body as you sit in your desk chair? (a) The normal force exerted by the chair (b) The force you exert downward on the seat of the chair (c) Neither of these forces.

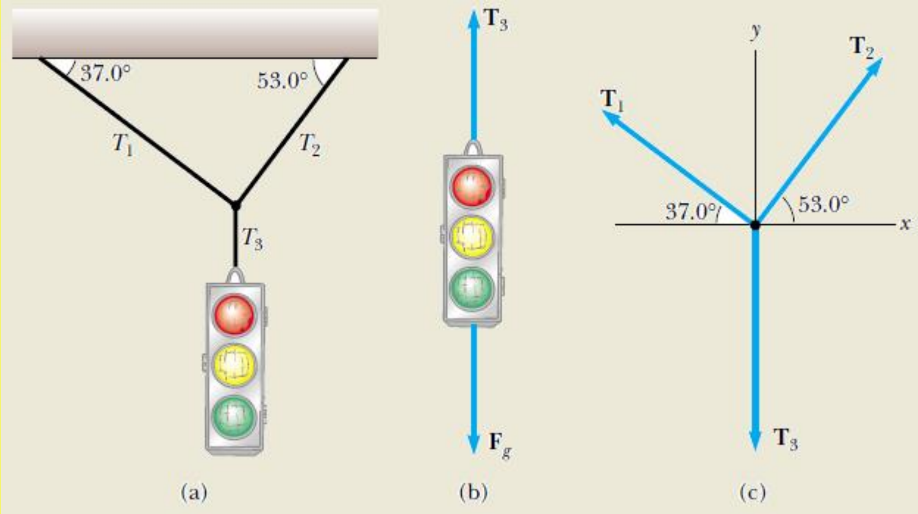
Quick Quiz 5.10 In a free-body diagram for a single object, you draw (a) the forces acting on the object and the forces the object exerts on other objects, or (b) only the forces acting on the object.

Example 5.4 A Traffic Light at Rest

- A traffic light weighing 122 N hangs from a cable tied to two other cables fastened to a support, as in Figure. The upper cables make angles of 37.0° and 53.0° with the horizontal. These upper cables are not as strong as the vertical cable, and will break if the tension in them exceeds 100 N. *Will the traffic light remain hanging in this situation, or will one of the cables break?*
- **Solution**:

We analyze forces as in the table Below.

Force	x Component	y Component
$\mathbf{T}_1$	$-T_1 \cos 37.0°$	$T_1 \sin 37.0°$
$\mathbf{T}_2$	$T_2 \cos 53.0°$	$T_2 \sin 53.0°$
$\mathbf{T}_3$	0	-122 N



Example 5.4 (continued)

- We should use the equilibrium conditions to solve this problem:

$$\sum F_x = 0 \qquad (1)$$

$$\sum F_y = 0 \qquad (2)$$

$$(1) \Rightarrow -T_1 \cos 37 + T_2 \cos 53 = 0 \qquad (3)$$

$$(2) \Rightarrow T_1 \sin 37 + T_2 \sin 53 - 122N = 0 \qquad (4)$$

$$(3) \Rightarrow T_2 = \frac{\cos 37}{\cos 53} T_1 = 1.33\ T_1 \qquad (5)$$

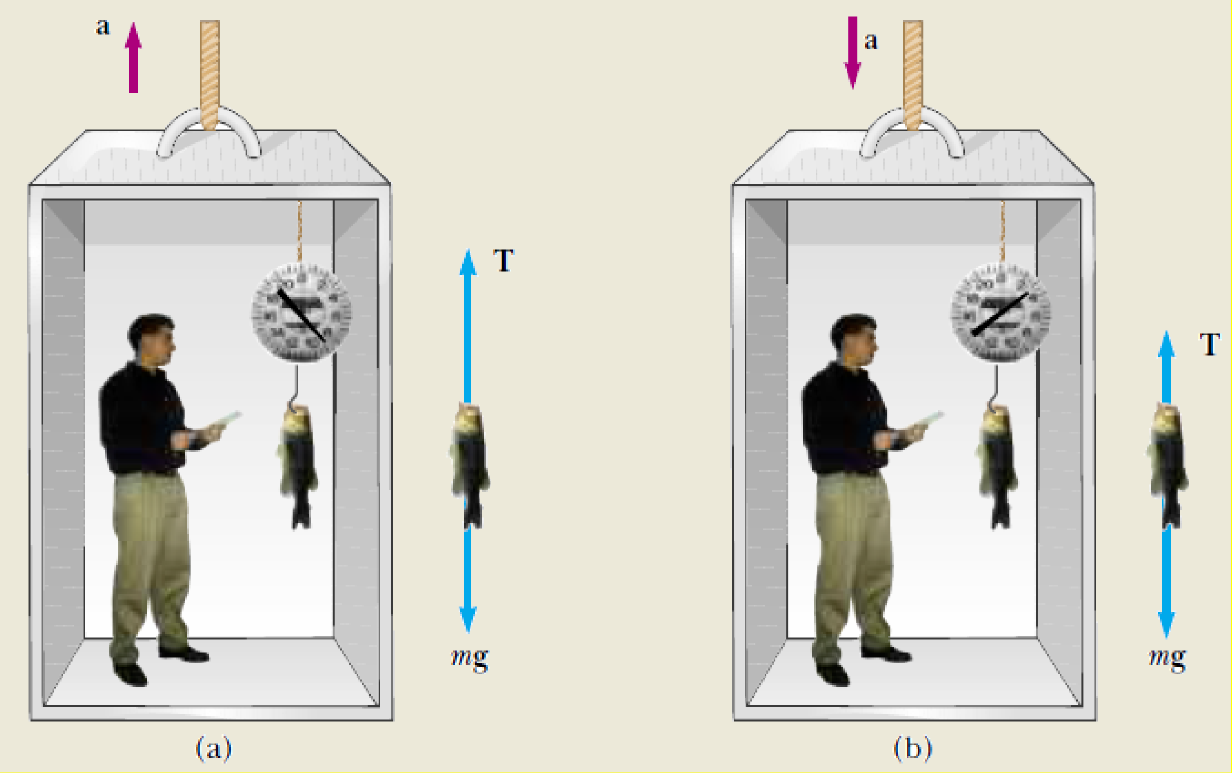
$$(5) \text{ in } (4) \Rightarrow T_1 \sin 37 + 1.33 \sin 53 T_1 = 122$$

$$\Rightarrow T_1 = 73.4 N \qquad (6)$$

$$(6) \text{ in } (5) \Rightarrow T_2 = 1.33 T_1 = 97.4 \text{ N}$$

Example 5.8 Weighing a Fish in an Elevator

- A person weighs a fish of mass m on a spring scale attached to the ceiling of an elevator, as illustrated in the figure. Show that if the elevator accelerates either upward or downward, the spring scale gives a reading that is different from the weight of the fish.



Example 5.8 Weighing a Fish in an Elevator

- **Solution**:
- We apply Newton 2nd law: $\mathbf{F}_{net} = m\mathbf{a}$

$$\sum F_y = T - mg = ma_y \qquad (1)$$

- Let us assume the weight of fish is: 40 N, and ay=±2 m/s^2

Case: $a_y = +2\ m/s^2$ (Upward):

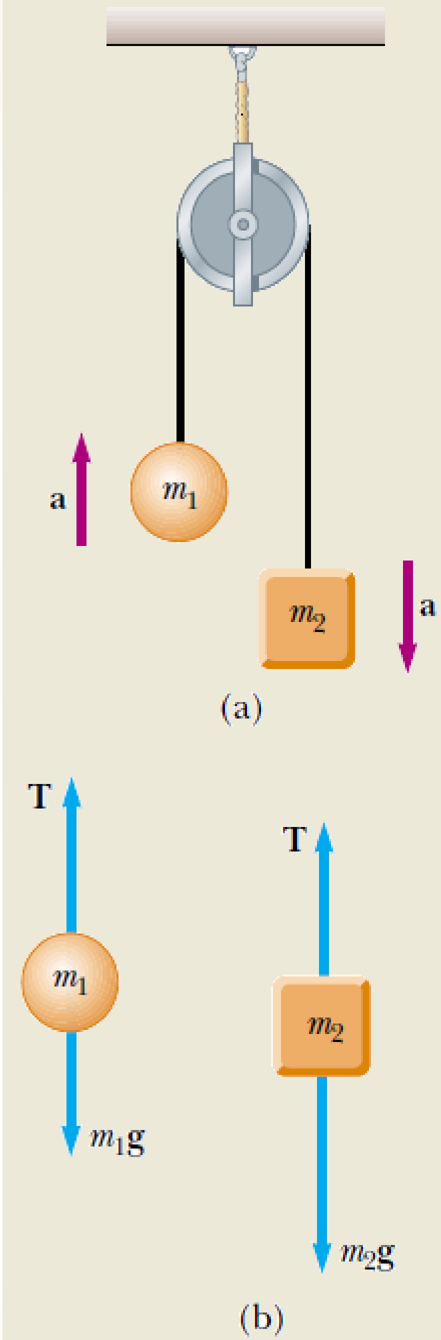
$$(1) \Rightarrow T = ma_y + mg = mg\left(\frac{a_y}{g} + 1\right) = 40\left(\frac{2}{9.8} + 1\right) = 48.2\,N$$

Case: $a_y = -2\ m/s^2$ (downward):

$$(1) \Rightarrow T = ma_y + mg = mg\left(\frac{-a_y}{g} + 1\right) = 40\left(\frac{2}{9.8} + 1\right) = 31.8\,N$$

Example 5.9 The Atwood Machine

- When two objects of unequal mass are hung vertically over a frictionless pulley of negligible mass, as in the figure, the arrangement is called an Atwood machine
- ***Solution***:
- We have in this example 2 objects. When we apply Newton's 2nd law, we get 2 equations (1for each object).
- We must assume a direction for the motion before we can setup the two equations.
- Let us assume Clockwise direction:
- Our strategy states that: *Net Force = ma* for each object. Please look at the free body diagram (b).



Example 5.9 (continued)

Solving for m_1 and m_2:

$$m_1: \sum F_y = T - m_1 g = m_1 a_y \quad (1)$$

$$m_2: \sum F_y = m_2 g - T = m_2 a_y \quad (2)$$

$$(1)+(2) \Rightarrow:$$

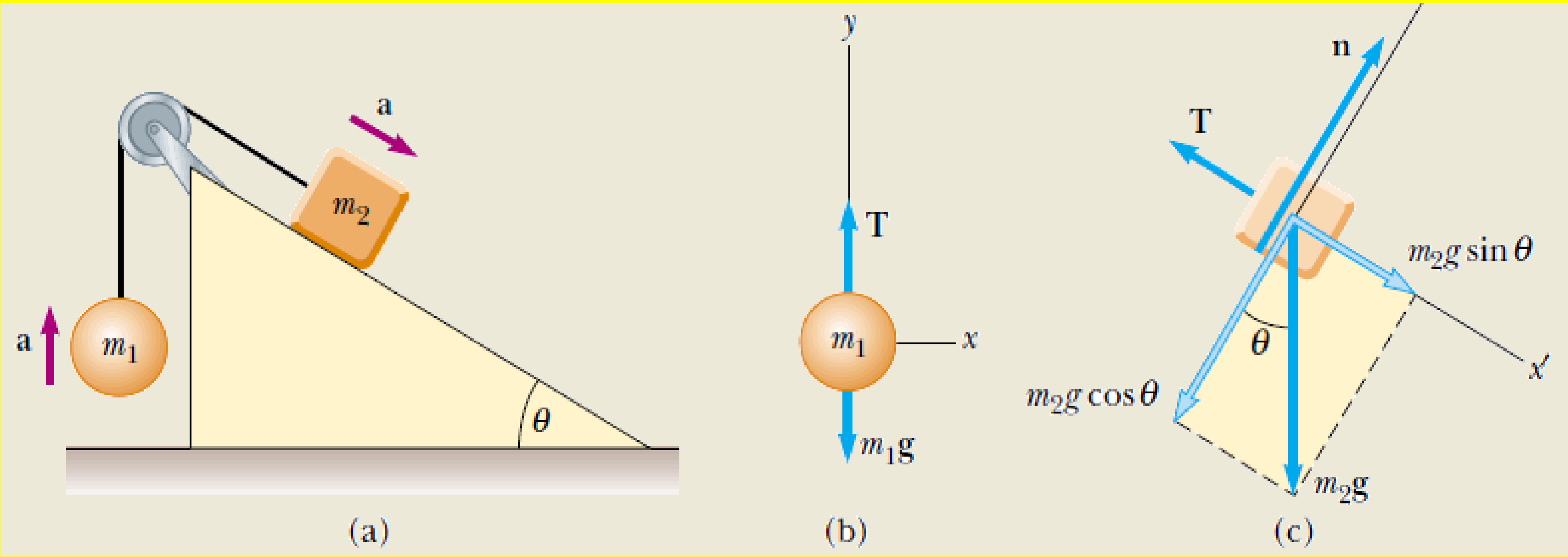
$$m_2 g - m_1 g = (m_1 + m_2) a_y$$

$$\Rightarrow a_y = \frac{m_2 g - m_1 g}{m_1 + m_2} \quad (3)$$

$$(3)\, in\, (1): T = \frac{2 m_1 m_2}{m_1 + m_2} g \quad (4)$$

Ex. 5.10 2 Obj. Connected by a Cord

- A ball of mass m_1 and a block of mass m_2 are attached by a lightweight cord that passes over a frictionless pulley of negligible mass, as in the figure . The block lies on a frictionless incline of angle θ. *Find the magnitude of the acceleration of the two objects and the tension in the cord.*



Ex. 5.10 2 Obj. Connected by a Cord

- Again: we have 2 bodies, thus we must have 2 equations. We call these equations: equations of motion. One equation is required for each body.
- We must also assume a direction for the motion. We select Clockwise.
- But first; we should analyze forces acting on each body.
  - Body m_1: T (up), m_1g (down)
  - Body m_2: from part (c) in previous figure: $m_2g\sin\theta$ (down the incline), T up the incline.
  - For : $m_2g\cos\theta$ component: this is not important unless there is a friction. We will get back to this issue when we consider the friction.
  - Also; the pulley is not considered now. If the pulley is not frictionless and thus rotates with motion; situation will be much different. In this case one more equation is to be added for the pulley. In this chapter; we always assume the pulley is frictionless.

Example 5.10 (continued)

$$m_1:\ T - m_1 g = m_1 a \qquad (1)$$

$$m_2:\ m_2 g \sin\theta - T = m_2 a \qquad (2)$$

$$(1)+(2):\ m_2 g \sin\theta - m_1 g = (m_1 + m_2)\,\mathrm{a}$$

$$\Rightarrow a = \frac{m_2 g \sin\theta - m_1 g}{m_1 + m_2} \qquad (3)$$

$$(3)\,\mathrm{in}\,(1):\ T - m_1 g = m_1\left(\frac{m_2 g \sin\theta - m_1 g}{m_1 + m_2}\right)$$

$$\Rightarrow T = \frac{m_1 m_2 g(\sin\theta + 1)}{m_1 + m_2} \qquad (4)$$

5.8 Forces of Friction

- When an object is in motion either on a surface or in a viscous medium such as air or water, there is resistance to the motion because the object interacts with its surroundings. We call such resistance a ***force of friction***
- There are two types of frictional forces:
  - Static: f_s and kinetic: f_k
- We define these two types as:

$$f_s = \mu_s n \qquad (1)$$

$$f_k = \mu_k n \qquad (1)$$

- μ_s is called coefficient of static friction, and μ_k is called coefficient of kinetic friction. $\mu_s > \mu_k$, $(0 \leq \mu \leq 1)$
- The direction of the friction force on an object is parallel to the surface with which the object is in contact and ***opposite*** to the actual motion.

Quizzes 5.1, to 5.13

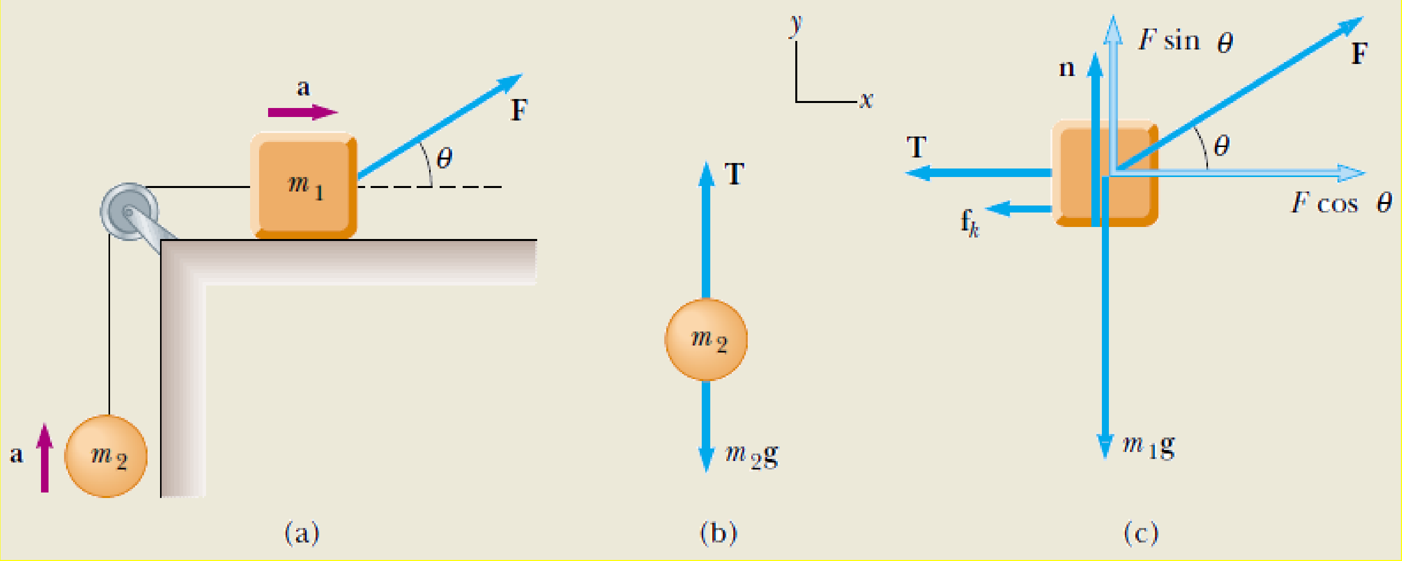
Quick Quiz 5.11 You press your physics textbook flat against a vertical wall with your hand. What is the direction of the friction force exerted by the wall on the book? (a) downward (b) upward (c) out from the wall (d) into the wall.

Quick Quiz 5.12 A crate is located in the center of a flatbed truck. The truck accelerates to the east, and the crate moves with it, not sliding at all. What is the direction of the friction force exerted by the truck on the crate? (a) to the west (b) to the east (c) No friction force exists because the crate is not sliding.

Quick Quiz 5.13 You place your physics book on a wooden board. You raise one end of the board so that the angle of the incline increases. Eventually, the book starts sliding on the board. If you maintain the angle of the board at this value, the book (a) moves at constant speed (b) speeds up (c) slows down (d) none of these.

Ex. 5.14 Two Connected Objects with Friction

- A block of mass m_1 on a rough, horizontal surface is connected to a ball of mass m_2 by a lightweight cord over a lightweight, frictionless pulley, as shown in the figure. A force of magnitude F at an angle θ with the horizontal is applied to the block as shown. The coefficient of kinetic friction between the block and surface is μ_k . *Determine the magnitude of the acceleration of the two objects*.



Example 5.14 (Continued)

- To solve, we use same steps in Example 5.9.

$$m_1: \quad F\cos\theta - f_k - T = m_1 a \qquad (1)$$

$$m_2: \quad T - m_2 g = m_2 a \qquad (2)$$

- It is our duty to find out abut $\boldsymbol{f}_k$.

$$\because \mathbf{f}_k = \mu_k n$$

$$\because n = m_1 g - F\sin\theta$$

$$\therefore \mathbf{f}_k = \mu_k (m_1 g - F\sin\theta) \qquad (3)$$

- Let use these values: m_1=10 kg, m_2=1 kg, μ_k =0.1, **F** = 30 N, θ =30^o .
- Using equations, (1), (2) and (3) we can find:
  - Acceleration: a
  - Tension: T

Example 5.14 (Continued)

- We can use these values to find:

$$\therefore f_k = \mu_k (m_1 g - F \sin\theta)$$

$$\Rightarrow f_k = 0.1(10\times 9.8 - 30\sin 30)$$

$$= 8.3\,\mathrm{N} \qquad (4)$$

$$(4)\,in\,(1):\ 30\times\cos 30 - 8.3 - T = 10a$$

$$\Rightarrow \quad 17.68 - T = 10a \qquad (5)$$

$$(2)\Rightarrow T - 1\times 9.8 = 1a \qquad (6)$$

$$(5)+(6){:}7.88=(11)\mathrm{a} \quad \Rightarrow a = 0.72\ m/s^2$$

$$\therefore T = 1\times 0.72 + 1\times 9.8 = 10.52\,N$$

Chapter Summary

- **Newton's first law** states that it is possible to find such a frame, or, equivalently, in the absence of an external force, when viewed from an inertial frame, an object at rest remains at rest and an object in uniform motion in a straight line maintains that motion.
- **Newton's second law** states that the acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass. The net force acting on an object equals the product of its mass and its acceleration: $\Sigma \mathbf{F} = m\boldsymbol{a}$. If the object is either stationary or moving with constant velocity, then the object is in equilibrium and the force vectors must cancel each other.
- **The gravitational force** exerted on an object is equal to the product of its mass (a scalar quantity) and the free-fall acceleration: $\mathbf{F}_g = mg$. The weight of an object is the magnitude of the gravitational force acting on the object.
- **Newton's third law** states that if two objects interact, the force exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force exerted by object 2 on object 1. Thus, an isolated force cannot exist in nature.

