Part 3: Properties of Laser Radiation

Physics 435: Laser Physics

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Outline

1. **Laser Linewidth**
2. Beam Divergence
3. Beam Coherence
4. Brightness
5. Focusing Properties of Laser Radiation
6. Q-Switching
7. Mode Locking
8. Frequency Doubling
9. Phase Conjugation
10. Problems

1.1 Recall Cavity Modes from Part 1

- In part 1, we discussed that the laser cavity supports specific resonant frequencies, known as **cavity modes**, which are determined by the physical dimensions of the cavity.
- It was shown that there are two types of cavity modes: **longitudinal modes** and **transverse modes**.
- The longitudinal modes correspond to standing waves along the length of the cavity, while the transverse modes correspond to the spatial distribution of the electromagnetic field in the plane perpendicular to the cavity axis.
- The presence of multiple cavity modes can lead to a broadening of the laser linewidth, as the laser may emit light at several closely spaced frequencies corresponding to these modes.

1.1 Recall Cavity Modes from Part 1

- It was also shown that the frequency spacing between adjacent longitudinal modes is given by:

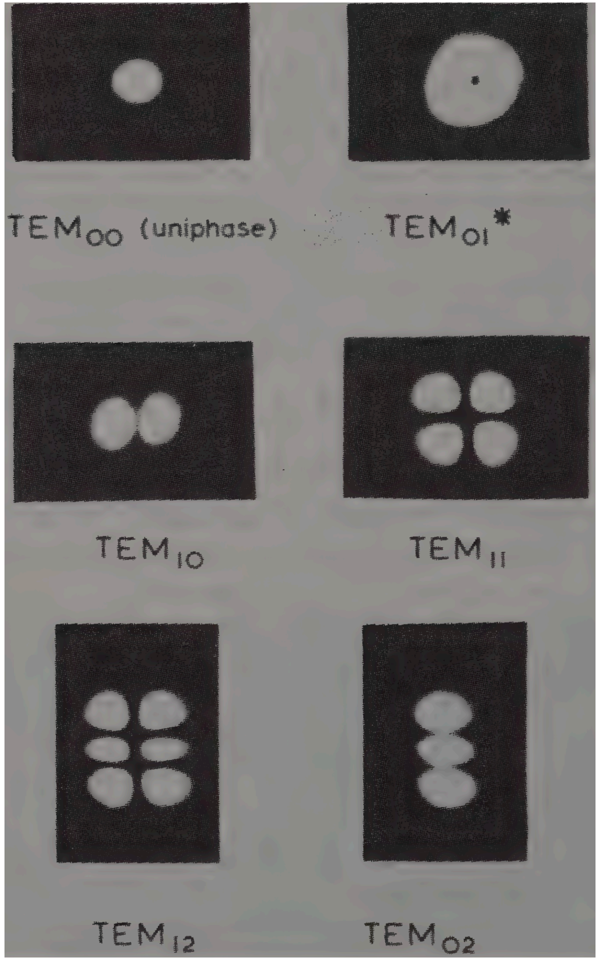
$$\Delta\nu = \frac{c}{2L}, \tag{1}$$

where L is the length of the cavity.

- Therefore, the frequency of the p mode (integer number) is given by:

$$\nu_p = p\,\frac{c}{2L}. \tag{2}$$

1.1 Recall Cavity Modes from Part 1



- It was shown that the **Transverse Modes** are typically denoted as $\mathrm{TEM}_{\mathrm{qr}}$, where q and r are integers representing the number of nodes in the transverse electric field distribution along the horizontal and vertical directions, respectively.
- It was mentioned that the fundamental transverse mode is the TEM_{00} mode, which has a Gaussian intensity profile and is often the most desirable mode for many applications due to its high beam quality.

1.2 Frequency of All Cavity Modes

- This part show that both longitudinal and transverse mode frequencies ν_{qrp} are related to the cavity's physical dimensions as:

$$\nu_{\mathrm{qrp}} = \frac{c}{2L}\left(p + \frac{1+q+r}{\pi}\cos^{-1}\left(g_1 g_2\right)^{1/2}\right), \tag{3}$$

where

$$g_1 = 1 - \frac{L}{r_1} \qquad \text{and} \qquad g_2 = 1 - \frac{L}{r_2}, \tag{4}$$

Here r_1 and r_2 are the radii of curvature of the cavity mirrors being positive for concave mirrors and negative for convex mirrors.

1.3 Example

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Example 1.1

Find the cavity modes ν_{qrp} for a cavity with concave mirrors at $r_1 = r_2 = 2L$

$$g_1 = g_2 = 1 - \frac{L}{2L} = \frac{1}{2}$$

$$\Rightarrow \quad \cos^{-1}\left(g_1 g_2\right)^{1/2} = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

$$\Longrightarrow \quad \nu_{\text{qrp}} = \frac{c}{2L}\left(p + \frac{1+q+r}{3}\right)$$

Note that many modes can have the same frequency, see the figure next page.

1.3 Example

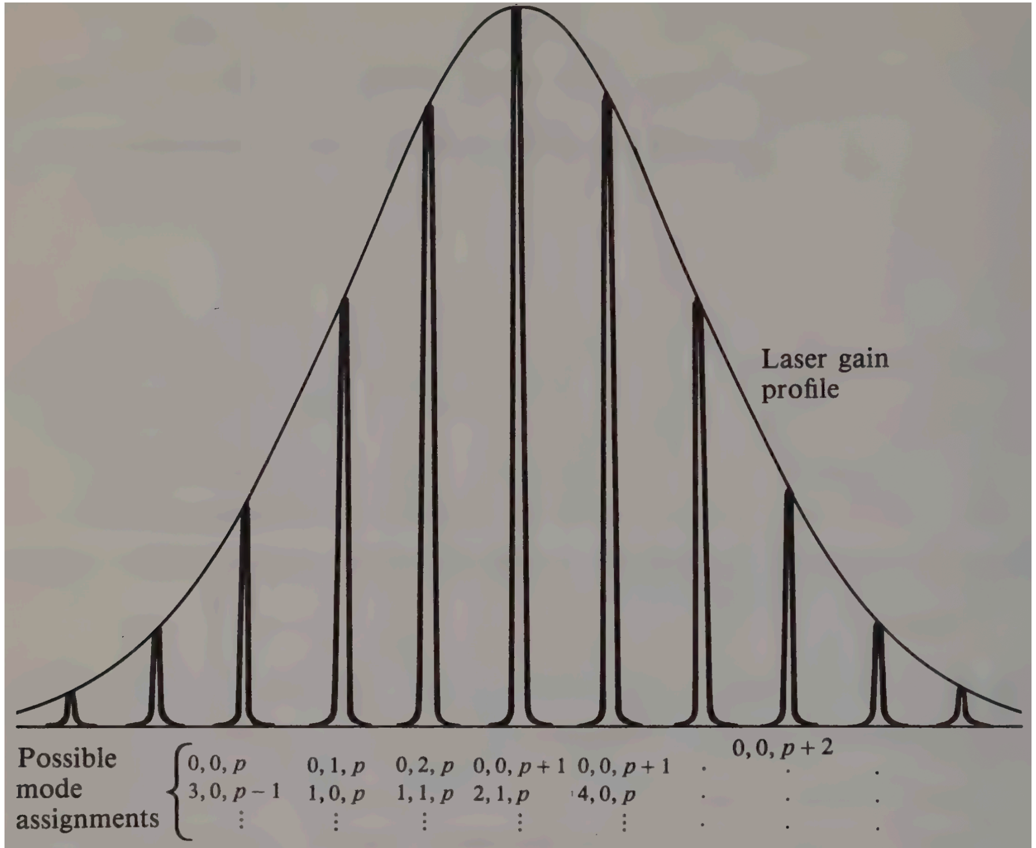

Fig. 3.2 The mode frequency spectrum for a cavity (where $r_1 = r_2 = 2L$) superimposed on the laser gain profile. Note that many modes have the same frequency (that is they are 'degenerate').

1.4 How to reduce the number of modes within the gain bandwidth?

- Some laser applications require a single longitudinal and transverse mode operation to achieve a narrow linewidth.
- There are several techniques to reduce the number of longitudinal modes within the gain bandwidth, such as:

1. Using a shorter cavity length to increase the mode spacing $\Delta\nu \propto 1/L$.
2. Introducing intracavity elements such as etalons or gratings to selectively enhance certain modes while suppressing others.
3. Employing active stabilization techniques to lock the laser frequency to a specific mode.