Chapter Summary

- **Newton's first law**: defined earlier.
- **Newton's second law**: defined earlier.
- **The gravitational force**: defined earlier.
- **Newton's third law**: defined earlier.
- **The maximum force of static friction** f_s,max between an object and a surface is proportional to the normal force acting on the object.
- In general, $f_s \leq \mu_s n$, where μ_s is the **coefficient of static friction** and n is the magnitude of the normal force.
- When an object slides over a surface, the direction of the force of kinetic friction f_k is **opposite** the direction of motion of the object relative to the surface and is also proportional to the magnitude of the normal force. The magnitude of this force is given by $f_k \leq \mu_k n$, where μ_k is the **coefficient of kinetic friction**.

Thanks

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PHYS 103 (GENERAL PHYSICS)

CHAPTER 6: THE LAWS OF MOTION (PART II)

THIS PRESENTATION HAS BEEN PREPARED BY: *DR. NASSR S. ALZAYED*

Chapter Outline

- Here is a quick list of the subjects that we will cover in this presentation. It is based on Serway, Ed. 6

- *Applications on Newton's Laws*
- *Newton's Second Law Applied to Uniform Circular Motion*
- *Example 6.2 The Conical Pendulum*
- *Example 6.4 What Is the Maximum Speed of the Car?*
- *Example 6.5 The Banked Exit Ramp*
- *Lecture Summary*
- *Interactive Quiz*
- *Interactive Flash*
- *End of Presentation*

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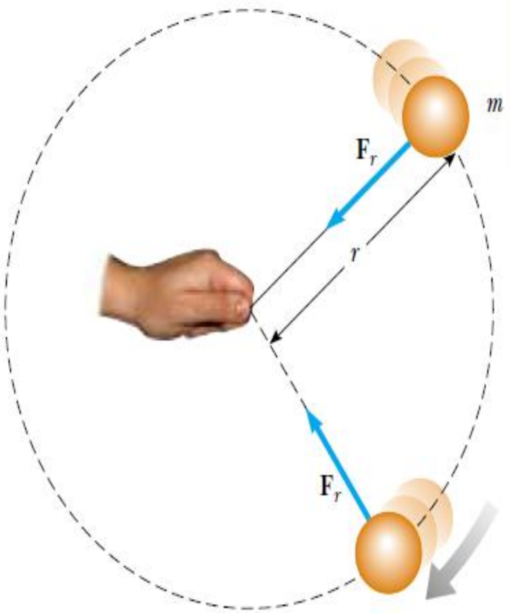
Newton's Second Law

- Applying Newton's Second Law to Uniform Circular Motion we get:

$$a_c = \frac{v^2}{r} \qquad \text{(centripetal acceleration)}$$

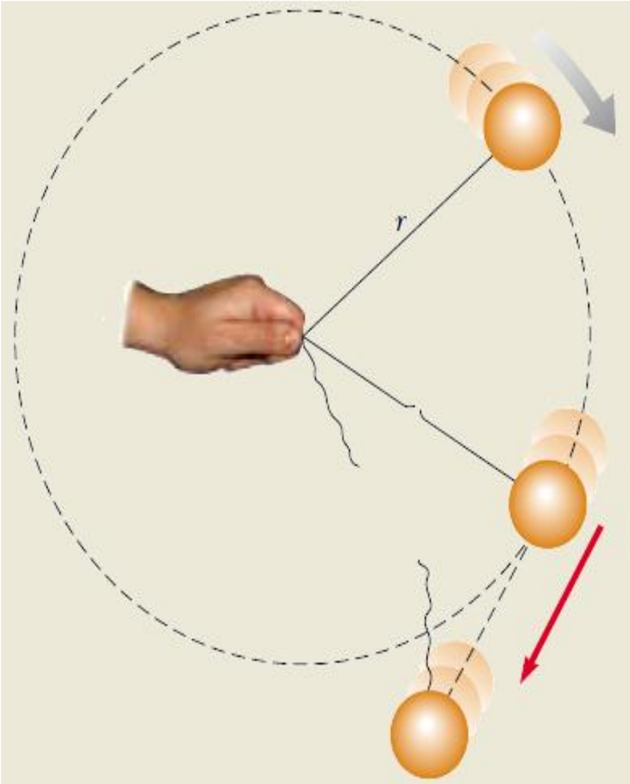
- The acceleration is called centripetal acceleration because ac is directed toward the center of the circle
- **a**$_c$ is always perpendicular to **v**
- If we apply Newton's second law along the radial direction, we find that the net force causing the centripetal acceleration can be evaluated:



- $$\sum \mathbf{F} = ma_c = m\frac{v^2}{r} \qquad (6.1)$$

Circular Motion

- A force causing a *centripetal acceleration* acts toward the *center of the circular path* and causes a change in the direction of the velocity vector.
- If that force should vanish, the object would no longer move in its circular path; instead, it would move along a straight-line path tangent to the circle.
- This idea is illustrated in the figure for the ball whirling at the end of a string in a horizontal plane. If the string breaks at some instant, the ball moves along the straight-line path tangent to the circle at the point where the string breaks.



Quick Quiz 6.1 and 6.2

Quick Quiz 6.1 You are riding on a Ferris wheel (Fig. 6.3) that is rotating with constant speed. The car in which you are riding always maintains its correct upward orientation—it does not invert. What is the direction of your centripetal acceleration when you are at the *top* of the wheel? (a) upward (b) downward (c) impossible to determine. What is the direction of your centripetal acceleration when you are at the *bottom* of the wheel? (d) upward (e) downward (f) impossible to determine.

Quick Quiz 6.2 You are riding on the Ferris wheel of Quick Quiz 6.1. What is the direction of the normal force exerted by the seat on you when you are at the *top* of the wheel? (a) upward (b) downward (c) impossible to determine. What is the direction of the normal force exerted by the seat on you when you are at the *bottom* of the wheel? (d) upward (e) downward (f) impossible to determine.

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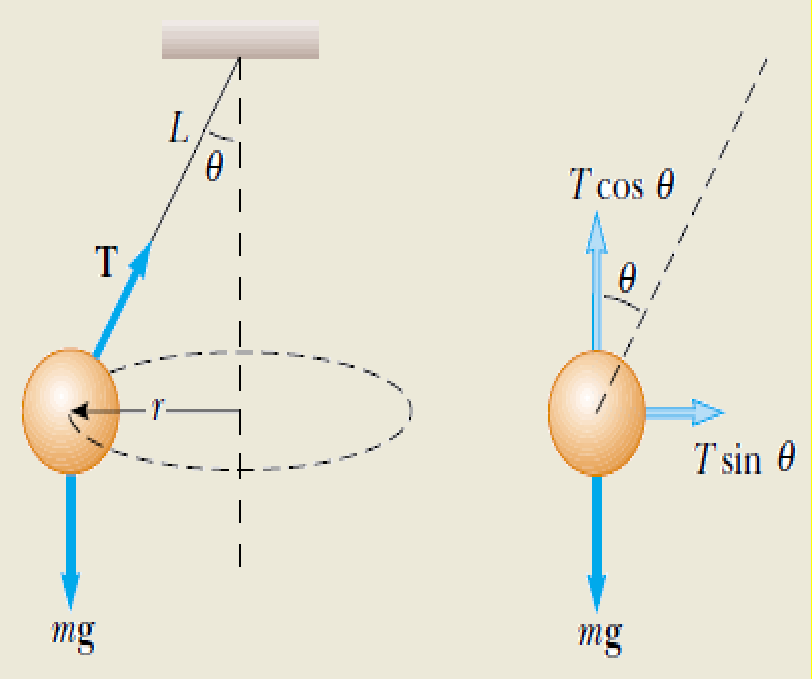
Example 6.2 The Conical Pendulum

- A small object of mass m is suspended from a string of length L. The object revolves with constant speed v in a horizontal circle of radius r, as shown in Figure. (Because the string sweeps out the surface of a cone, the system is known as a conical pendulum.) *Find an expression for v*.
- ***Solution***:
- We shall apply Newton's 2^{nd} law as we did before.
- We need first to analyze forces and apply the law in x, then y directions.

$$\sum \boldsymbol{F}_x = m a_x \qquad (1)$$

$$\sum \boldsymbol{F}_y = m a_y \qquad (2)$$



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Example 6.2 (continued)

- Solving we get:

$$(1) \Rightarrow T\ \cos\theta = mg \qquad (3)$$

$$(2) \Rightarrow T\ \sin\theta = \mathrm{ma_c} = m\frac{v^2}{r} \qquad (4)$$

$$(4) \div (3): \Rightarrow \frac{T\ \sin\theta}{T\ \cos\theta} = \frac{m\frac{v^2}{r}}{mg} \Rightarrow \tan\theta = \frac{v^2}{gr}$$

$$\therefore \mathrm{v} = \sqrt{gr\ \tan\theta} \qquad (5)$$

$$\because r = L\sin\theta$$

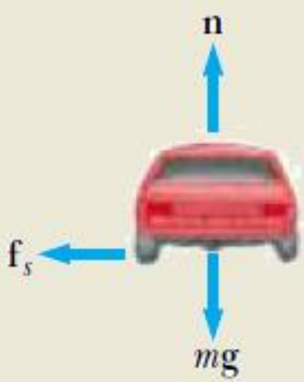
$$\therefore (5) \Rightarrow \mathrm{v} = \sqrt{Lg\sin\theta\tan\theta}$$

Example 6.4 A car on a Curve

- A 1 500-kg car moving on a flat, horizontal road negotiates a curve, as shown in Figure. If the radius of the curve is 35.0 m and the coefficient of static friction between the tires and dry pavement is 0.500, *find the maximum speed the car can have and still make the turn successfully.*

- ***Solution***: In this case, the force that enables the car to remain in its circular path is the force of *static friction*. (Static because no slipping occurs at the point of contact between road and tires.
- We shall apply Newton's 2nd law.

(a)



(b)

Example 6.4 (continued)

- Solving we get:

$$m = 1500\,kg,\ r = 35\,m,\ \mu_s = 0.5$$

$$\because \sum \mathbf{F_x} = m a_x \qquad (1)$$

$$\therefore f_s = m\frac{v^2}{r} \qquad (2)$$

$$\because f_s = \mu_s n = \mu_s (mg) \qquad (3)$$

$$\therefore \mu_s (\cancel{m} g) = \cancel{m}\frac{v^2}{r}$$

$$\Rightarrow v = \sqrt{r \mu_s g} \qquad (4)$$

$$\therefore v = \sqrt{(35)(0.5)(9.8)} = 13.1\,\text{m/s}$$

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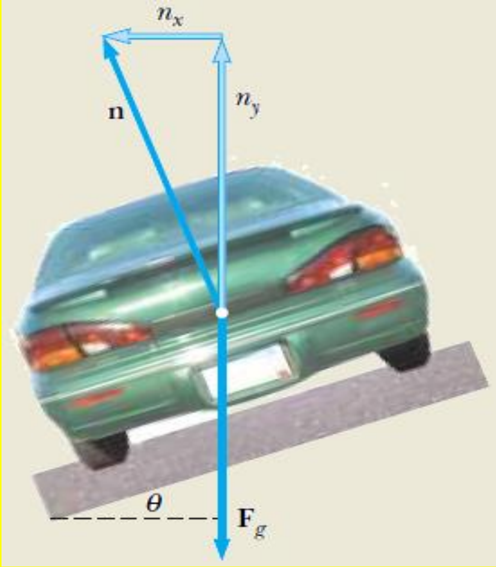
Example 6.5 The Banked Exit Ramp

- A civil engineer wishes to design a curved exit ramp for a highway in such a way that a car will not have to rely on friction to round the curve without skidding. In other words, a car moving at the designated speed can negotiate the curve even when the road is covered with ice. Such a ramp is usually *banked*. Suppose the designated speed for the ramp is to be 13.4 and the radius of the curve is 50.0 m. At what angle should the curve be banked?
- ***Solution***: We shall apply Newton's 2nd law:



$$\sum F_x = ma_x \qquad (1)$$

$$\sum F_y = ma_y \qquad (2)$$

Example 6.5 (continued)

- Solving we get:

$$r = 50\,m, v = 13.4\,m/s$$

$$(1) \Rightarrow n\sin\theta = m\frac{v^2}{r} \qquad (3)$$

$$(2) \Rightarrow n\cos\theta = mg \qquad (4)$$

$$(3) \div (4) \Rightarrow: \tan\theta = \frac{v^2}{gr}$$

$$\therefore \theta = \tan^{-1}\left(\frac{v^2}{gr}\right) = \tan^{-1}\left(\frac{13.4^2}{(50)(9.8)}\right) = 20.1^o$$

- A driver who attempts to negotiate the curve at a speed greater than 13.4 m/s has to depend on friction to keep from sliding up the bank.

Chapter Summary

- Newton's second law applied to a particle moving in uniform circular motion states that the net force causing the particle to undergo a centripetal acceleration is:

$$\sum \mathbf{F} = ma_c = m\frac{v^2}{r} \qquad (6.1)$$

- A particle moving in a uniform circular motion has the centripetal acceleration give by:

$$a_c = \frac{v^2}{r} \qquad \text{(centripetal acceleration)}$$

Thanks

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PHYS 103 (GENERAL PHYSICS)

CHAPTER 7: ENERGY AND ENERGY TRANSFER

Nasser S. Alzayed

THIS PRESENTATION HAS BEEN PREPARED BY: *DR. NASSR S. ALZAYED*

Chapter Outline

- Here is a quick list of the subjects that we will cover in this presentation. It is based on Serway, Ed. 6

Introduction

- The concept of energy is one of the most important topics in science and engineering.
- In everyday life, we think of energy in terms of fuel for transportation and heating, electricity for lights and appliances, and foods for consumption. However, these ideas do not really define energy
- Energy is present in the Universe in various forms. Every physical process that occurs in the Universe involves energy and energy transfers
- The notion of energy is more abstract, although we do have experiences with energy, such as running out of gasoline, or losing our electrical service if we forget to pay the utility bill.
- Our problem-solving techniques presented in earlier chapters were based on the motion of a particle. This was called the particle model. We begin our new approach by focusing our attention on a system and developing techniques to be used in a system model.

7.1 Systems and Environments

- In the system model, we focus our attention on a small portion of the Universe—the system—and ignore details of the rest of the Universe outside of the system.
- A valid system may:
  - *be a single object or particle*
  - *be a collection of objects or particles*
  - *be a region of space*
  - *vary in size and shape*
- As an example, imagine a force applied to an *object* in empty space. We can define the object as the system. The force applied to it is an influence on the system from the environment that acts across the system boundary
- We shall find that there are a number of mechanisms by which a system can be influenced by its environment. The first of these that we shall investigate is ***work***.

7.2 Work Done by a Constant Force

- The work W done on a system by an agent exerting a constant force on the system is the product of the magnitude **F** of the force, the magnitude **Δr** of the displacement of the point of application of the force, and cos θ, where θ is the angle between the force and displacement vectors:

$$W = F\Delta r\cos\theta \qquad (7.1)$$

- if θ = 90°, then W = 0 because cos 90° = 0
- If an applied force **F** is in the same direction as the displacement **Δr**, then θ = 0 and cos 0 = 1. In this case, Equation 7.1 gives:

$$W = F\Delta r$$

- Work is a *scalar quantity*, and its units are force multiplied by length. Therefore, the SI unit of work is the newton.meter (N. m). This combination of units is used so frequently that it has been given a name of its own: the *joule* (J).

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Example 7.1 Mr. Clean

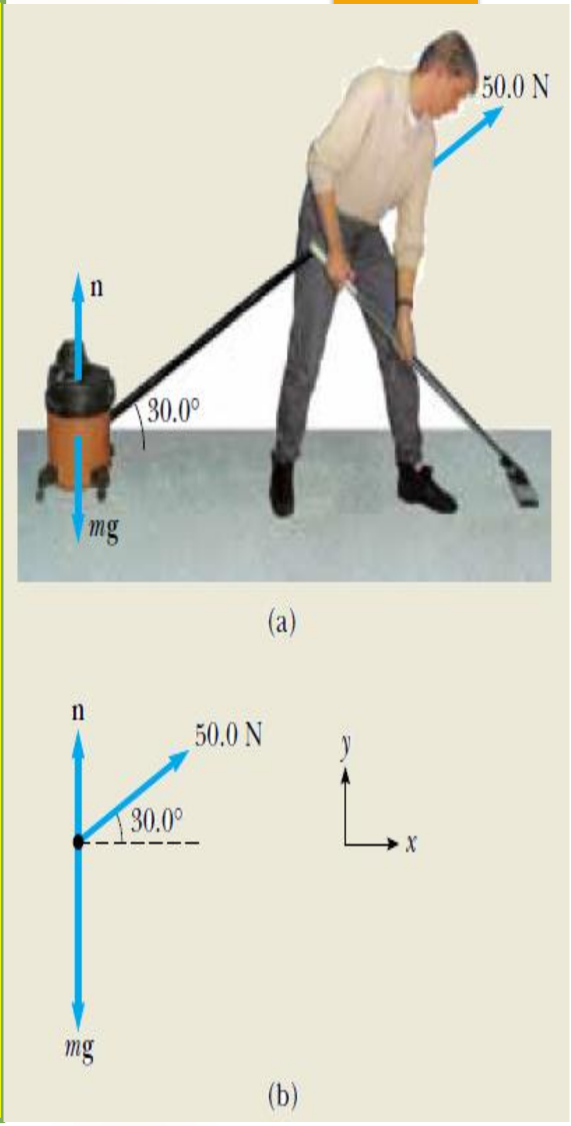
- A man cleaning a floor pulls a vacuum cleaner with a force of magnitude **F** = 50.0 N at an angle of 30.0° with the horizontal. Calculate the work done by the force on the vacuum cleaner as the vacuum cleaner is displaced 3.00 m to the right.
- ***Solution***:

$$F = 50\,N\,,\theta = 30^{o}\,,\Delta r = 3\,m$$

$$\because W = F\,\Delta r\cos\theta$$

$$\therefore W = (50)(3)(\cos 30) = 130\,J$$

- Notice: in this situation the normal force **n** and $\mathbf{F}_g$ = mg do no work because θ=90°.



Quiz 7.1 and 7.2

Quick Quiz 7.1 The gravitational force exerted by the Sun on the Earth holds the Earth in an orbit around the Sun. Let us assume that the orbit is perfectly circular. The work done by this gravitational force during a short time interval in which the Earth moves through a displacement in its orbital path is (a) zero (b) positive (c) negative (d) impossible to determine.

Quick Quiz 7.2 Figure 7.4 shows four situations in which a force is applied to an object. In all four cases, the force has the same magnitude, and the displacement of the object is to the right and of the same magnitude. Rank the situations in order of the work done by the force on the object, from most positive to most negative.