1.4 How to reduce the number of modes within the gain bandwidth?

- Additionally, there are techniques to reduce the number of transverse modes, such as:

  1. Using apertures or spatial filters to block higher-order transverse modes since they have larger spacial spread compared to the fundamental mode TEM_{00}.
  2. Designing the cavity geometry (e.g., mirrors) to favor the fundamental mode.
  3. Employing gain media with a small cross-sectional area to limit the number of supported transverse modes.

1.5 Example

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Example 1.2

Find the cavity modes ν_{qrp} for a confocal cavity with flat mirrors ($r_1 = r_2 = \infty$)

$$g_1 = g_2 = 1 - \frac{L}{\infty} = 1$$

$$\Rightarrow \quad \cos^{-1}(g_1 g_2)^{1/2} = \cos^{-1}(1) = 0$$

$$\Longrightarrow \quad \nu_{\mathrm{qrp}} = \frac{c}{2L}\left(p + \frac{1+q+r}{\pi}(0)\right) = \frac{c}{2L}\,p$$

Notice that the transverse modes do not contribute to the frequency of the cavity modes when the mirrors are flat.

1.6 Quality Factor (Q-Factor) of a Laser Cavity

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- The quality factor, or Q-factor, of a laser cavity is a dimensionless parameter that characterizes the sharpness of the resonance of the cavity modes, defined as:

$$Q = \frac{\text{resonant frequency}}{\text{linewidth}} = \frac{\nu}{\Delta\nu} \tag{5}$$

- A higher Q-factor indicates a narrower linewidth and a more stable laser output, while a lower Q-factor indicates a broader linewidth and a less stable output.
- The Q-factor can be influenced by various factors, including the cavity design, mirror reflectivity, and the gain medium properties.

1.6 Quality Factor (Q-Factor) of a Laser Cavity

- Alternative definition of the Q-factor is:

$$Q = \frac{2\pi \times \text{energy stored in the resonator at resonance}}{\text{energy dissipated per cycle}} \quad (6)$$

- Following this definition one can verify that the linewidth and Q-factor can be expressed as:

$$\Delta\nu = \frac{\nu}{Q} = \frac{c(1 - R_1 R_2)}{4\pi L}. \quad (7)$$

1.6 Quality Factor (Q-Factor) of a Laser Cavity

Example 1.3

Estimate the linewidth of a laser where $R_1 = R_2 = 0.99$ and $L = 0.5$ m.

$$\Delta\nu = \frac{c(1 - R_1 R_2)}{4\pi L} = \frac{3 \times 10^8 \times (1 - 0.99^2)}{4 \times \pi \times 0.5} = 950 \text{ kHz}$$

- Interestingly, a linewidth of as small as 1 Hz has been reported!
- A common issue with very narrow linewidth is that the single mode frequency may fluctuate due to environmental factors such as temperature changes, vibrations, and air currents that affect the cavity properties and stability.

1. Laser Linewidth

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3. Beam Coherence

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2.1 What is Beam Divergence?

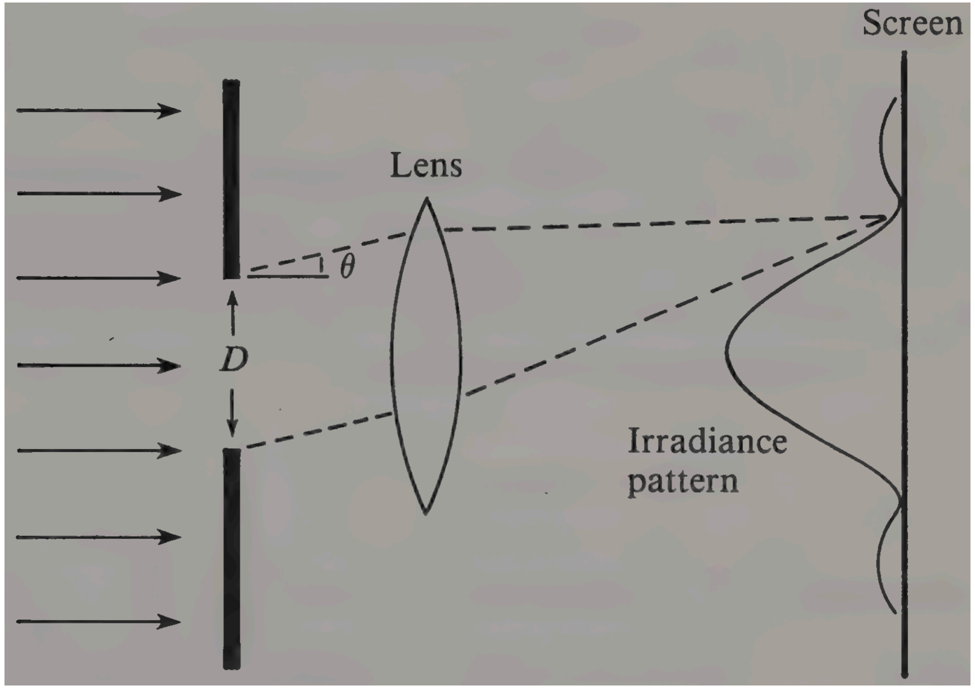
- Typical light sources such as light bulbs emit light in **all directions**, resulting in a beam that spreads out significantly as it propagates in space.
- One of the main properties of laser radiation is that its beam output is highly **collimated**, meaning that the light rays are parallel and do not spread out significantly as they propagate.
- However, laser beams are **not perfectly collimated** and do exhibit some degree of divergence, which is the gradual spreading of the beam as it travels through space.
- The divergence of a laser beam is **caused by** the wave nature of light and the finite size of the laser aperture.

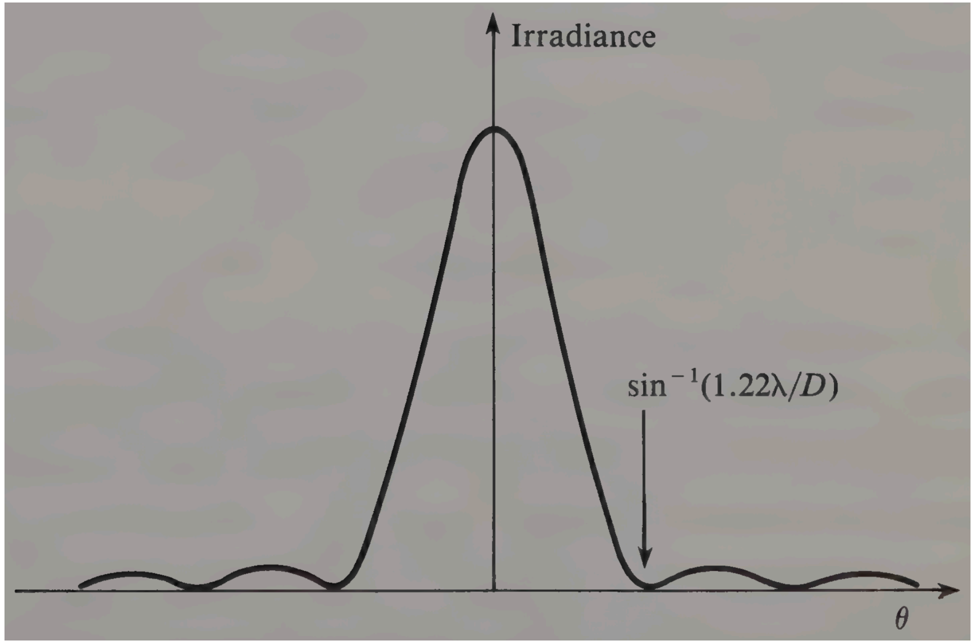
2.1 What is Beam Divergence?

- Beam divergence is a **disadvantage** as it causes the intensity of the laser beam to decrease with distance, which can limit the effective range and applications of the laser.
- Beam divergence can be **reduced** by using beam collimating optics such as lenses or mirrors to focus the beam and reduce its spread, or by using laser cavities with specific designs (such as using large apertures) that minimize divergence.

2.2 Calculating Beam Divergence

- Consider a laser beam approximated as a plane waves with a Gaussian intensity profile (TEM_{00}) passing through a circular aperture of diameter D.