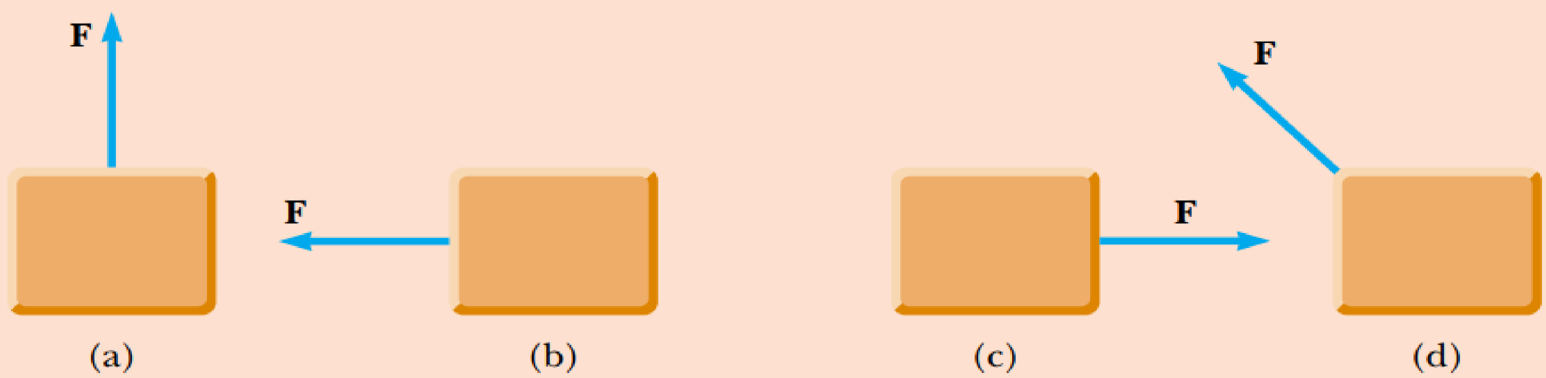


Figure 7.4 (Quick Quiz 7.2)

7.3 The Scalar Product of Two Vectors

- Because of the way the force and displacement vectors are combined in Equation 7.1, it is helpful to use a convenient mathematical tool called the scalar product of two vectors.
- In general; for any two vectors **A** and **B**; Scalar product is defined as:

$$A.B = AB\cos\theta \qquad (7.2)$$

$$\therefore W = F\Delta r\cos\theta = F.\Delta r \qquad (7.3)$$

- In other words, **F.Δr** (“F dot Δr ”) is a shorthand notation for *F*Δr *cos* θ.
- Please note that the scalar product is commutative. That is:

$$A.B = B.A$$

- Although (7.3) defines the work in terms of two vectors, *work is a scalar. All types of* energy and energy transfer are scalars. This is a major advantage of the energy approach. We don’t need vector calculations!

7.3 The Scalar Product of Two Vectors

Quick Quiz 7.3 Which of the following statements is true about the relationship between $\mathbf{A}\cdot\mathbf{B}$ and $(-\mathbf{A})\cdot(-\mathbf{B})$? (a) $\mathbf{A}\cdot\mathbf{B} = -[(-\mathbf{A})\cdot(-\mathbf{B})]$; (b) If $\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$, then $(-\mathbf{A})\cdot(-\mathbf{B}) = AB\cos(\theta + 180°)$; (c) Both (a) and (b) are true. (d) Neither (a) nor (b) is true.

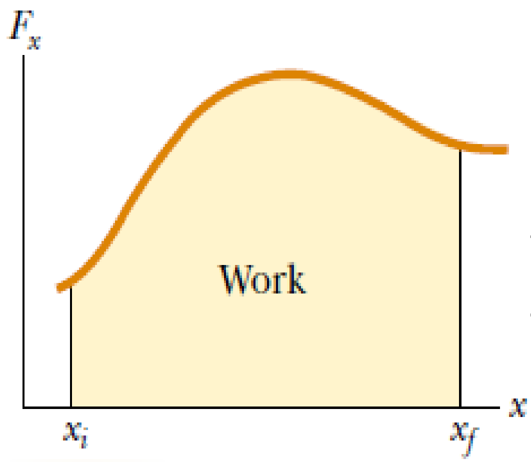
Quick Quiz 7.4 Which of the following statements is true about the relationship between the dot product of two vectors and the product of the magnitudes of the vectors? (a) $\mathbf{A}\cdot\mathbf{B}$ is larger than AB; (b) $\mathbf{A}\cdot\mathbf{B}$ is smaller than AB; (c) $\mathbf{A}\cdot\mathbf{B}$ could be larger or smaller than AB, depending on the angle between the vectors; (d) $\mathbf{A}\cdot\mathbf{B}$ could be equal to AB.

7.4 Work Done by a Varying Force

- If a force $\mathbf{F}_x$ is varying with position, x, we can express the work done by $\mathbf{F}_x$ as the particle moves from x_i to x_f as:

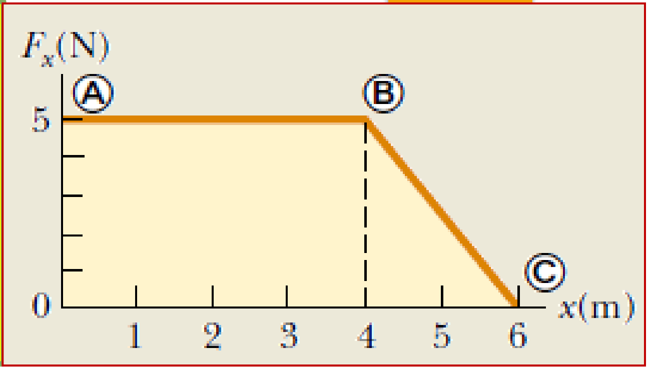
$$W = \int_{x_i}^{x_f} F_x dx \qquad (7.7)$$

- The work done by the component $\boldsymbol{F}_x$ *of* the varying force as the particle moves from x_i to x_f is exactly equal to the area under this curve.



Ex. 7.4 Calculating Work Done from a Graph

- A force acting on a particle varies with x, as shown in figure. Calculate the work done by the force as the particle moves from x = *0 to* x = *6.0 m.*



- ***Solution***:

$$\because W = \int_{x_i}^{x_f} F_x dx = area$$

$$\therefore W = Area_{A-B} + Area_{B-C} = (5\times 4) + (\frac{1}{2}\times 2\times 5)$$

$$\Rightarrow W = 20 + 5 = 25\ J$$

Work Done by a Spring

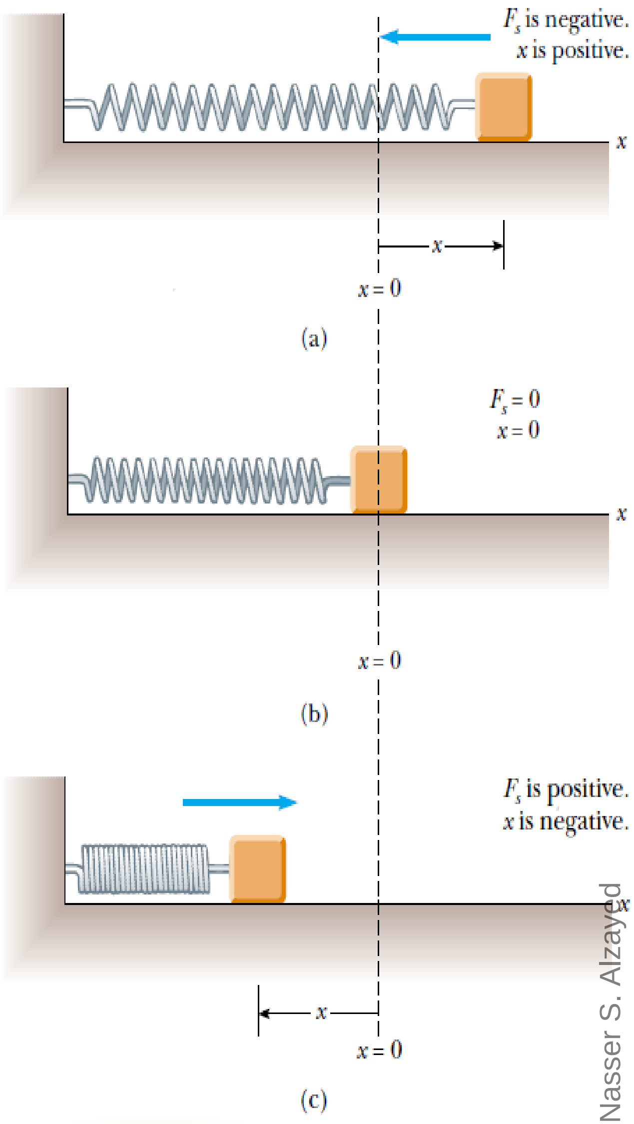
- If the spring is either stretched or compressed a small distance from its equilibrium configuration, it exerts on the block a force that can be expressed as:

$$F_s = -kx \qquad (7.9)$$

- This force law for springs is known as **Hooke's law**.

$$\because W = \int_{x_i}^{x_f} F_s dx = \int_{-x}^{0} (-kx) dx$$

$$\therefore W = \frac{1}{2} kx^2 \qquad (7.10)$$



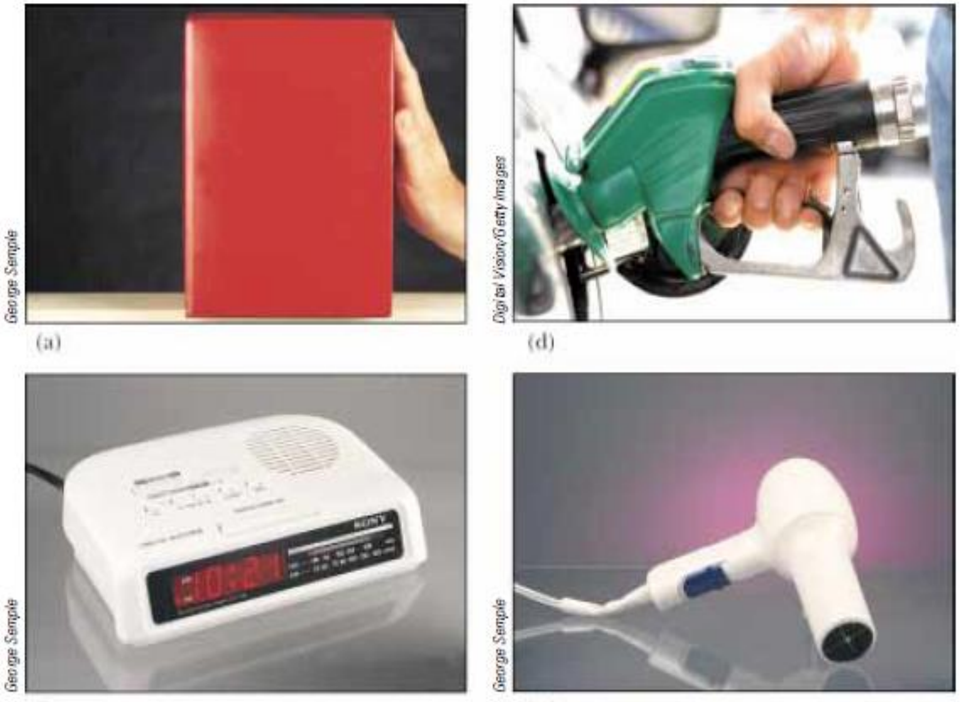
Work Done by a Spring

Quick Quiz 7.5 A dart is loaded into a spring-loaded toy dart gun by pushing the spring in by a distance d. For the next loading, the spring is compressed a distance $2d$. How much work is required to load the second dart compared to that required to load the first? (a) four times as much (b) two times as much (c) the same (d) half as much (e) one-fourth as much.

7.6 The Nonisolated System-Conservation of Energy

- A particle, that is acted on by various forces, resulting in a change in its kinetic energy is an example of nonisolated system.
- Another example: when a body slides on a surface, heat will be generated although kinetic energy of the surface has not changed.
- Methods of Energy Transfer:
  - Work
  - Mechanical Waves
  - Heat
  - Matter transfer
  - Electrical Transmission
  - Electromagnetic radiation

7.6 The Nonisolated System-Conservation of Energy



Example 7.6 Measuring k for a Spring

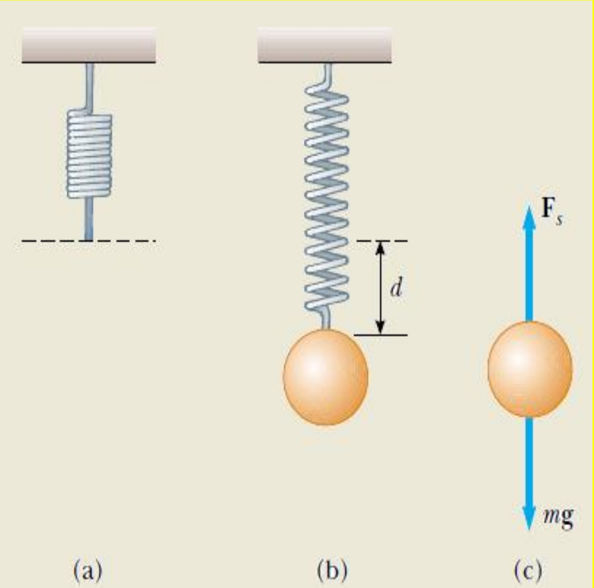
- A common technique used to measure the force constant of a spring is demonstrated by the setup in Figure. The spring is hung vertically, and an object of mass m is attached to its lower end. Under the action of the "load" mg, the spring stretches a distance d from its equilibrium position.
- *(A) If a spring is stretched 2.0 cm by a suspended object having a mass of 0.55 kg, what is the force constant of the spring?*
- ***Solution***:

$$m = 0.55\,kg\,,\; x = 2cm$$

$$\because |F_s| = kx = mg$$

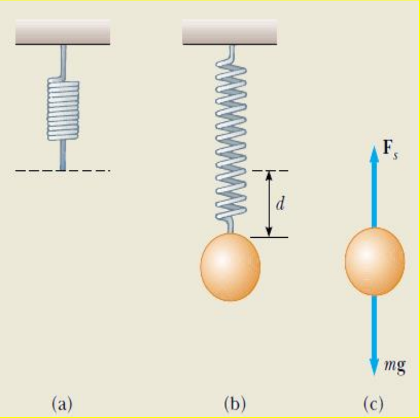
$$\therefore k\,(2\times10^{-2}) = 0.55\times9.8$$

$$\Rightarrow k = \frac{0.55\times9.8}{2\times10^{-2}} = 2.7\times10^{2}\ \text{N/m}$$



Example 7.6 Measuring k for a Spring

(B) How much work is done by the spring as it stretches through this distance?



Solution:

$$W_s = -\frac{1}{2}kx^2$$

$$W_s = -\frac{1}{2}(2.7\times 10^2)(0.02)^2 = -5.4\times 10^{-2} J$$

- tive sign is due to the fact that the spring is now loosing energy that will be subsides when get back to its original position.
- Or:

$$\because W = \int_{x_i}^{x_f} F_s dx = \int_0^{-x}(-kx)dx$$

$$\therefore W = -\frac{1}{2}kx^2$$

7.5 Kinetic Energy

- changes from initial value (K_i) to final value (K_f) so that:

$$W = K_f - K_i$$

$$\text{we define K as: } K = \frac{1}{2}mv^2$$

$$\therefore W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \qquad (7.14)$$

- the **work–kinetic energy theorem** is defined as:

$$W = K_f - K_i = \Delta K \qquad (7.15)$$

- This theorem indicates that the speed of a particle will **increase** if the net work done on it is **positive**, because the final kinetic energy will be greater than the initial kinetic energy. The speed will **decrease** if the net work is **negative**

Quiz 7.5 and 7.6

Quick Quiz 7.6 A dart is loaded into a spring-loaded toy dart gun by pushing the spring in by a distance d. For the next loading, the spring is compressed a distance $2d$. How much faster does the second dart leave the gun compared to the first? (a) four times as fast (b) two times as fast (c) the same (d) half as fast (e) one-fourth as fast.

7.6 The Nonisolated System-Conservation of Energy

- ***we can neither create nor destroy energy—energy is always conserved. Thus, if the total amount of energy in a system changes, it can only be due to the fact that energy has crossed the boundary of the system by a transfer mechanism such as one of the methods listed above. This is a general statement of the principle of conservation of energy.***

$$\Delta E_{system} = \sum T \qquad (7.17)$$

- Change in the total energy of the system

  = the amount of energy transferred across the system boundary by some mechanism

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7.7 Situations Involving Kinetic Friction

- Change in Kinetic energy is linked to the work done by a frictional force as:

$$-f_k d = \Delta K \qquad (7.20)$$

or :

$$\Delta E_{\text{int}} = f_k d \qquad (7.22)$$

- *the result of a friction force is to transform kinetic energy into internal energy, and the increase in internal energy is equal to the decrease in kinetic energy.*

Example 7.10 (Conceptual)

- A car traveling at an initial speed v slides a distance *d* to a halt after its brakes lock. Assuming that the car's initial speed is instead 2v at the moment the brakes lock, estimate the distance it slides

- ***Solution***:

$$\because -f_k d = \Delta K = \frac{1}{2}mv^2$$

$$-f_k d_1 = \frac{1}{2}mv^2$$

$$-f_k d_2 = \frac{1}{2}m4v^2$$

$$\rightarrow \frac{-f_k d_2}{-f_k d_1} = \frac{\frac{1}{2}m4v^2}{\frac{1}{2}mv^2}$$

$$\therefore d_2 = 4d_1$$

Quizes 7.7-7.10

Quick Quiz 7.7 By what transfer mechanisms does energy enter and leave (a) your television set; (b) your gasoline-powered lawn mower; (c) your hand-cranked pencil sharpener?

Quick Quiz 7.8 Consider a block sliding over a horizontal surface with friction. Ignore any sound the sliding might make. If we consider the system to be the *block*, this system is (a) isolated (b) nonisolated (c) impossible to determine.

Quick Quiz 7.9 If we consider the system in Quick Quiz 7.8 to be the *surface*, this system is (a) isolated (b) nonisolated (c) impossible to determine.

Quick Quiz 7.10 If we consider the system in Quick Quiz 7.8 to be the *block and the surface*, this system is (a) isolated (b) nonisolated (c) impossible to determine.

7.8 Power

- Average power is defined as:

$$\bar{p} = \frac{W}{\Delta t} \qquad (7.23)$$

- instantaneous power is:

$$p = \frac{dW}{dt}$$

$$\because dW = \boldsymbol{F}.d\boldsymbol{r}$$

$$\rightarrow p = \frac{\boldsymbol{F}.d\boldsymbol{r}}{dt} = \boldsymbol{F}.\frac{d\boldsymbol{r}}{dt} = \boldsymbol{F}.\boldsymbol{v} \qquad (7.23)$$

- instantaneous power is: *Applied force × velocity*
- The SI unit of power is joules per second (J/s), also called the watt (W)
- *Or horsepower: 1hp = 746 W*

Chapter Summary

- *The work W done on a system by an agent exerting a constant force on the system is the product of the magnitude **F** of the force, the magnitude Δr of the displacement of the point of application of the force, and cos θ, where θ is the angle between the force and displacement vectors:*

$$W = F\Delta r\cos\theta \qquad (7.1)$$

- *The scalar product (dot product) of two vectors A and B is defined by the relationship:*

$$A.B = AB\cos\theta \qquad (7.2)$$

- *If a force $\mathbf{F_x}$ is varying with position, x, we can express the work done by $\mathbf{F}_x$ as the particle moves from x_i to x_f as:*

$$W = \int_{x_i}^{x_f} F_x dx \qquad (7.7)$$

Chapter Summary (continued)

- The kinetic energy of a particle of mass m moving with a speed v is:

$$K = \frac{1}{2} m v^2 \qquad (7.14)$$

- The work–kinetic energy theorem states that if work is done on a system by external forces and the only change in the system is in its speed, then

$$W = K_f - K_i = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = \Delta K \qquad (7.14,16)$$

- Work Done by a Spring:

$$\because W = \int_{x_i}^{x_f} F_s dx = \int_{-x}^{0} (-kx) dx$$

$$\therefore W = \frac{1}{2} k x^2 \qquad (7.10)$$

Thanks

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PHYS 103 (GENERAL PHYSICS)

CHAPTER 8: POTENTIAL ENERGY

THIS PRESENTATION HAS BEEN PREPARED BY: *DR* ***NASSR S. ALZAYED***

Chapter Outline

- Here is a quick list of the subjects that we will cover in this presentation. It is based on Serway, Ed. 6
- *Introduction*
- *8.1 The System–Conservation of Energy*
- *8.2 Potential Energy of a System*
- *Potential Energy of a Spring*
- *Example 8.2 Ball in Free Fall*
- *Example 8.3 The Pendulum*
- *Example 8.5 The Spring-Loaded Popgun*
- *Chapter Summary*
- *End of Presentation*

Introduction

- The potential energy concept can be used only when dealing with a special class of forces called *conservative forces*.
- When only conservative forces act within an isolated system, the kinetic energy gained (or lost) by the system is balanced by an equal loss (or gain) in potential energy.
- This balancing of the two forms of energy is known as the *principle of conservation of mechanical energy*.
- Potential energy is present in the Universe in various forms, including gravitational, electromagnetic, chemical, and nuclear.
- Furthermore, one form of energy in a system can be converted to another. For example, when a system consists of an electric motor connected to a battery, the chemical energy in the battery is converted to kinetic
- Energy.