2.2 Calculating Beam Divergence

- The beam will diverge and show interference patterns at a screen placed at a distance z from the aperture.
- The interference pattern consists of a central bright spot (known as the **Airy disk**) containing most of the light energy (about 84%) surrounded by concentric rings of decreasing intensity.
- The angle of divergence θ can be calculated using the formula:

$$\theta = \sin^{-1}\left(\frac{k\lambda}{D}\right), \tag{8}$$

where k is a constant that depends on the geometry of the aperture and for a circular aperture, k is approximately 1.22.

2.2 Calculating Beam Divergence

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Example 2.4

Calculate the divergence angle of a GaAs laser beam with a wavelength of 900 nm that has an active region with cross-sectional dimensions of 3 um by 10 um. Assume that $k \approx 1$

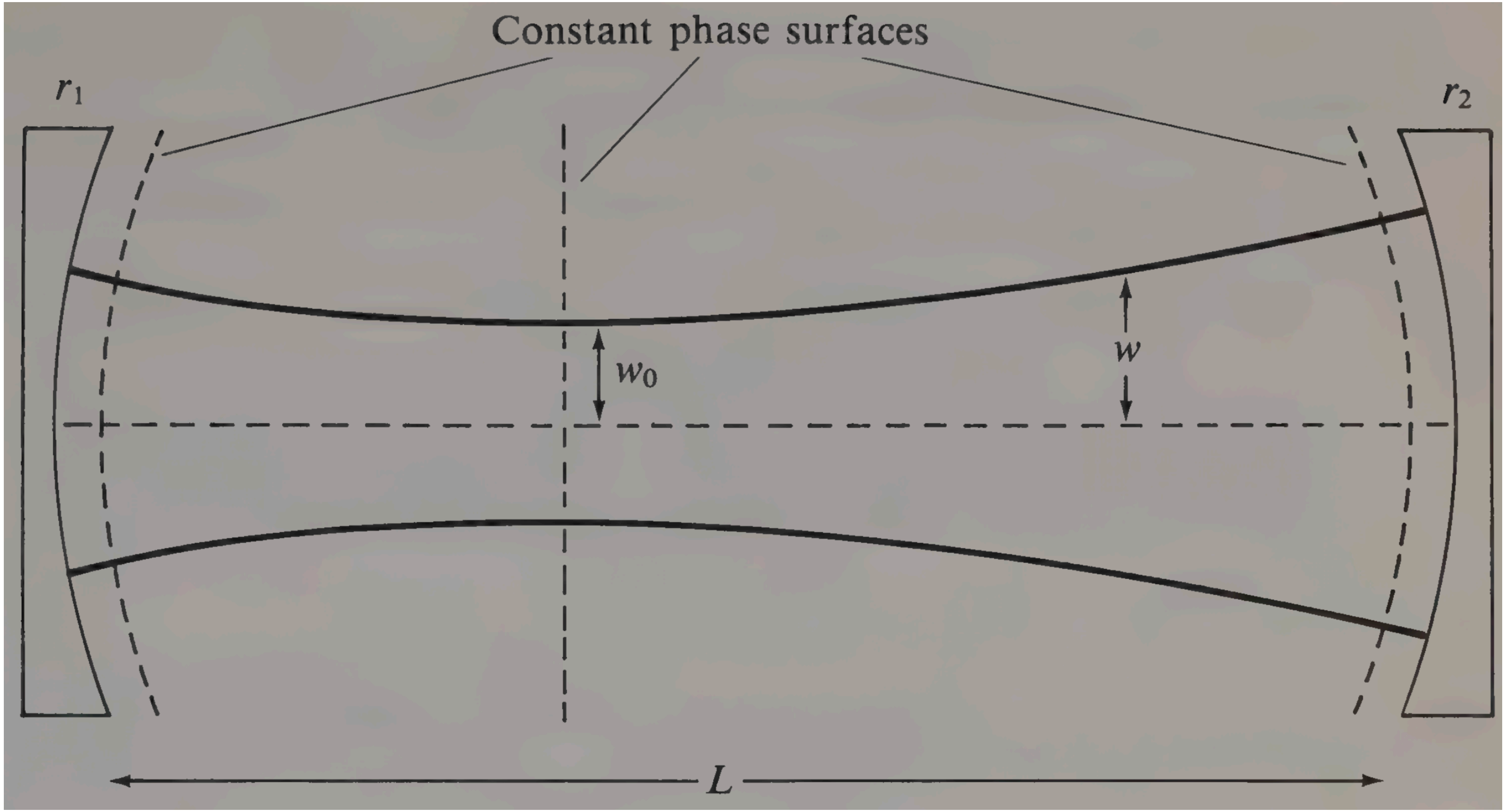
$$\theta_1 = \sin^{-1}\left(\frac{k\lambda}{D}\right) = \sin^{-1}\left(\frac{900 \times 10^{-9}}{3 \times 10^{-6}}\right) = \texttt{17.46deg}$$