8.1 The System–Conservation of Energy

- Let us now derive an expression for the gravitational potential energy (U_g) associated with an object (m) at a given location (y) above the surface of the Earth

$$U_g = mgy \qquad (8.2)$$

- Mathematical description of the work done on a system that changes the gravitational pot. energy of the system is given by:

$$W = \Delta U_g \qquad (8.3)$$

- The gravitational potential energy depends only on the vertical height of the object above the surface of the Earth. The same amount of work must be done on an object–Earth system whether the object is lifted vertically from the Earth or is pushed starting from the same point up a frictionless incline, ending up at the same height

8.2 Potential Energy of a System

- As the book, shown in the figure, falls back to its original height, from y_b to y_a, the work done by the gravitational force on the book is:

$$W = mgy_b - mgy_a \quad (8.4)$$

$$= \Delta K$$

$$\therefore \Delta K = mgy_b - mgy_a \quad (8.5)$$

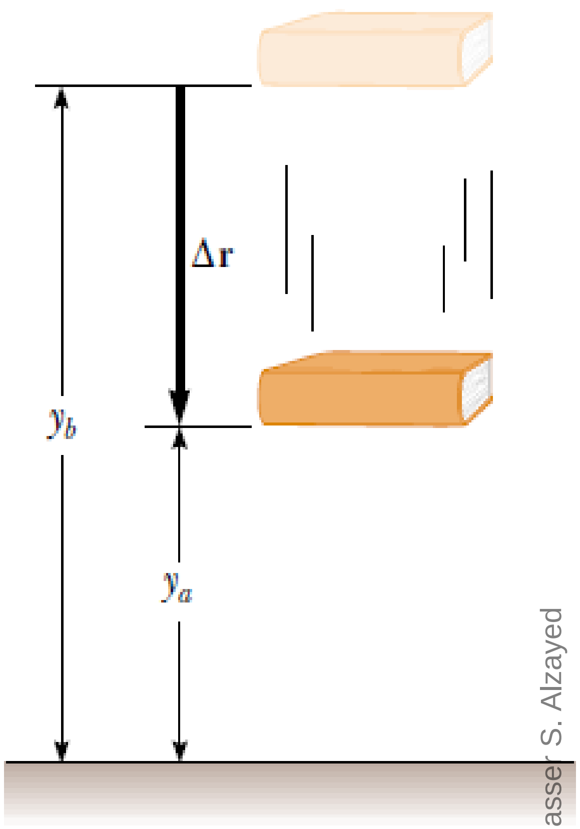
$$= -\Delta U_g \quad (8.6)$$

$$\therefore \Delta K + \Delta U_g = 0 \quad (8.7)$$

- ***Mechanical energy*** is defined as:

$$E_{mech} = K + U_g$$

Or, in general: $E_{mech} = K + U \quad (8.8)$



8.2 Potential Energy (continued)

- Let us now write the changes in energy in Equation 8.7 explicitly:

$$(K_f - K_i) + (U_f - U_i) = 0$$

$$\therefore \; K_f + U_f = K_i + U_i \qquad (8.9)$$

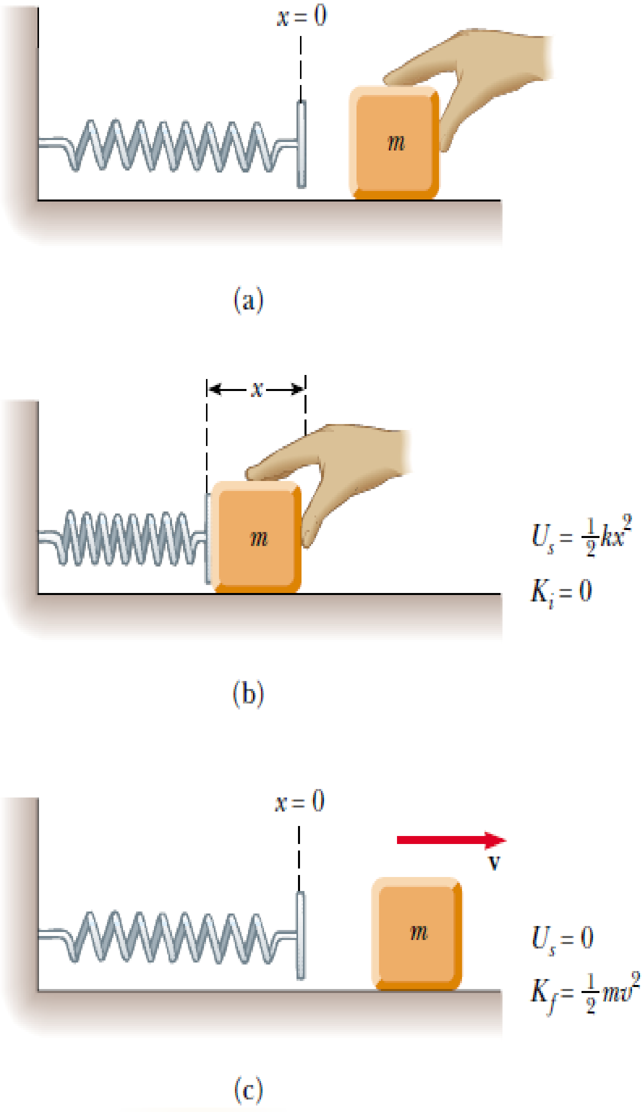
- Equation 8.9 is a statement of conservation of mechanical energy for an isolated system.
- An isolated system is one for which there are no energy transfers across the boundary.
- The energy in such a system is conserved—the sum of the kinetic and Potential energies remains constant.
- This statement assumes that no nonconservative forces act within the system

Potential Energy of a Spring

- Potential Energy of a Spring is given by:

$$U_s = \frac{1}{2}Kx^2 \qquad (8.11)$$

- When the block is released from rest, the spring exerts a force on the block and returns to its original length.
- The stored elastic potential energy is transformed into kinetic energy of the block.
- The ***elastic potential energy*** stored in a spring is zero when: x = 0
- Energy is stored in the spring only when the spring is either stretched or compressed.



Example 8.2 Ball in Free Fall

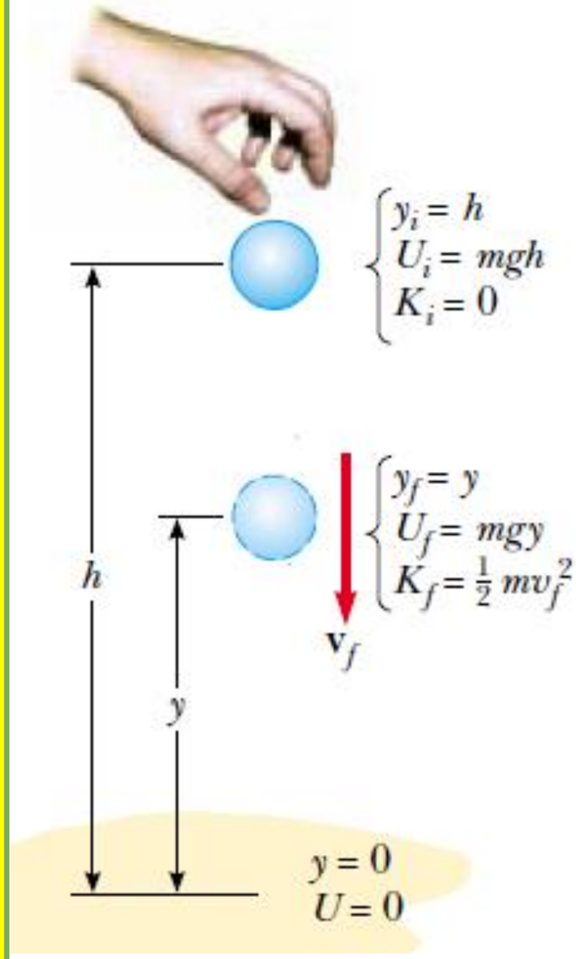
- A ball of mass m is dropped from a height h above the ground, as shown in Figure .
- *Neglecting air resistance, determine the speed of the ball when it is at a height y above the ground.*
- **Solution**:

$$\because K_f + U_f = K_i + U_i$$

$$\therefore \frac{1}{2} m v_f^{\,2} + mgy = 0 + mgh$$

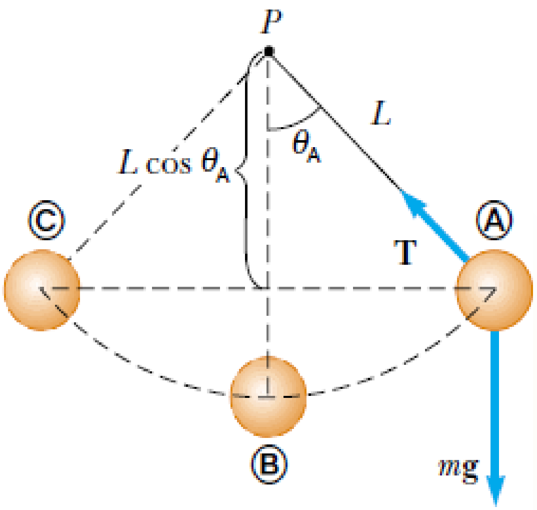
$$\Rightarrow v_f^{\,2} = 2g(h-y)$$

$$\therefore v_f = \sqrt{2g(h-y)}$$



Example 8.3 The Pendulum

- A pendulum consists of a sphere of mass m = 200 gm attached to a light cord of length L =50 cm, as shown in Figure. The sphere is released from rest at point A when the cord makes an angle θ_A =37° with the vertical.
- (A) Find the speed of the sphere when it is at the lowest point B.



- **Solution:**

$$\because K_B + U_B = K_A + U_A$$

$$\therefore \frac{1}{2} m v_B^2 - mgL = 0 - mgL\cos\theta_A$$

$$\therefore v_B = \sqrt{2gL(1-\cos\theta_A)} = \sqrt{2(9.8)(0.5)(1-\cos 37)} = 1.4\, m/s$$

Example 8.3 The Pendulum (continued)

- (B) What is the tension T_B in the cord at B?
- **Solution:**
- Newton's second law gives:

$$\sum F_r = T_B - mg = ma_r = m\frac{v_B^{\ 2}}{L}$$

$$\Rightarrow T_B = mg + m\frac{v_B^{\ 2}}{L}$$

$$\therefore T_B = 0.2 * 9.8 + 0.2\frac{1.98}{0.5}$$

$$\Rightarrow T_B = 1.96 + 0.79 = 2.75\,N$$

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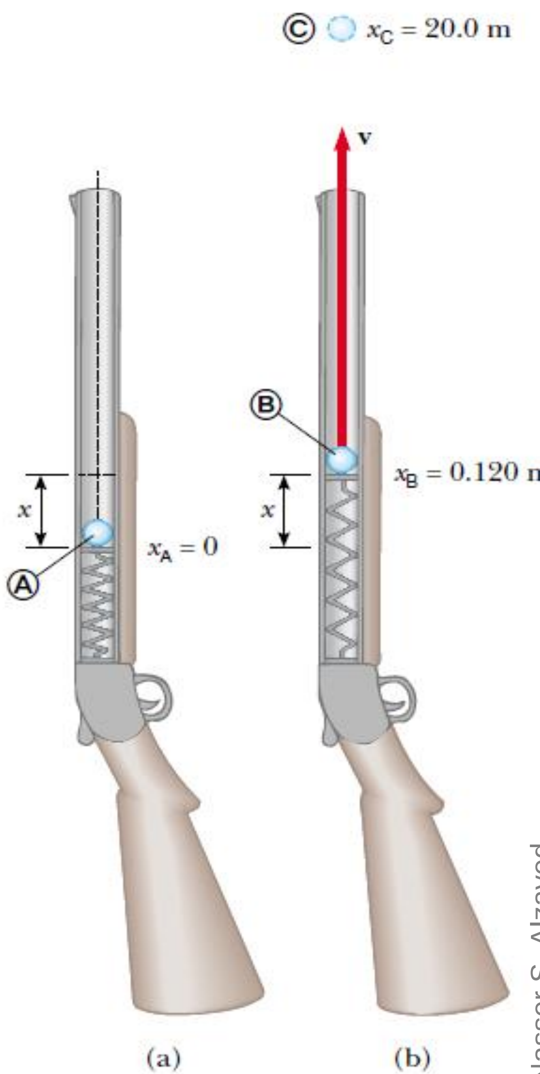
Example 8.5 The Spring-Loaded Popgun

- The launching mechanism of a toy gun consists of a spring of unknown spring constant. When the spring is compressed 0.120 m, the gun, when fired vertically, is able to launch a 35.0-g projectile to a maximum height of 20.0 m above the position of the projectile before firing.
- *(A) Neglecting all resistive forces, determine the spring constant.*
- **Solution**:
- Total energy at position (c) for the projectile + spring = Total energy at position (A)
- Hence: $[K_{projectile} + U_{projectile} + U_{spring}]_C$
- $= [K_{projectile} + U_{projectile} + U_{spring}]_A$



Example 8.5 (Continued)

$$\therefore K_C + U_{gC} + U_{sC} = K_A + U_{gA} + U_{sA}$$

$$\Rightarrow \quad 0 + mgh + 0 = 0 + 0 + \frac{1}{2}kx^2$$

$$\therefore k = \frac{2mgh}{x^2} = \frac{(2)(0.035)(9.8)(20)}{0.12^2} = 953\,\text{N/m}$$

- Find the speed of the projectile as it moves through the equilibrium position of the spring (where x_B = 0.120m).

$$\therefore K_B + U_{gB} + U_{sB} = K_A + U_{gA} + U_{sA}$$

$$\Rightarrow \quad \frac{1}{2}mv_B^2 + mgx_B + 0 = 0 + 0 + \frac{1}{2}kx^2$$

$$\therefore v_B = \sqrt{\frac{kx^2}{m} - 2gx_B} = \sqrt{\frac{(953)(0.12)^2}{0.035} - 2(9.8)(0.12)} = 19.7\,\text{m/s}$$

Quizzes 8.1 to 8.3

Quick Quiz 8.1 Choose the correct answer. The gravitational potential energy of a system (a) is always positive (b) is always negative (c) can be negative or positive.

Quick Quiz 8.2 An object falls off a table to the floor. We wish to analyze the situation in terms of kinetic and potential energy. In discussing the kinetic energy of the system, we (a) must include the kinetic energy of both the object and the Earth (b) can ignore the kinetic energy of the Earth because it is not part of the system (c) can ignore the kinetic energy of the Earth because the Earth is so massive compared to the object.

Quick Quiz 8.3 An object falls off a table to the floor. We wish to analyze the situation in terms of kinetic and potential energy. In discussing the potential energy of the system, we identify the system as (a) both the object and the Earth (b) only the object (c) only the Earth.

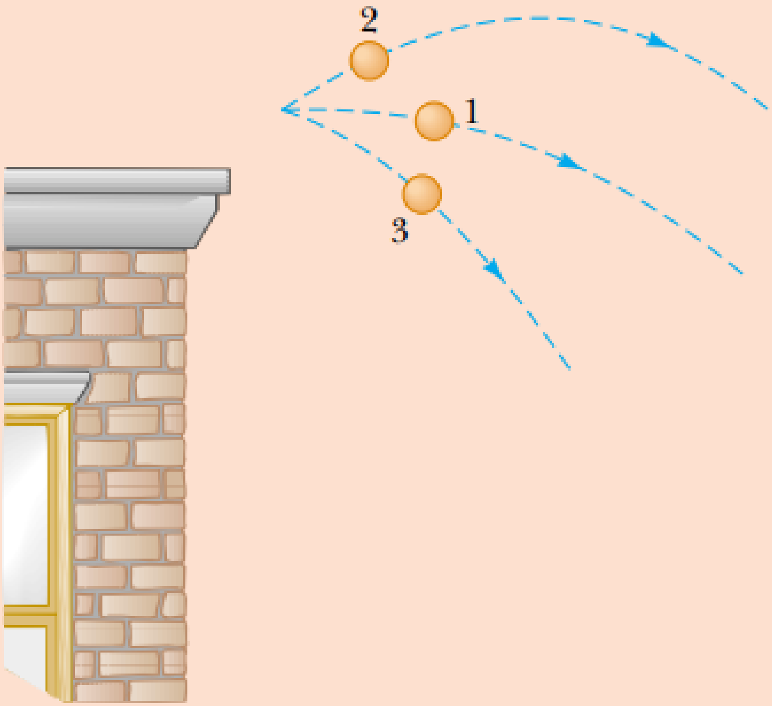
Quizzes 8.4 to 8.5

Quick Quiz 8.4 In an isolated system, which of the following is a correct statement of the quantity that is conserved? (a) kinetic energy (b) potential energy (c) kinetic energy plus potential energy (d) both kinetic energy and potential energy.

Quick Quiz 8.5 A rock of mass m is dropped to the ground from a height h. A second rock, with mass $2m$, is dropped from the same height. When the second rock strikes the ground, its kinetic energy is (a) twice that of the first rock (b) four times that of the first rock (c) the same as that of the first rock (d) half as much as that of the first rock (e) impossible to determine.

Quizz 8.6

Quick Quiz 8.6 Three identical balls are thrown from the top of a building, all with the same initial speed. The first is thrown horizontally, the second at some angle above the horizontal, and the third at some angle below the horizontal, as shown in Figure 8.3. Neglecting air resistance, rank the speeds of the balls at the instant each hits the ground.



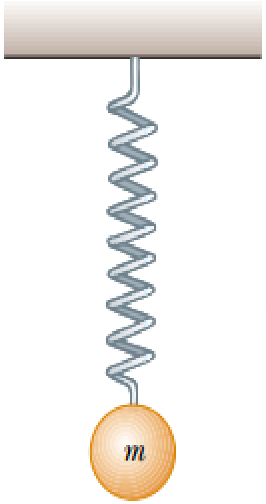
Active Figure 8.3 (Quick Quiz 8.6) Three identical balls are thrown with the same initial speed from the top of a building.

At the Active Figures link at http://www.pse6.com, you can throw balls at different angles from the top of the building and compare the trajectories and the speeds as the balls hit the ground.

Quizzes 8.7 to 8.8

Quick Quiz 8.7 A ball is connected to a light spring suspended vertically, as shown in Figure 8.5. When displaced downward from its equilibrium position and released, the ball oscillates up and down. In the system of *the ball, the spring, and the Earth,* what forms of energy are there during the motion? (a) kinetic and elastic potential (b) kinetic and gravitational potential (c) kinetic, elastic potential, and gravitational potential (d) elastic potential and gravitational potential.

Quick Quiz 8.8 Consider the situation in Quick Quiz 8.7 once again. In the system of *the ball and the spring,* what forms of energy are there during the motion? (a) kinetic and elastic potential (b) kinetic and gravitational potential (c) kinetic, elastic potential, and gravitational potential (d) elastic potential and gravitational potential.



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Chapter Summary

- If a particle of mass *m is at a distance y above the Earth's surface, the gravitational potential* energy of the particle–Earth system is

$$U_g = mgy \qquad (8.2)$$

- The elastic potential energy stored in a spring of force constant *k is*

$$U_s = \frac{1}{2}Kx^2 \qquad (8.11)$$

- *Total Energy of A system is:*

$$\therefore\ K_f + U_f = K_i + U_i \qquad (8.9)$$

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Thanks

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PHYS 103 (GENERAL PHYSICS)

CHAPTER 9: LINEAR MOMENTUM

THIS PRESENTATION HAS BEEN PREPARED BY: *DR* ***NASSR S. ALZAYED***

Lecture Outline

- Here is a quick list of the subjects that we will cover in this presentation. It is based on Serway, Ed. 6
- *Introduction*
- *9.1 Linear Momentum and Its Conservation*
- *9.2 Impulse and Momentum*
- *9.3 Collisions in One Dimension*
- *9.4 Two-Dimensional Collisions*
- *Chapter Summary*

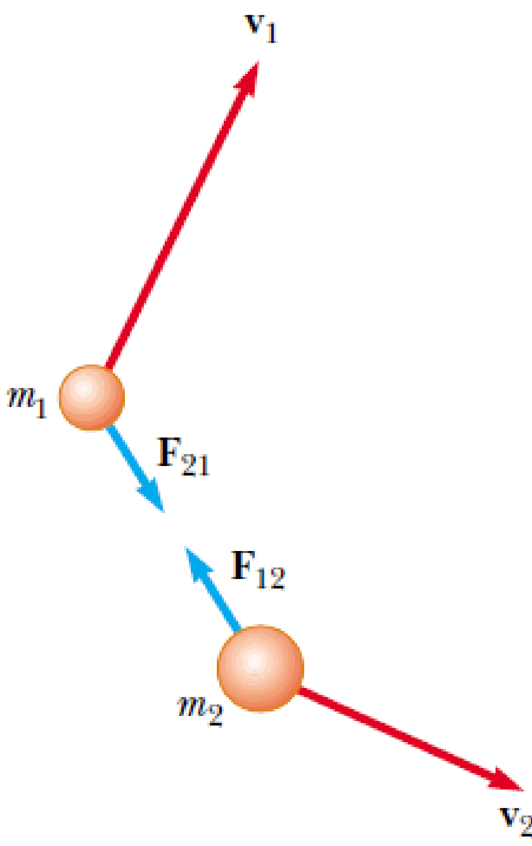
Introduction

Consider what happens when a bowling ball strikes a pin, as in the opening photograph. The pin is given a large velocity as a result of the collision; consequently, it flies away and hits other pins or is projected toward the backstop. Because the average force exerted on the pin during the collision is large, the pin achieves the large velocity very rapidly and experiences the force for a very short time interval. According to Newton’s third law, the pin exerts a reaction force on the ball that is equal in magnitude and opposite in direction to the force exerted by the ball on the pin. This reaction force causes the ball to accelerate, but because the ball is so much more massive than the pin, the ball’s acceleration is much less than the pin’s acceleration.

9.1 Linear Momentum and Its Conservation

- Consider two particles m_1 and m_2 with v_1 and v_2 collide as in figure:
- If a force from particle 1 acts on particle 2, then there must be a second force—equal in magnitude but opposite in direction—that particle 2 exerts on particle 1. That is, they form a Newton's third law action–reaction pair, so that $F_{12}=-F_{21}$. We can express this condition as:
- $\mathbf{F}_{21} + \mathbf{F}_{12} = 0$
- Using Newton's 2nd law:
- $m_1\mathbf{a}_1 + m_2\mathbf{a}_2 = 0$

$$\Rightarrow m_1 \frac{dv_1}{dt} + m_2 \frac{dv_2}{dt} = 0$$



9.1 Linear Mom. and Its Cons. (Continued)

- If the masses m_1 and m_2 are constant, we can bring them into the derivatives, which gives:

$$\Rightarrow \frac{d(m_1 v_1)}{dt} + \frac{d(m_2 v_2)}{dt} = 0$$

$$\therefore \frac{d}{dt}\left(m_1 v_1 + m_2 v_2\right) = 0 \qquad (9.1)$$

- To finalize this discussion, note that the derivative of the sum $(m_1 v_1 + m_2 v_2)$ with respect to time is zero. Consequently, this sum must be constant. We learn from this discussion that the quantity mv for a particle is important, in that the sum of these quantities for an isolated system is conserved. We call this quantity linear momentum
- linear momentum of a particle or an object is defined as:
- $\mathbf{p} = m\mathbf{v}$ (9.2)

9.1 Linear Mom. and Its Cons. (Continued)

- If a particle is moving in 3-D then:
- $p_x = mv_x$ $p_y = mv_y$ $p_z = mv_z$
- Using Newton's second law of motion, we can relate the linear momentum of a particle to the resultant force acting on the particle:

$$\sum F_x = ma = m\frac{dv}{dt}$$

- In Newton's second law, the mass m is assumed to be constant. Thus, we can bring m inside the derivative notation to give us:

$$\sum F_x = \frac{d(mv)}{dt} = \frac{dp}{dt} \qquad (9.3)$$

- *This shows that the time rate of change of the linear momentum of a particle is equal to the net force acting on the particle.*

9.1 Linear Mom. and Its Cons. (Continued)

- Using the definition of momentum, Equation 9.1 can be written:

$$\frac{d}{dt}(p_1 + p_2) = 0$$

$$\because \frac{d}{dt} p_{tot} = \frac{d}{dt}(p_1 + p_2) = 0$$

$$\therefore p_{tot} = p_1 + p_2 = \text{constant} \qquad (9.4)$$

$$\Rightarrow p_{1i} + p_{2i} = p_{1f} + p_{2f} \qquad (9.5)$$

- *Whenever two or more particles in an isolated system interact, the total momentum of the system remains constant.*
- This law tells us that the total momentum of an isolated system at all times equals its initial momentum.

Quiz 9.1+9.2

Quick Quiz 9.1 Two objects have equal kinetic energies. How do the magnitudes of their momenta compare? (a) $p_1 < p_2$ (b) $p_1 = p_2$ (c) $p_1 > p_2$ (d) not enough information to tell.

Quick Quiz 9.2 Your physical education teacher throws a baseball to you at a certain speed, and you catch it. The teacher is next going to throw you a medicine ball whose mass is ten times the mass of the baseball. You are given the following choices: You can have the medicine ball thrown with (a) the same speed as the baseball (b) the same momentum (c) the same kinetic energy. Rank these choices from easiest to hardest to catch.

Quiz 9.3+9.4

Quick Quiz 9.3 A ball is released and falls toward the ground with no air resistance. The isolated system for which momentum is conserved is (a) the ball (b) the Earth (c) the ball and the Earth (d) impossible to determine.

Quick Quiz 9.4 A car and a large truck traveling at the same speed make a head-on collision and stick together. Which vehicle experiences the larger change in the magnitude of momentum? (a) the car (b) the truck (c) The change in the magnitude of momentum is the same for both. (d) impossible to determine.

9.2 Impulse and Momentum

- To build a better, let us assume that a single force **F** acts on a particle and that this force may vary with time. According to Newton's second law:

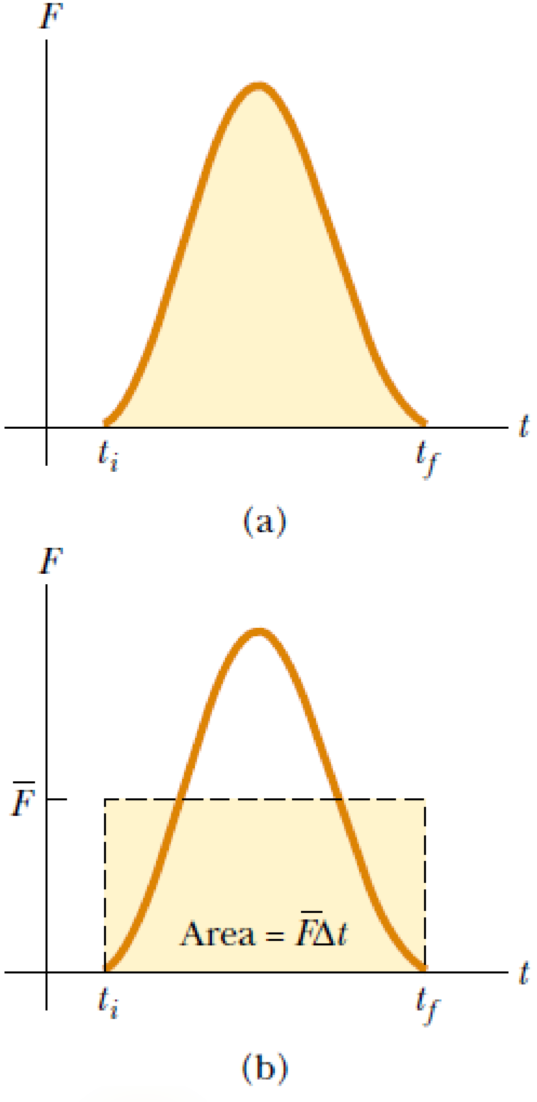
$$F = \frac{dp}{dt}$$

$$\Rightarrow dp = F dt \qquad (9.7)$$

- Integrating for time t_i to t_f :

$$\Delta p = p_f - p_i = \int_{t_i}^{t_f} F dt \qquad (9.8)$$

OR: $$\mathrm{I} = \int_{t_i}^{t_f} F dt \qquad (9.9)$$



9.2 Impulse and Momentum (continued)

- The quantity in (9.9) is called: Impulse. (9.8) is called: **Impulse-Momentum Theorem**.
- The impulse of the force **F** acting on a particle equals the change in the momentum of the particle.
- Because the force imparting an impulse can generally vary in time, it is convenient to define a time-averaged force:

$$\bar{F} = \frac{1}{\Delta t}\int_{t_i}^{t_f} F dt \qquad (9.10)$$

OR: $$\mathrm{I} = \bar{F}\Delta t \qquad (9.11)$$

- In principle, if **F** is known as a function of time, the impulse can be calculated from Equation 9.9. The calculation becomes especially simple if the force acting on the particle is constant. In this case Equation 9.11 becomes: **I = F**Δt

Quiz 9.5 - 9.7

Quick Quiz 9.5 Two objects are at rest on a frictionless surface. Object 1 has a greater mass than object 2. When a constant force is applied to object 1, it accelerates through a distance d. The force is removed from object 1 and is applied to object 2. At the moment when object 2 has accelerated through the same distance d, which statements are true? (a) $p_1 < p_2$ (b) $p_1 = p_2$ (c) $p_1 > p_2$ (d) $K_1 < K_2$ (e) $K_1 = K_2$ (f) $K_1 > K_2$.

Quick Quiz 9.6 Two objects are at rest on a frictionless surface. Object 1 has a greater mass than object 2. When a force is applied to object 1, it accelerates for a time interval Δt. The force is removed from object 1 and is applied to object 2. After object 2 has accelerated for the same time interval Δt, which statements are true? (a) $p_1 < p_2$ (b) $p_1 = p_2$ (c) $p_1 > p_2$ (d) $K_1 < K_2$ (e) $K_1 = K_2$ (f) $K_1 > K_2$.

Quick Quiz 9.7 Rank an automobile dashboard, seatbelt, and airbag in terms of (a) the impulse and (b) the average force they deliver to a front-seat passenger during a collision, from greatest to least.

Example 9.4 How Good Are the Bumpers?

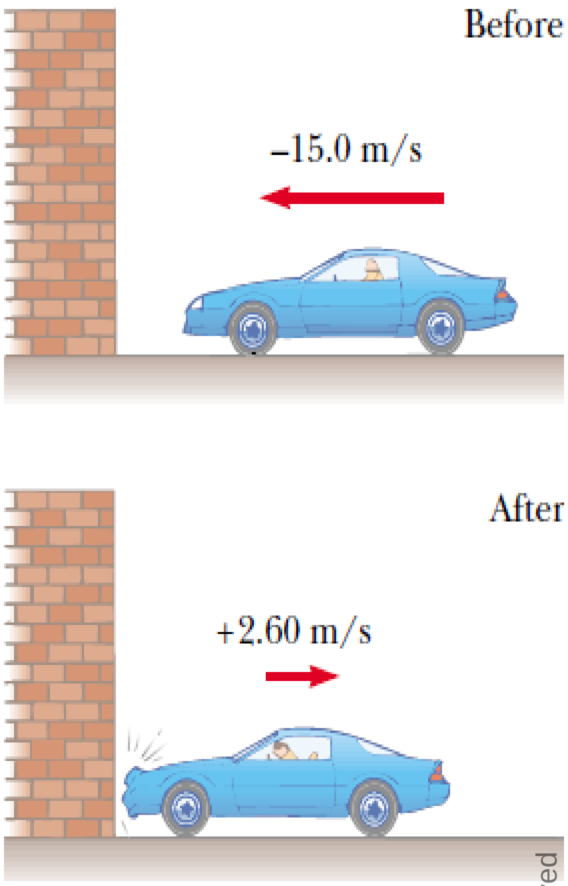
- In a particular crash test, a car of mass 1 500 kg collides with a wall, as shown in Figure 9.6. The initial and final velocities of the car are $v_i = -15$ m/s and $v_f = 2.6$ m/s , respectively. If the collision lasts for 0.150 s, *find the impulse caused by the collision and the average force exerted on the car.*
- ***Solution:***

$$\because I = \Delta p = p_f - p_i$$

$$= mv_f - mv_i = (1500)(2.6\hat{i}) - (1500)(-15\hat{i})$$

$$= 2.64\times10^4\,\hat{i}\ \mathrm{kg.m/s}$$

$$\because \bar{F} = \frac{\Delta p}{\Delta t} = \frac{2.64\times10^4}{0.15} = 1.76\times10^5\ N$$



9.3 Collisions in One Dimension

- The total kinetic energy of the system of particles may or may not be conserved, depending on the type of collision. In fact, whether or not kinetic energy is conserved is used to classify collisions as either ***elastic or inelastic*****.**
- An elastic collision between two objects is one in which the total kinetic energy (as well as total momentum) of the system is the same before and after the collision.
- An inelastic collision is one in which the total kinetic energy of the system is not the same before and after the collision (even though the momentum of the system is conserved).
- Inelastic collisions are of two types. When the colliding objects stick together after the collision, the collision is called **perfectly inelastic**, When the colliding objects do not stick together, but some kinetic energy is lost, the .

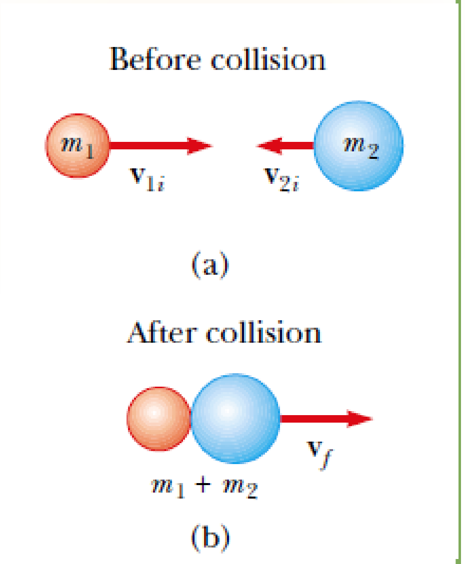
9.3 Perfectly Inelastic Collisions

- Consider two particles of masses m_1 and m_2 moving with initial velocities v_{1i} and v_{2i} along the same straight line, as shown in Figure. The two particles collide head-on, stick together, and then move with some common velocity v_f after the collision.

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f \qquad (9.13)$$

$$\Rightarrow$$

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} \qquad (9.14)$$



- This is true only if the two objects Stick together in one-object.

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9.3 Perfectly Elastic Collisions

- For this type of collisions: kinetic energy and liner momentum are conserved:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \qquad (9.15)$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \qquad (9.16)$$

- We can use (9.15) and (9.16) directly to solve our problems or simplify (9.16) to go directly to some special cases:

$$(9.16) \rightarrow m_1 v_{1i}^2 + m_2 v_{2i}^2 = m_1 v_{1f}^2 + m_2 v_{2f}^2$$

$$\therefore m_1 (v_{1i}^2 - v_{1f}^2) = m_2 (v_{2f}^2 - v_{2i}^2)$$

$$\therefore m_1 (v_{1i} - v_{1f})(v_{1i} + v_{1f}) = m_2 (v_{2f} - v_{2i})(v_{2f} + v_{2i}) \qquad (9.17)$$

$$(9.15) \rightarrow m_1 (v_{1i} - v_{1f}) = m_2 (v_{2f} - v_{2i}) \qquad (9.18)$$

9.3 Perfectly Elastic Collisions (continued)

- To obtain our final result, we divide Equation 9.17 by Equation 9.18 and obtain:

$$v_{1i} + v_{1f} = v_{2i} + v_{2f}$$

$$v_{1i} - v_{2i} = -(v_{1f} - v_{2f}) \qquad (9.19)$$

- Suppose that the masses and initial velocities of both particles are known:

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i} \qquad (9.20)$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2i} \qquad (9.21)$$

- Let us consider some special cases. If $m_1 = m_2$, then Equations 9.20 and 9.21 show us that $v_{1f} = v_{2i}$ and $v_{2f} = v_{1i}$.

9.3 Perfectly Elastic Collisions (continued)

- If m_2 is initially at rest → $v_{2i} = 0$
- and (9.20) (9.21) becomes:

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_{1i} \qquad (9.22)$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2} \right) v_{1i} \qquad (9.23)$$

- If m_1 is much greater than m_2 and $v_{2i} = 0$, we see from Equations 9.22 and 9.23 that $v_{1f} \approx v_{1i}$ and $v_{2f} \approx 2v_{1i}$. That is, when a very heavy particle collides head-on with a very light one that is initially at rest, the heavy particle continues its motion unaltered after the collision and the light particle rebounds with a speed equal to about twice the initial speed of the heavy particle.

Example 9.6 Carry Collision

- An 1800-kg car stopped at a traffic light is struck from the rear by a 900-kg car, and the two become entangled, moving along the same path as that of the originally moving car. If the smaller car were moving at 20.0 m/s before the collision, *what is the velocity of the entangled cars after the collision?*
- ***Solution:***

$$\because m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\therefore (1800)(0) + (900)(20) = (1800 + 900) v_f$$

$$\Rightarrow v_f = \frac{900 \times 20}{2700} = 6.67\, m/s$$

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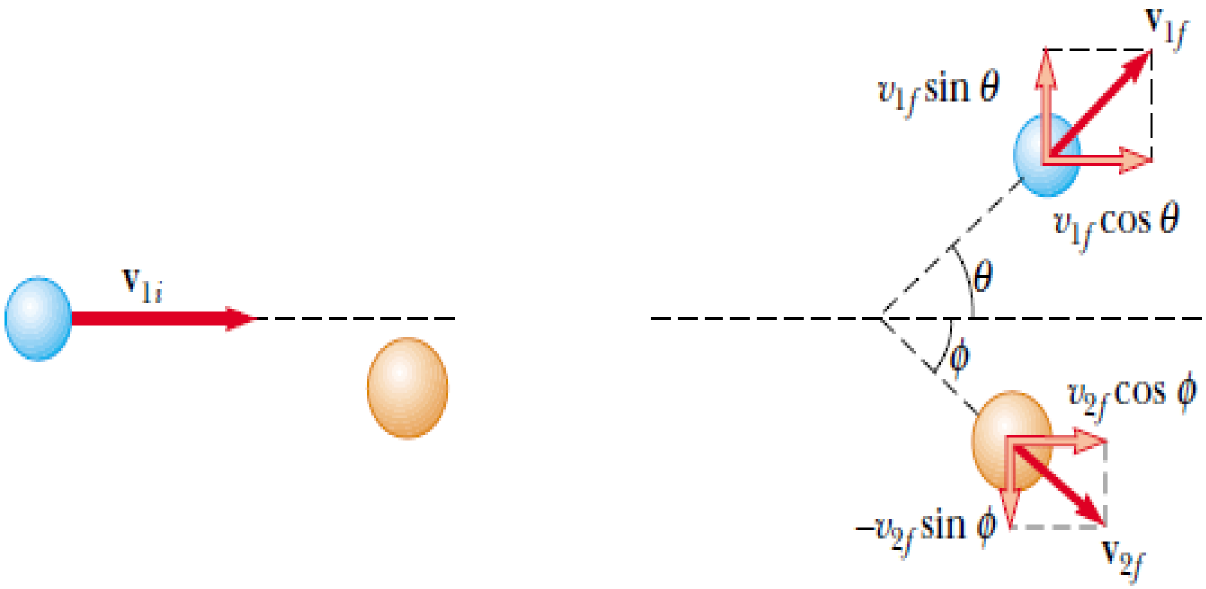
9.4 Two-Dimensional Collisions

- For two dimensional collisions, we obtain two component equations for conservation of momentum:

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

$$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$

consider a 2-D problem in which particle 1 of mass *m_1 collides* with particle 2 of mass m_2 , *where particle 2 is initially at rest, as in Figure*



(a) Before the collision (b) After the collision

9.4 2-Dimensional Collisions (continued)

- Applying the law of conservation of momentum in component form and noting that the initial y component of the momentum of the two-particle system is zero, we obtain:

$$m_1 v_{1i} = m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\varphi \qquad (9.24)$$

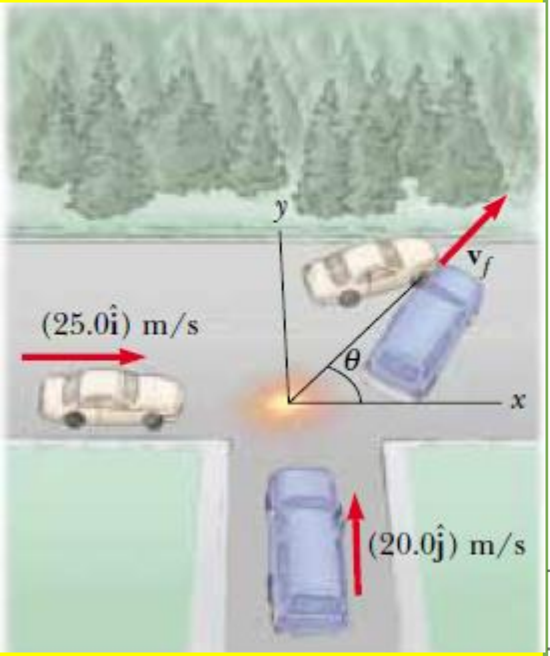
$$0 = m_1 v_{1f} \sin\theta - m_2 v_{2f} \sin\varphi \qquad (9.25)$$

- where the minus sign in Equation 9.25 comes from the fact that after the collision, particle 2 has a y component of velocity that is downward.
- If the collision is elastic, we can also use Equation 9.16 (conservation of kinetic ene

$$\frac{1}{2} m_1 {v_{1i}}^2 = \frac{1}{2} m_1 {v_{1f}}^2 + \frac{1}{2} m_2 {v_{2f}}^2 \qquad (9.26)$$

Example 9.10 Collision at an

A 1 500-kg car traveling east with a speed of 25.0 m/s collides at an intersection with a 2 500-kg van traveling north at a speed of 20.0 m/s, as shown in Figure 9.14. *Find the direction and magnitude of the velocity of the wreckage after the collision, assuming that the vehicles undergo a perfectly inelastic collision (that is, they stick together).*



- ***Solution***:
- We shall apply the conservation of momentum in each direction.

$$x: \; m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx} \quad (1)$$

$$y: \; m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy} \quad (2)$$

Example 9.10 (continued)

- Solving to find final velocity and direction:

$$(1)\rightarrow:\ (1500)(25)+(2500)(0)=(1500+2500)v_{fx} \qquad (3)$$

$$\therefore v_{fx}=\frac{37500}{4000}=9.37\,m/s$$

$$(2)\rightarrow:\ (1500)(0)+(2500)(20)=(1500+2500)v_{fy} \qquad (4)$$

$$\therefore v_{fy}=\frac{50000}{4000}=12.5\,m/s$$

$$(1)+(2)\rightarrow: v_f=\sqrt{v_{fx}^2+v_{fy}^2}=\sqrt{9.37^2+12.5^2}=15.6\,m/s$$

$$\theta=\tan^{-1}\left(\frac{v_{fy}}{v_{fx}}\right)=\tan^{-1}\left(\frac{12.5}{9.37}\right)=53.1^o$$

PROBLE M-SOLV ING HI NTS

- Set up a coordinate system and define your velocities with respect to that system.
- In your sketch of the coordinate system, draw and label all velocity vectors and include all the given information.
- Write expressions for the x *and y components of the momentum of each* object before and after the collision.
- Write expressions for the total momentum of the system in the x *direction* before and after the collision and equate the two.
- If the collision is inelastic, kinetic energy of the system is *not conserved, and* additional information is probably required.
- If the collision is *perfectly* inelastic, the final velocities of the two objects are equal. Solve the momentum equations for the unknown quantities.
- If the collision is *elastic, kinetic energy of the system is conserved, and you can* equate the total kinetic energy before the collision to the total kinetic energy after the collision to obtain an additional relationship between the velocities.