$$\theta_2 = \sin^{-1}\left(\frac{k\lambda}{D}\right) = \sin^{-1}\left(\frac{900 \times 10^{-9}}{10 \times 10^{-6}}\right) = \texttt{5.16deg}$$

2.3 Divergence of Gaussian Beams inside a Laser Cavity

- To keep the laser beam **confined** within the cavity and maintain a **stable** cavity operation, the cavity must be designed such that the beam waist (the location where the beam is narrowest) is appropriately positioned and the curvature of the mirrors is chosen to fit the gain medium.
- Consider two concave mirrors with radii of curvature r_1 and r_2 separated by a distance L.

2.3 Divergence of Gaussian Beams inside a Laser Cavity



2.3 Divergence of Gaussian Beams inside a Laser Cavity

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- The beam waist w_0 is located at the center of the cavity and can be calculated using the formula:

$$w_0^2 = \frac{\lambda}{\pi} \sqrt{\frac{L(r_1 - L)(r_2 - L)(r_1 + r_2 - L)}{(r_1 + r_2 - 2L)^2}} \tag{9}$$

- In the case of nearly confocal cavity where $r = r_1 = r_2 \approx L$, the beam waist is simplified by:

$$w_0^2 = \frac{\lambda r}{2\pi}. \tag{10}$$

2.3 Divergence of Gaussian Beams inside a Laser Cavity

- The variation of the beam radius $w(z)$ along the cavity axis can be calculated using the formula:

$$w(z) = w_0 \sqrt{1 + \left(\frac{z\lambda}{\pi w_0^2}\right)^2}, \tag{11}$$

2.3 Divergence of Gaussian Beams inside a Laser Cavity

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Example 2.5

(A) Calculate the beam waist w_0 of HeNe laser for a confocal cavity with $r_1 = r_2 = L = 0.5$ m and a wavelength of 633 nm. (B) Then calculate the beam radius $w(z)$ at the mirror surfaces.

$$w_0 = \sqrt{\frac{\lambda r}{2\pi}} = \sqrt{\frac{633 \times 10^{-9} \times 0.5}{2\pi}} = 224\ \mu\text{m}$$

For the mirror surfaces, $z = L/2 = 0.25$ m, therefore:

$$w(z) = w_0\sqrt{1 + \left(\frac{z\lambda}{\pi w_0^2}\right)^2} = 312\ \mu\text{m}$$

2.3 Divergence of Gaussian Beams inside a Laser Cavity

- To calculate the angular divergence inside the cavity as the beam expands from the beam waist, we can use the formula:

$$\theta = \sin^{-1}\left(\frac{w}{z}\right), \tag{12}$$

where w is the beam radius at distance z from the beam waist. This equation can be approximated to:

$$\theta_{\text{confocal}} = \sin^{-1}\left(\frac{\lambda}{\pi w_0}\right), \tag{13}$$

2.3 Divergence of Gaussian Beams inside a Laser Cavity

Example 2.6

Calculate the angular divergence of the HeNe laser beam inside the confocal cavity from the previous example.

$$\theta = \sin^{-1}\left(\frac{\lambda}{\pi w_0}\right) = \sin^{-1}\left(\frac{633 \times 10^{-9}}{\pi \times 224 \times 10^{-6}}\right) = 0.052^{\circ}$$

2.4 Conditions for making a stable laser cavity

- A stable laser cavity is one in which the light remains confined within the cavity and does not diverge excessively, allowing for sustained lasing action.
- The stability of a laser cavity can be determined by the **g-parameters** defined as:

$$0 \leq g_1 g_2 \leq 1 \tag{14}$$

where $g_1 = 1 - \frac{L}{r_1}$ and $g_2 = 1 - \frac{L}{r_2}$.

- Outside this range, the cavity is unstable, and the light will diverge and escape from the cavity, preventing lasing action.