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Chapter Summary

- The linear momentum **p** of a particle , m moving with a velocity **v** is: **p** = m**v** (9.2)
- The law of conservation of linear momentum indicates that the total momentum of an isolated system is conserved. If two particles form an isolated system, the momentum of the system is conserved regardless of the nature of the force between them. Therefore, the total momentum of the system at all times equals its initial total momentum, or

$$p_{1i} + p_{2i} = p_{1f} + p_{2f} \qquad (9.5)$$

- The impulse imparted to a particle by a force **F is equal to the change in the momentum** of the particle:

$$\mathrm{I} = \int_{t_i}^{t_f} F dt = \Delta p \qquad (9.9)$$

- This is known as the impulse–momentum theorem.

Chapter Summary

- Perfectly Inelastic Collisions:

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f \qquad (9.13)$$

- Perfectly Elastic Collisions:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \qquad (9.15)$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \qquad (9.16)$$

- When two particles collide, the total momentum of the isolated system before the collision always equals the total momentum after the collision, regardless of the nature of the collision. An inelastic collision is one for which the total kinetic energy of the system is not conserved. A perfectly inelastic collision is one in which the colliding bodies stick together after the collision. An elastic collision is one in which the kinetic energy of the system is conserved.

Chapter Summary

- For two dimensional collisions, we obtain two component equations for conservation of momentum:

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

$$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$

- The center of mass of the pair of particles:

$$x_{CM} = \frac{\sum_i m_i x_i}{M} \qquad y_{CM} = \frac{\sum_i m_i y_i}{M} \qquad z_{CM} = \frac{\sum_i m_i z_i}{M}$$

- When mass is distributed uniformly over the space:

$$x_{CM} = \frac{1}{M}\int x\,dm \qquad y_{CM} = \frac{1}{M}\int y\,dm \qquad z_{CM} = \frac{1}{M}\int z\,dm$$

Thanks

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PHYS 103 (GENERAL PHYSICS)

CHAPTER 10

ROTATION OF A RIGID OBJECT ABOUT A FIXED AXIS

THIS PRESENTATION HAS BEEN PREPARED BY: *DR. NASSR S. ALZAYED*

Chapter Outline

- *We shall discuss the following topics:*
- *10.1 Angular Position, Velocity, and Acceleration*
- *10.2 Rotational Motion with Cons. Ang. Acc.*
- *10.3 Angular and Linear Quantities*
- *10.4 Rotational Kinetic Energy*
- *10.5 Calculation of Moments of Inertia*
- *10.6 Torque*
- *10.7 Relationship Between Torque and Angular Acceleration*
- *10.8 Work, Power, and Energy in Rotational Motion*
- *Chapter Summary*
- *End of Presentation*

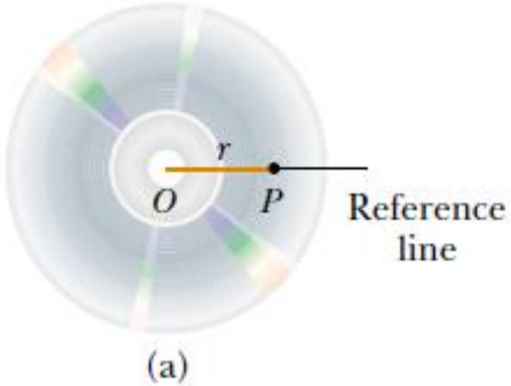
10.1 Angular Position

- Consider a particle at P is at a fixed distance r from the origin and rotates about it in a circle of radius r.
- The particle moves through an arc of length s, as in Figure . The arc length *s* is related to the angle θ through the relationship:

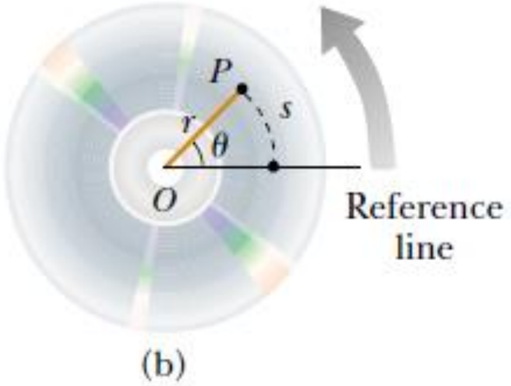
$$s = r\theta \qquad (10.1a)$$

$$\Rightarrow \theta = \frac{s}{r} \qquad (10.1b)$$

- Note the dimensions of θ in Equation 10.1b. Because θ is the ratio of an arc length and the radius of the circle, it is a pure number. However, we commonly give ! the artificial unit radian (rad).



(a)



(b)

10.1 Angular Speed

- As a particle travels from position 1 to position 2 in a time interval Δ, the reference line of length *r* sweeps out an angle $\Delta\theta = \theta_f - \theta_i$. This quantity Δθ is defined as the ***angular displacement*** of the rigid object:

$$\Delta\theta = \theta_f - \theta_i$$

- We define the average angular speed as:

$$\bar{\omega} = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t} \qquad (10.2)$$

- the instantaneous angular speed is:

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} \qquad (10.3)$$

Angular speed has units of radians per second (rad/s)

10.1 Angular Acceleration

- The average angular acceleration of a rotating rigid object is defined as:

$$\bar{\alpha} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t} \qquad (10.3)$$

- the instantaneous angular acceleration is defined as:

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} \qquad (10.5)$$

- Angular acc. has units of radians per second squared (rad/s^2)
- When a rigid object is rotating about a *fixed axis*, every particle on the object rotates through the same angle in a given time interval and has the same angular speed and the same angular acceleration.
- That is, θ, ω, and **α** characterize the rotational motion of the entire rigid object as well as individual particles in the object.

Quizes 10.1, 10.2 & 10.3

Quick Quiz 10.1 A rigid object is rotating in a counterclockwise sense around a fixed axis. Each of the following pairs of quantities represents an initial angular position and a final angular position of the rigid object. Which of the sets can *only* occur if the rigid object rotates through more than 180°? (a) 3 rad, 6 rad (b) −1 rad, 1 rad (c) 1 rad, 5 rad.

Quick Quiz 10.2 Suppose that the change in angular position for each of the pairs of values in Quick Quiz 10.1 occurs in 1 s. Which choice represents the lowest average angular speed?

Quick Quiz 10.3 A rigid object is rotating with an angular speed $\omega < 0$. The angular velocity vector $\boldsymbol{\omega}$ and the angular acceleration vector $\boldsymbol{\alpha}$ are antiparallel. The angular speed of the rigid object is (a) clockwise and increasing (b) clockwise and decreasing (c) counterclockwise and increasing (d) counterclockwise and decreasing.

10.2 Rotational Motion with Cons. Ang. Acc.

- In our study of linear motion, we found that the simplest form of motion to analyze is motion under constant linear acceleration.
- Likewise, for rotational motion about a fixed axis, the simplest motion to analyze is motion under constant angular acceleration.

with $\theta_i = 0$, $\alpha = \text{constant}$

$$\omega_f = \omega_i + \alpha t \qquad (10.6)$$

$$\theta_f = \omega_i t + \frac{1}{2}\alpha t^2 \qquad (10.7)$$

$$\omega_f^{\,2} = \omega_i^{\,2} + 2\alpha\theta_f \qquad (10.8)$$

Kinematic Equations for Rotational and Linear Motion Under Constant Acceleration

Rotational Motion About Fixed Axis	Linear Motion
$\omega_f = \omega_i + \alpha t$	$v_f = v_i + at$
$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$	$x_f = x_i + v_i t + \frac{1}{2}at^2$
$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$	$v_f^2 = v_i^2 + 2a(x_f - x_i)$
$\theta_f = \theta_i + \frac{1}{2}(\omega_i + \omega_f)t$	$x_f = x_i + \frac{1}{2}(v_i + v_f)t$

- Notice that these expressions are of the same mathematical form as those for linear motion under constant linear acceleration with the change:

x → θ, v → ω, a → α

Quizes 10.4

Quick Quiz 10.4 Consider again the pairs of angular positions for the rigid object in Quick Quiz 10.1. If the object starts from rest at the initial angular position, moves counterclockwise with constant angular acceleration, and arrives at the final angular position with the same angular speed in all three cases, for which choice is the angular acceleration the highest?

Example 10.1 Rotating Wheel

- A wheel rotates with a constant angular acceleration of 3.50 rad/s^2.

(A) If the angular speed of the wheel is 2.00 rad/s at $t_i = 0$, through what angular displacement does the wheel rotate in 2.00 s?

$$\because \quad \theta_i = 0$$

$$\therefore \Delta\theta = \theta_f = (2)(2) + \frac{1}{2}(3.5)(2)^2 = 11\,rad = 630^o$$

(B) Through how many revolutions has the wheel turned during this time interval?

$$\Delta\theta = 630^o \left(\frac{1 \ \ \text{rev}}{360^o}\right) = 1.75 \ \ \text{rev}$$

(C) What is the angular speed of the wheel at t = 2.00 s?

$$\because \omega_f = \omega_i + \alpha t$$

$$\therefore \omega_f = 2 + (3.5)(2) = 9\,rad\,/\,s$$

10.3 Angular and Linear Quantities

- We shall find relations between linear and angular quantities:

$$\because \omega_f = \omega_i + \alpha t$$

$$\because v = \frac{ds}{dt} = \frac{d}{dt} r\theta = r\frac{d\theta}{dt} = r\omega \qquad (10.10)$$

$$\because a_t = \frac{dv}{dt} = \frac{d}{dt} r\omega = r\frac{d\omega}{dt} = r\alpha \qquad (10.11)$$

$$\because a_c = \frac{v^2}{r} = \frac{r^2\omega^2}{r} = r\omega^2 \qquad (10.12)$$

$$\therefore a = \sqrt{a_t^{\,2} + a_c^{\,2}} = \sqrt{r^2\alpha^2 + r^2\omega^4}$$

$$\therefore a = r\sqrt{\alpha^2 + \omega^4} \qquad (10.13)$$

- a_t : tangential acceleration, a_c: central acceleration, a: total acceleration

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10.4 Rotational Kinetic Energy

- Let us consider an object as a collection of particles and assume that it rotates about a fixed z axis with an angular speed **ω** If the mass of the i[th] particle is m_i and its tangential speed is v_i , its kinetic energy is:

$$K_i = \frac{1}{2} m_i v_i^{\,2}$$

$$\because v_i = r_i \omega$$

$$\therefore K_R = \sum_i K_i = \sum_i \frac{1}{2} m_i v_i^{\,2} = \frac{1}{2} \sum_i m_i r_i^{\,2} \omega^2 = \frac{1}{2} \left(\sum_i m_i r_i^{\,2} \right) \omega^2$$

define the moment of inertia I as: $I = \sum_i m_i r_i^{\,2}$ (10.15)

$$\therefore K_R = \frac{1}{2} I \omega^2 \qquad (10.16)$$

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Quizes 10.5, 10.6 & 10.7

Quick Quiz 10.5 Andy and Charlie are riding on a merry-go-round. Andy rides on a horse at the outer rim of the circular platform, twice as far from the center of the circular platform as Charlie, who rides on an inner horse. When the merry-go-round is rotating at a constant angular speed, Andy's angular speed is (a) twice Charlie's (b) the same as Charlie's (c) half of Charlie's (d) impossible to determine.

Quick Quiz 10.6 Consider again the merry-go-round situation in Quick Quiz 10.5. When the merry-go-round is rotating at a constant angular speed, Andy's tangential speed is (a) twice Charlie's (b) the same as Charlie's (c) half of Charlie's (d) impossible to determine.

Quick Quiz 10.7 A section of hollow pipe and a solid cylinder have the same radius, mass, and length. They both rotate about their long central axes with the same angular speed. Which object has the higher rotational kinetic energy? (a) the hollow pipe (b) the solid cylinder (c) they have the same rotational kinetic energy (d) impossible to determine.

Example 10.3 The Oxygen

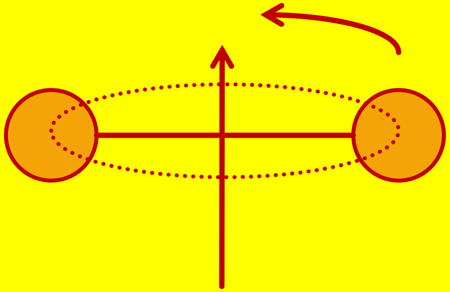
- Consider an oxygen molecule (O_2) rotating in the xy plane about the z axis. The rotation axis passes through the center of the molecule, perpendicular to its length. The mass of each oxygen atom is 2.66×10^{-26} kg, and at room temperature the average separation between the two atoms is $d = 1.21\times10^{-10}$ m. (The atoms are modeled as particles.)

(A) *Calculate the moment of inertia of the molecule about the z axis.*

$$\because\ I=\sum_i m_i r_i^2 = m\left(\frac{d}{2}\right)^2 = m\left(\frac{d}{2}\right)^2 = \frac{1}{2}md^2 \qquad (1)$$

$$\therefore I = \frac{1}{2}(2.66\times10^{-26})(1.21\times10^{-10})^2 = 1.95\times10^{-46}\,kg.m^2 \qquad (2)$$

This is a very small number, consistent with the Minuscule masses and distances involved



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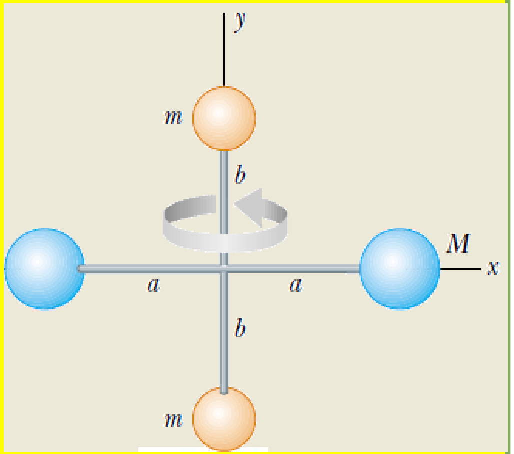
Example 10.4 Four Rotating

- Four tiny spheres are fastened to the ends of two rods of negligible mass lying in the xy plane. We shall assume that the radii of the spheres are small compared with the dimensions of the rods.

- *(A) If the system rotates about the y axis with an angular speed ω, find the moment of inertia and the rotational kinetic energy about this axis.*

$$\because\ I_y = \sum_i m_i r_i^2 = Ma^2 + Ma^2 + m(0) + m(0) = 2Ma^2 \quad (1)$$

$$\therefore K_R = \frac{1}{2} I_y \omega^2 = \frac{1}{2}(2Ma^2)\omega^2 = Ma^2\omega^2 \quad (2)$$



- *(B) Same but in the xy plane about the z axis*

$$I_z = \sum_i m_i r_i^2 = Ma^2 + Ma^2 + mb^2 + mb^2 = 2Ma^2 + 2\text{m}b^2$$

$$\therefore K_R = \frac{1}{2} I_z \omega^2 = \frac{1}{2}(2Ma^2 + 2\text{m}b^2)\omega^2 = (Ma^2 + 2\text{m}b^2)\omega^2$$

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10.5 Calculation of Moment of Inertia

- The definition of moment of inertia is

$$I = \sum_i r_i^2 m_i$$

- The dimensions of moment of inertia are ML^2 and its SI units are $kg \cdot m^2$
- We can calculate the moment of inertia of an object more easily by assuming it is divided into many small volume elements, each of mass Δm_i

10.5 Calculation of Moment of Inertia

- We can rewrite the expression for *I* in terms of Δm

$$I = \lim_{\Delta m_i \to 0} \sum_i r_i^2 \Delta m_i = \int r^2 dm$$

- With the small volume segment assumption,

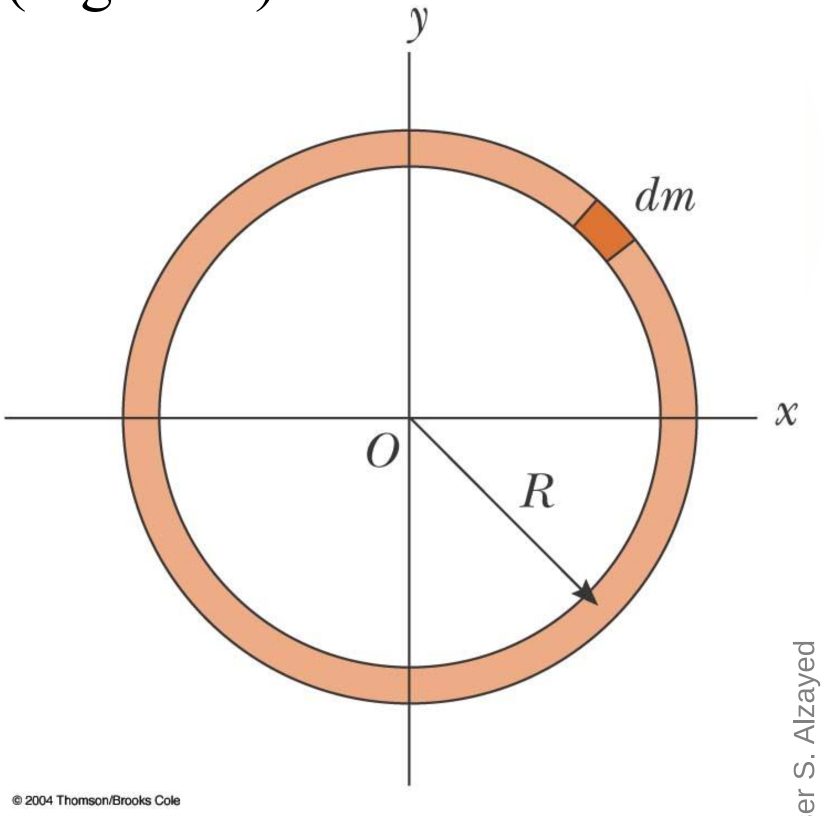
$$I = \int \rho r^2 dV$$

- If ρ is constant, the integral can be evaluated with known geometry, otherwise its variation with position must be known

Example 10.5 Uniform Thin Hoop

Find the moment of inertia of a uniform thin hoop of mass M and radius R about an axis perpendicular to the plane of the hoop and passing through its center (Fig. 10.9).

- Since this is a thin hoop, all mass elements are the same distance from the center



$$I = \int r^2 dm = R^2 \int dm$$

$$I = MR^2$$

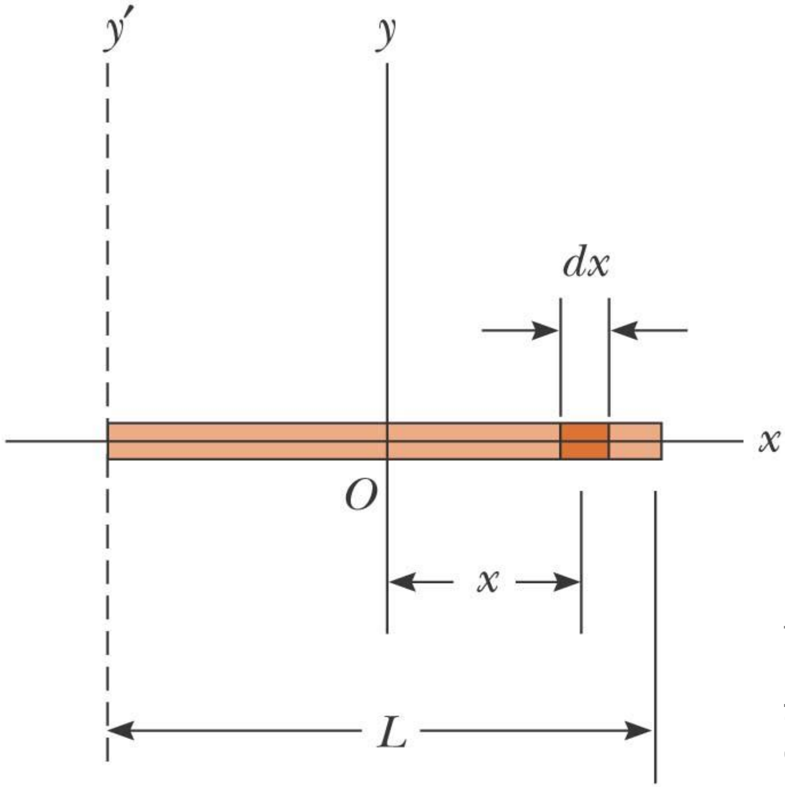
Example 10.6 Uniform Rigid Rod

Calculate the moment of inertia of a uniform rigid rod of length L and mass M (Fig. 10.10) about an axis perpendicular to the rod (the y axis) and passing through its center of mass.