2.4 Conditions for making a stable laser cavity

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Example 2.7

Is the confocal cavity with $r_1 = r_2 = L$ stable?

$$g_1 = g_2 = 1 - \frac{L}{L} = 0$$

$$\Rightarrow \quad g_1 g_2 = 0$$

Therefore, the confocal cavity is stable since $0 \le g_1 g_2 \le 1$.

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3. **Beam Coherence**

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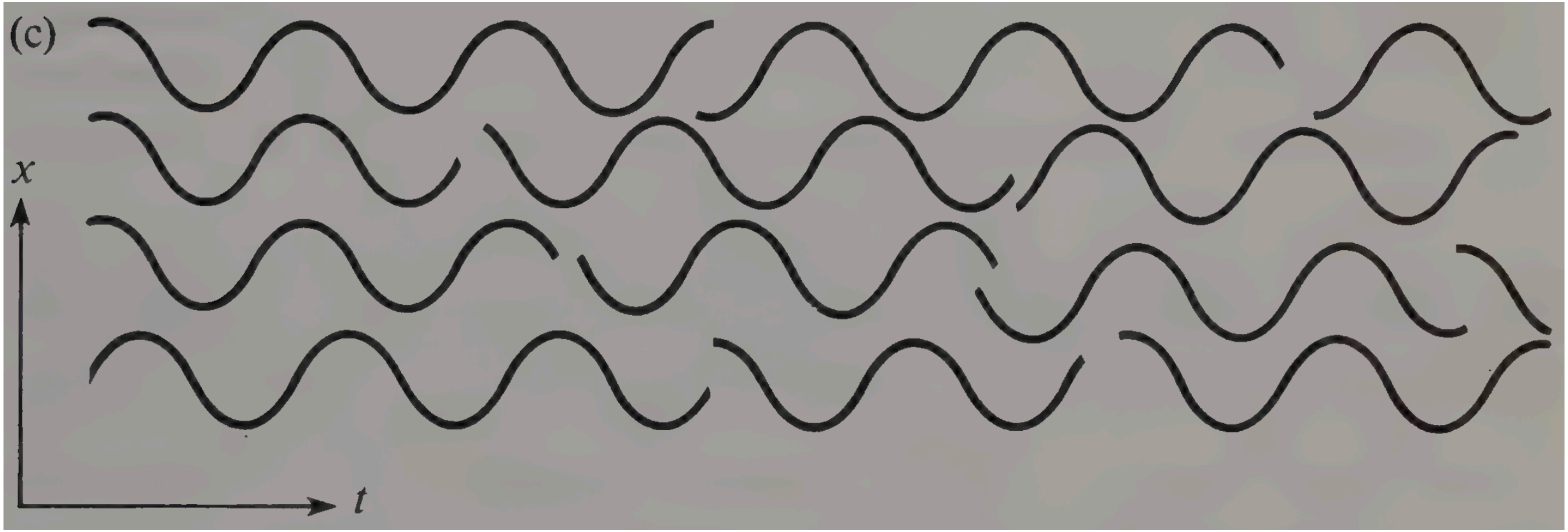
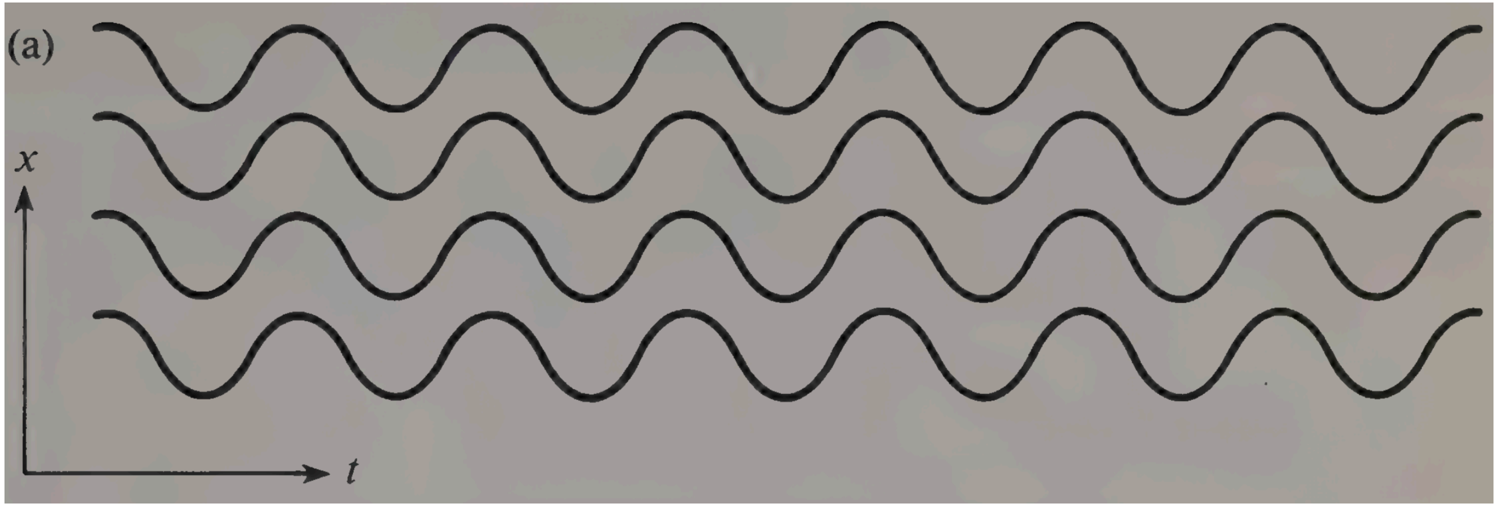
3.1 What is Coherence?