- The shaded area has a mass
  - $dm = \lambda\, dx$
- Then the moment of inertia is

$$I = \int r^2 dm = \int_{-L/2}^{L/2} x^2 \frac{M}{L} dx$$

$$I = \frac{1}{12} ML^2$$



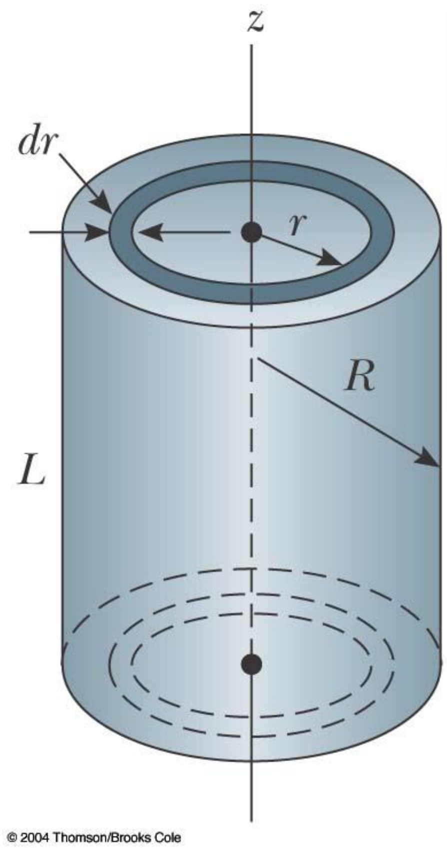
Example:10.7 Uniform Solid Cylinder

A uniform solid cylinder has a radius R, mass M, and length L. Calculate its moment of inertia about its central axis (the z axis in Fig. 10.11).

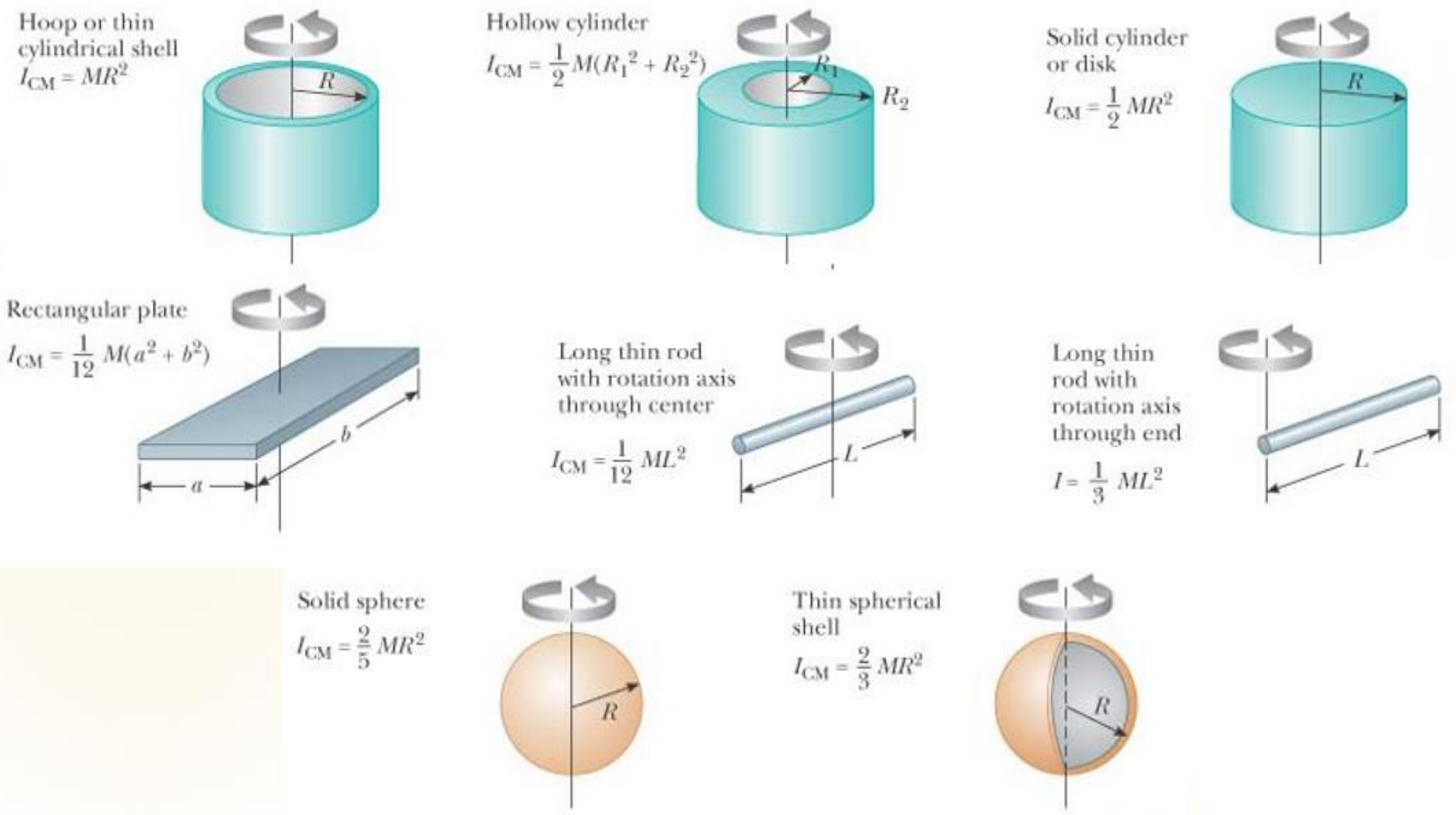
- Divide the cylinder into concentric shells with radius *r*, thickness *dr* and length *L*
- Then for *I*

$$I = \int r^2 dm = \int r^2 (2\pi\rho Lr\, dr)$$

$$I_z = \frac{1}{2} MR^2$$



10.5 Moments of Inertia of Various Rigid Objects



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10.5 Parallel-Axis Theorem

- In the previous examples, the axis of rotation coincided with the axis of symmetry of the object
- For an arbitrary axis, the parallel-axis theorem often simplifies calculations
- The theorem states $I = I_{CM} + MD^2$
  - I is about any axis parallel to the axis through the center of mass of the object
  - I_{CM} is about the axis through the center of mass
  - D is the distance from the center of mass axis to the arbitrary axis

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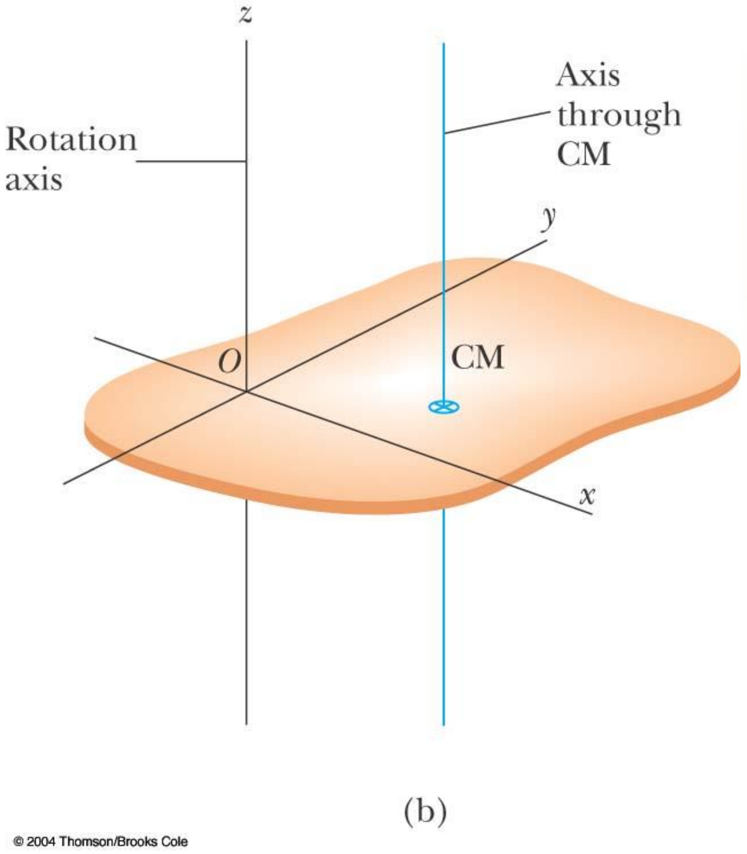
10.5 Parallel-Axis Theorem Example

- The axis of rotation goes through O
- The axis through the center of mass is shown
- The moment of inertia about the axis through O would be

$$I_O = I_{CM} + MD^2$$



10.5 Moment of Inertia for a Rod Rotating Around One End

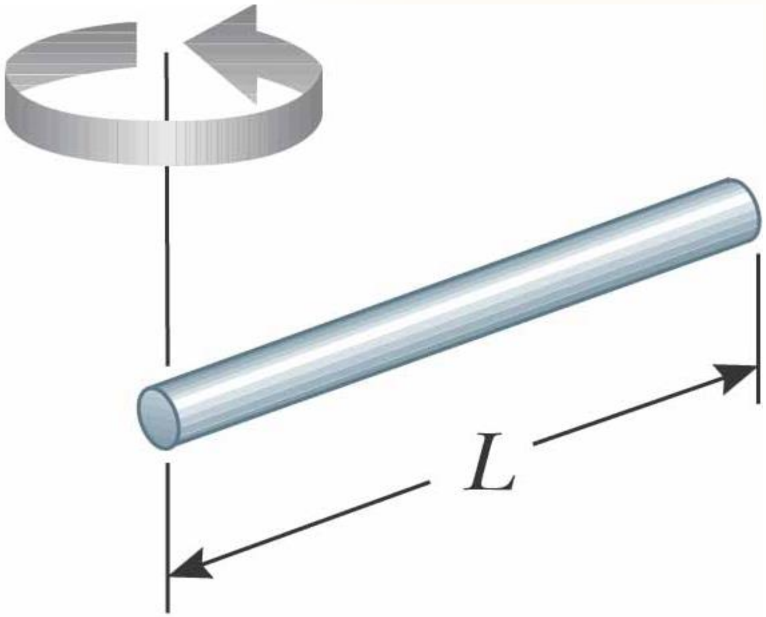
Consider once again the uniform rigid rod of mass *M* and length *L* shown in Figure 10.10. Find the moment of inertia of the rod about an axis perpendicular to the rod through one end (the *y* axis in Fig. 10.10).

$$I_{CM} = \frac{1}{12} ML^2$$

- *D* is ½ *L*
- Therefore,

$$I = I_{\text{CM}} + MD^2$$

$$I = \frac{1}{12} ML^2 + M\left(\frac{L}{2}\right)^2 = \frac{1}{3} ML^2$$



10.6 Torque

- Torque, τ, is the tendency of a force to rotate an object about some axis
  - Torque is a vector
  - $\boldsymbol{\tau = r\,F \sin\phi = F\,d}$
    - **F** is the force
    - ϕ is the angle the force makes with the horizontal
    - *d* is the *moment arm* (or lever arm)

$$\sum \tau = \tau_1 + \tau_2 = F_1 d_1 - F_2 d_2$$

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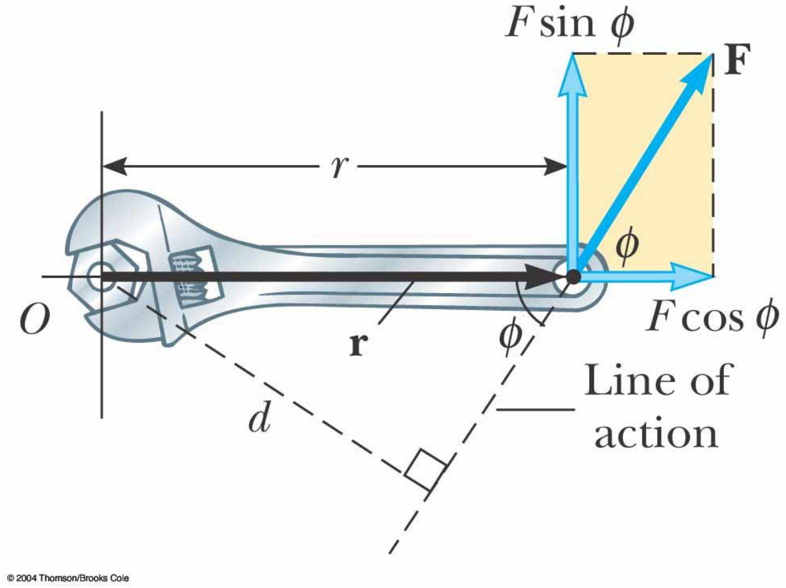
10.6 Torque, cont

- The moment arm, *d*, is the *perpendicular* distance from the axis of rotation to a line drawn along the direction of the force
  - ***d* = *r* sin *Φ***

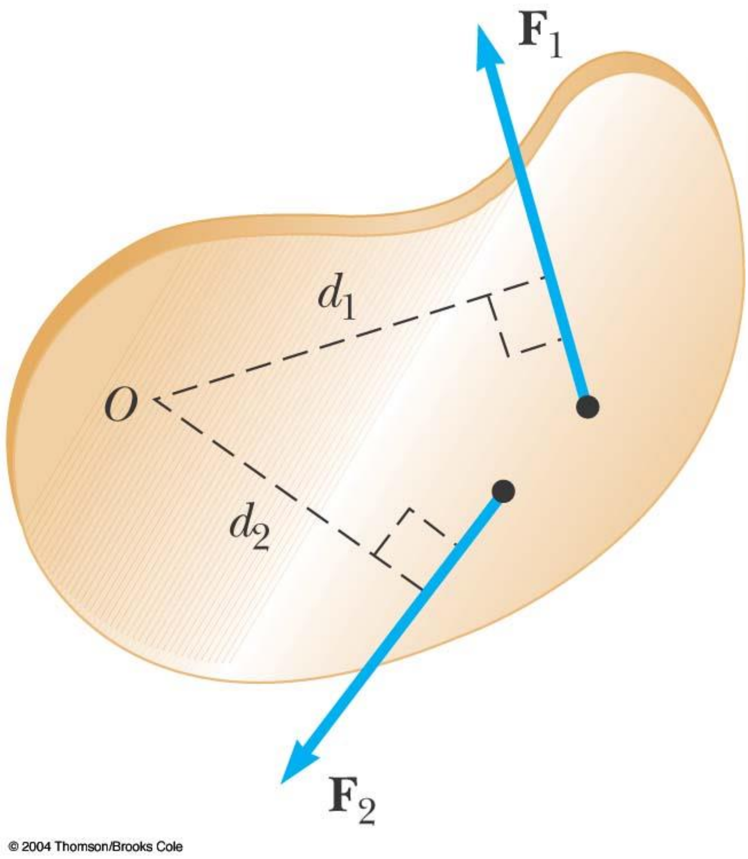


10.6 Torque, final

- The horizontal component of **F** ($F \cos \phi$) has no tendency to produce a rotation
- Torque will have direction
  - If the turning tendency of the force is counterclockwise, the torque will be *positive*
  - If the turning tendency is clockwise, the torque will be *negative*

10.6 Net Torque

- The force $\mathbf{F}_1$ will tend to cause a counterclockwise rotation about O
- The force $\mathbf{F}_2$ will tend to cause a clockwise rotation about O
- $\Sigma\tau = \tau_1 + \tau_2 = F_1 d_1 - F_2 d_2$



10.6 Quiz 10.8 & 10.9

Quick Quiz 10.8 If you are trying to loosen a stubborn screw from a piece of wood with a screwdriver and fail, should you find a screwdriver for which the handle is (a) longer or (b) fatter?

Quick Quiz 10.9 If you are trying to loosen a stubborn bolt from a piece of metal with a wrench and fail, should you find a wrench for which the handle is (a) longer (b) fatter?

Example 10.9 The Net Torque on a Cylinder

A one-piece cylinder is shaped as shown in Figure 10.15, with a core section protruding from the larger drum. The cylinder is free to rotate about the central axis shown in the drawing. A rope wrapped around the drum, which has radius R_1, exerts a force $\mathbf{T}_1$ to the right on the cylinder. A rope wrapped around the core, which has radius R_2, exerts a force $\mathbf{T}_2$ downward on the cylinder.

(A) What is the net torque acting on the cylinder about the rotation axis (which is the z axis in Figure 10.15)?

Solution The torque due to $\mathbf{T}_1$ is $-R_1T_1$. (The sign is negative because the torque tends to produce clockwise rotation.) The torque due to $\mathbf{T}_2$ is $+R_2T_2$. (The sign is positive because the torque tends to produce counterclockwise rotation.) Therefore, the net torque about the rotation axis is

$$\sum \tau = \tau_1 + \tau_2 = R_2T_2 - R_1T_1$$

We can make a quick check by noting that if the two forces are of equal magnitude, the net torque is negative because $R_1 > R_2$. Starting from rest with both forces of equal magnitude acting on it, the cylinder would rotate clockwise because $\mathbf{T}_1$ would be more effective at turning it than would $\mathbf{T}_2$.

(B) Suppose $T_1 = 5.0$ N, $R_1 = 1.0$ m, $T_2 = 15.0$ N, and $R_2 = 0.50$ m. What is the net torque about the rotation axis, and which way does the cylinder rotate starting from rest?

Solution Evaluating the net torque,

$$\sum \tau = (15\text{ N})(0.50\text{ m}) - (5.0\text{ N})(1.0\text{ m}) = 2.5\text{ N}\cdot\text{m}$$

Because this torque is positive, the cylinder will begin to rotate in the counterclockwise direction.

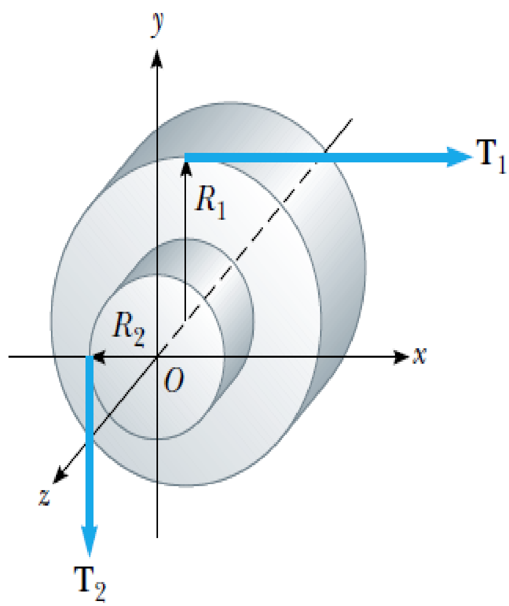
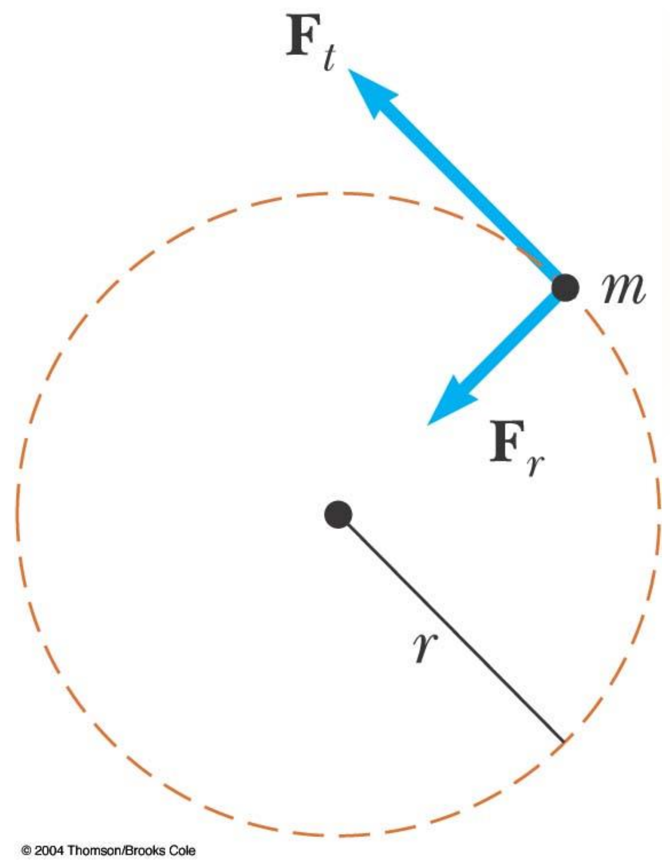


Figure 10.15 (Example 10.9) A solid cylinder pivoted about the z axis through O. The moment arm of $\mathbf{T}_1$ is R_1, and the moment arm of $\mathbf{T}_2$ is R_2.

10.7 Torque and Angular Acceleration

- Consider a particle of mass *m* rotating in a circle of radius *r* under the influence of tangential force $\mathbf{F}_t$
- The tangential force provides a tangential acceleration:
  - $F_t = ma_t$



10.7 Torque and Angular Acceleration, Particle cont.

- The magnitude of the torque produced by $\mathbf{F}_t$ around the center of the circle is
  - $\tau = F_t\, r = (ma_t)\, r$
- The tangential acceleration is related to the angular acceleration
  - $\tau = (ma_t)\, r = (mr\alpha)\, r = (mr^2)\, \alpha$
- Since mr^2 is the moment of inertia of the particle,
  - $\boldsymbol{\tau = I\alpha}$
  - The torque is directly proportional to the angular acceleration and the constant of proportionality is the moment of inertia

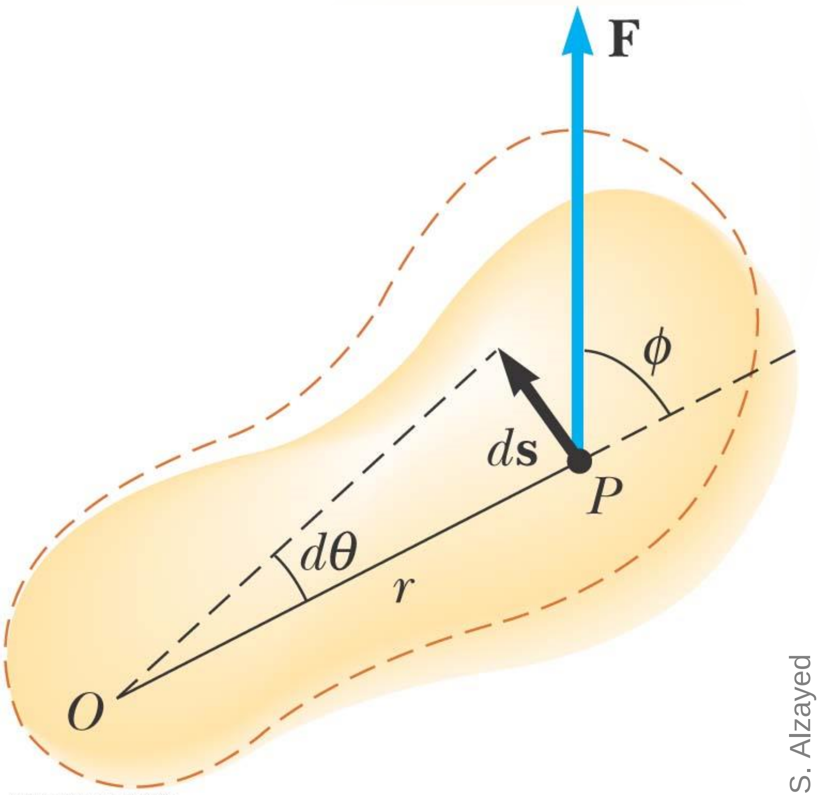
10.8 Work in Rotational Motion

- Find the work done by **F** on the object as it rotates through an infinitesimal distance $ds = r\,d\theta$
- $dW = \mathbf{F} \cdot d\,\mathbf{s}$

  $= (F \sin\phi)\, r\, d\theta$

  $dW = \tau\, d\theta$

  The radial component of **F** does no work because it is perpendicular to the displacement

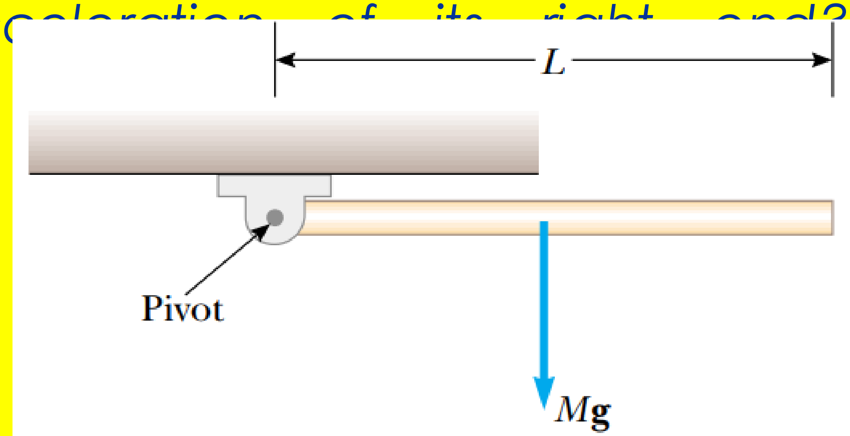


Quiz 10.10

Quick Quiz 10.10 You turn off your electric drill and find that the time interval for the rotating bit to come to rest due to frictional torque in the drill is Δt. You replace the bit with a larger one that results in a doubling of the moment of inertia of the entire rotating mechanism of the drill. When this larger bit is rotated at the same angular speed as the first and the drill is turned off, the frictional torque remains the same as that for the previous situation. The time for this second bit to come to rest is (a) $4\Delta t$ (b) $2\Delta t$ (c) Δt (d) $0.5\Delta t$ (e) $0.25\Delta t$ (f) impossible to determine.

Example 10.10 Rotating Rod

▶*A uniform rod of length L and mass M is attached at one end to a frictionless pivot and is free to rotate about the pivot in the vertical plane, as in Figure 10.18. The rod is released from rest in the horizontal position. What is the initial angular acceleration of the rod and the initial linear acceleration of its right end?*



$$\tau = Mg\left(\frac{L}{2}\right)$$

$$\sum \tau = I\alpha \quad and \quad I = \frac{1}{3}ML^2$$

$$\alpha = \frac{\tau}{I} = Mg\left(\frac{L}{2}\right) / \frac{1}{3}ML^2 = \frac{3g}{2L}$$

$$\rightarrow a_t = L\alpha = \frac{3}{2}g$$

10.8 Power in Rotational Motion

- The rate at which work is being done in a time interval *dt* is

$$\text{Power} = \frac{dW}{dt} = \tau \frac{d\theta}{dt} = \tau\omega$$

- This is analogous to $P = Fv$ in a linear system

- The work-kinetic energy theorem for rotational motion states that the net work done by external forces in rotating a symmetrical rigid object about a fixed axis equals the change in the object's rotational kinetic energy

$$\sum W = \int_{\omega_i}^{\omega_f} I\omega \, d\omega = \frac{1}{2} I\omega_f^2 - \frac{1}{2} I\omega_i^2$$

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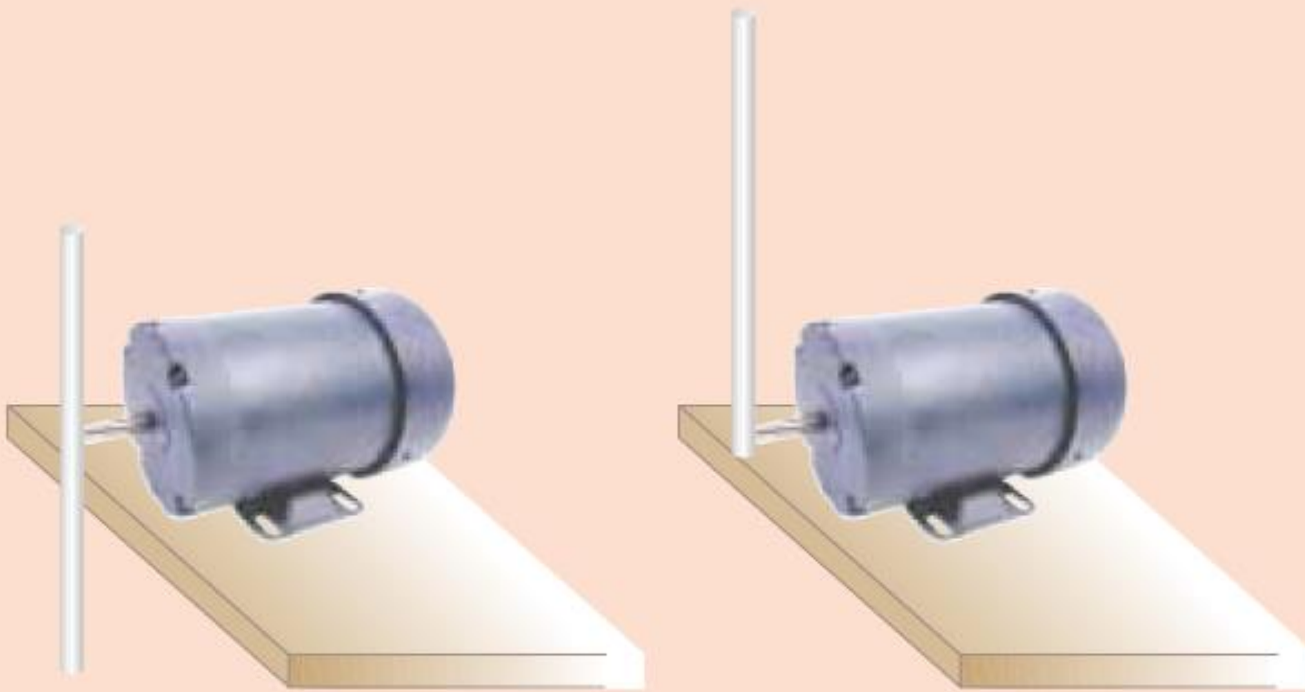
Quick Quiz 10.11 A rod is attached to the shaft of a motor at the center of the rod so that the rod is perpendicular to the shaft, as in Figure 10.23a. The motor is turned on and performs work W on the rod, accelerating it to an angular speed ω. The system is brought to rest, and the rod is attached to the shaft of the motor at one end of the rod as in Figure 10.23b. The motor is turned on and performs work W on the rod. The angular speed of the rod in the second situation is (a) 4ω (b) 2ω (c) ω (d) 0.5ω (e) 0.25ω (f) impossible to determine.



(a) (b)

Figure 10.23 (Quick Quiz 10.11) (a) A rod is rotated about its midpoint by a motor. (b) The rod is rotated about one of its ends.

Summary of Useful Equations

Rotational Motion About a Fixed Axis	Linear Motion
Angular speed $\omega = d\theta/dt$	Linear speed $v = dx/dt$
Angular acceleration $\alpha = d\omega/dt$	Linear acceleration $a = dv/dt$
Net torque $\Sigma\tau = I\alpha$	Net force $\Sigma F = ma$
If $\alpha =$ constant $\begin{cases} \omega_f = \omega_i + \alpha_t \\ \theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 \\ \omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i) \end{cases}$	If $a =$ constant $\begin{cases} v_f = v_i + at \\ x_f = x_i + v_i t + \frac{1}{2}at^2 \\ v_f^2 = v_i^2 + 2a(x_f - x_i) \end{cases}$
Work $W = \int_{\theta_i}^{\theta_f} \tau\, d\theta$	Work $W = \int_{x_i}^{x_f} F_x\, dx$
Rotational kinetic energy $K_R = \frac{1}{2}I\omega^2$	Kinetic energy $K = \frac{1}{2}mv^2$
Power $\mathcal{P} = \tau\omega$	Power $\mathcal{P} = Fv$
Angular momentum $L = I\omega$	Linear momentum $p = mv$
Net torque $\Sigma\tau = dL/dt$	Net force $\Sigma F = dp/dt$

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Total Kinetic Energy of a Rolling Object

- The total kinetic energy of a rolling object is the sum of the translational energy of its center of mass and the rotational kinetic energy about its center of mass
  - $K = \frac{1}{2} I_{CM} \omega^2 + \frac{1}{2} M v_{CM}^2$

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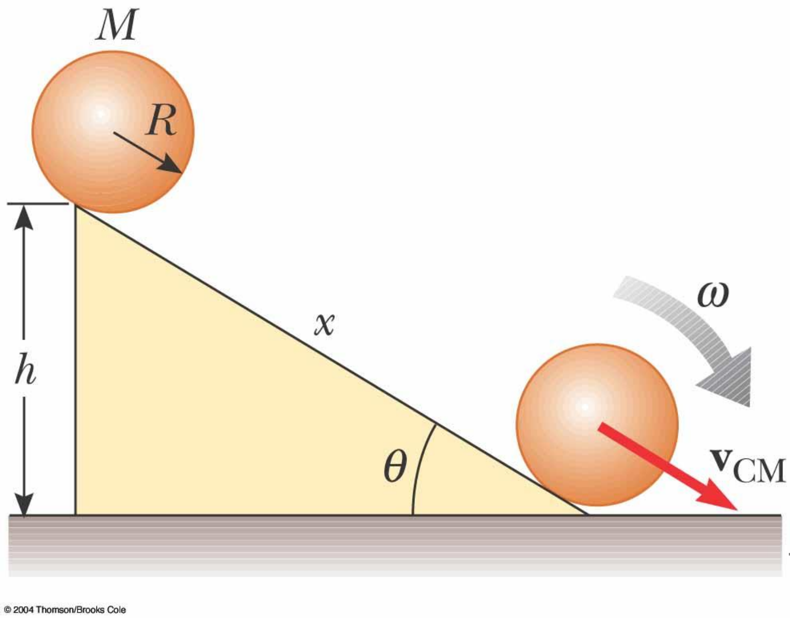
Total Kinetic Energy, Example

- Accelerated rolling motion is possible only if friction is present between the sphere and the incline
  - The friction produces the net torque required for rotation



Total Kinetic Energy, Example cont

- Despite the friction, no loss of mechanical energy occurs because the contact point is at rest relative to the surface at any instant
- Let $U = 0$ at the bottom of the plane
- $K_f + U_f = K_i + U_i$
- $K_f = ½ (I_{CM} / R^2) v_{CM}^2 + ½ Mv_{CM}^2$
- $U_i = Mgh$
- $U_f = K_i = 0$

Quick Quiz 10.12 A ball rolls without slipping down incline A, starting from rest. At the same time, a box starts from rest and slides down incline B, which is identical to incline A except that it is frictionless. Which arrives at the bottom first? (a) the ball (b) the box (c) Both arrive at the same time. (d) impossible to determine

Quick Quiz 10.13 Two solid spheres roll down an incline, starting from rest. Sphere A has twice the mass and twice the radius of sphere B. Which arrives at the bottom first? (a) sphere A (b) sphere B (c) Both arrive at the same time. (d) impossible to determine

Quick Quiz 10.14 Two spheres roll down an incline, starting from rest. Sphere A has the same mass and radius as sphere B, but sphere A is solid while sphere B is hollow. Which arrives at the bottom first? (a) sphere A (b) sphere B (c) Both arrive at the same time. (d) impossible to determine

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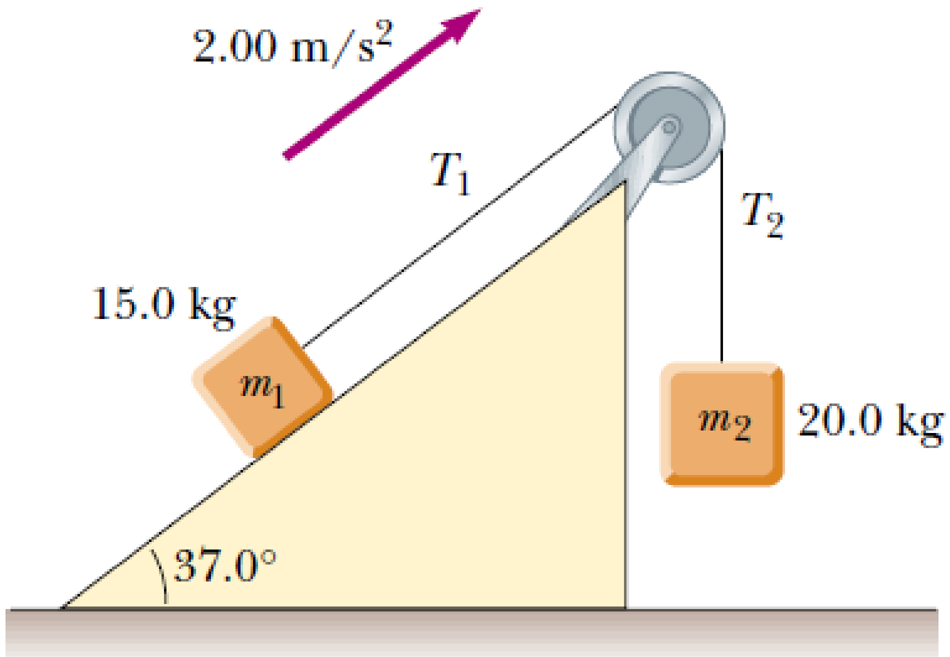
Two blocks, as shown in Figure P10.71, are connected by a string of negligible mass passing over a pulley of radius 0.250 m and moment of inertia I. The block on the frictionless incline is moving up with a constant acceleration of 2.00 m/s². (a) Determine T_1 and T_2, the tensions in the two parts of the string. (b) Find the moment of inertia of the pulley.



$m_2 g - T_2 = m_2 a$ --- (1)

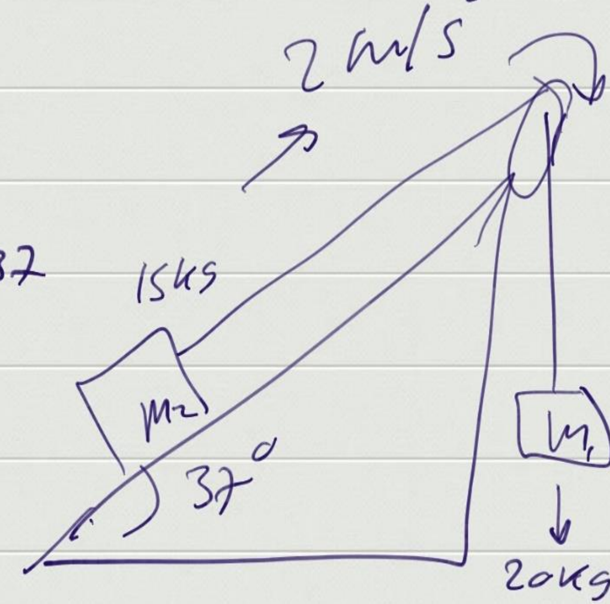
$\rightarrow T_2 = m_2(g - a) = 20(9.8 - 2)$

$= 156\ N$ #

$T_1 - m_1 g \sin\theta = m_1 a$ (2)

$\rightarrow T_1 = m_1 a + m_1 g \sin\theta = 15 \times 2 + 15 \times 9.8 \times \sin 37$

$= 118\ N$ #



b) ∵ ∴ $\tau_{net} = I\alpha \rightarrow (T_2 - T_1)R = I\alpha$

$\rightarrow (156 - 118)R = I\left(\frac{a}{R}\right)$

$\rightarrow I = R^2(156 - 118) \div a$

$= (0.25)^2(156 - 118) \div 2 = 1.17\ kg \cdot m^2$

Chapter Summary

- If a particle moves in a circular path of radius *r through an angle θ (measured in radians),* the arc length it moves through is $s = r\theta$.
- The angular position of a rigid object is defined as the angle *θ* between a reference line attached to the object and a reference line fixed in space. The angular displacement of a particle moving in a circular path or a rigid object rotating about a fixed axis is $\Delta\theta = \theta_f - \theta_i$.
- The instantaneous angular speed of a particle moving in a circular path or of a rigid object rotating about a fixed axis is: $\omega = d\theta/dt$
- The instantaneous angular acceleration of a particle moving in a circular path or a rotating rigid object is: $\alpha = d\omega/dt$
- When a rigid object rotates about a fixed axis, every part of the object has the same angular speed and the same angular acceleration.

Chapter Summary (Continued)

If an object rotates about a fixed axis under constant angular acceleration, one can apply equations of kinematics that are analogous to those for linear motion under constant linear acceleration:

$$\text{with } \theta_i = 0,\ \alpha = \text{constant}$$

$$\omega_f = \omega_i + \alpha t \qquad (10.6)$$

$$\theta_f = \omega_i t + \frac{1}{2}\alpha t^2 \qquad (10.7)$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\theta_f \qquad (10.8)$$

- Relationships between linear and rotational quantities:

$$s = r\theta, \qquad v = r\omega, \qquad a_t = r\alpha$$

- The moment of inertia of a system of particles is defined as

$$I = \sum_i m_i r_i^2$$

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Thanks

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