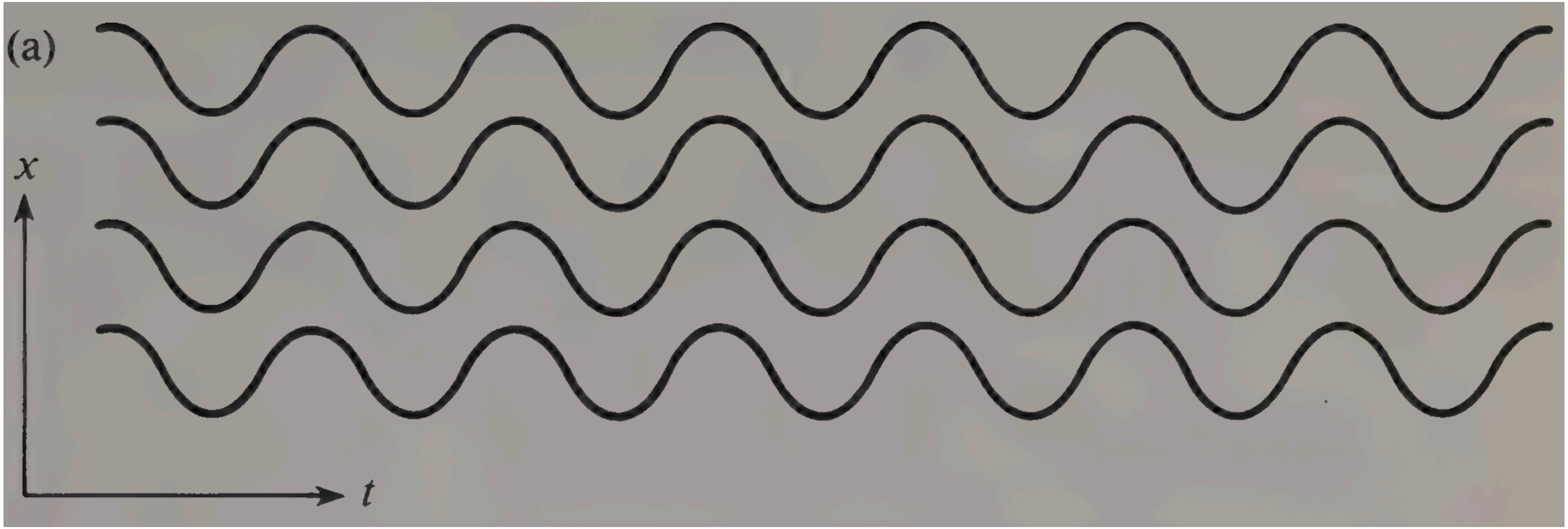
Conventional light sources such as light bulbs emit light that is **incoherent**, meaning that the emitted light waves have random phases, directions, and frequencies, resulting in a lack of correlation between different points in space and time.

3.1 What is Coherence?



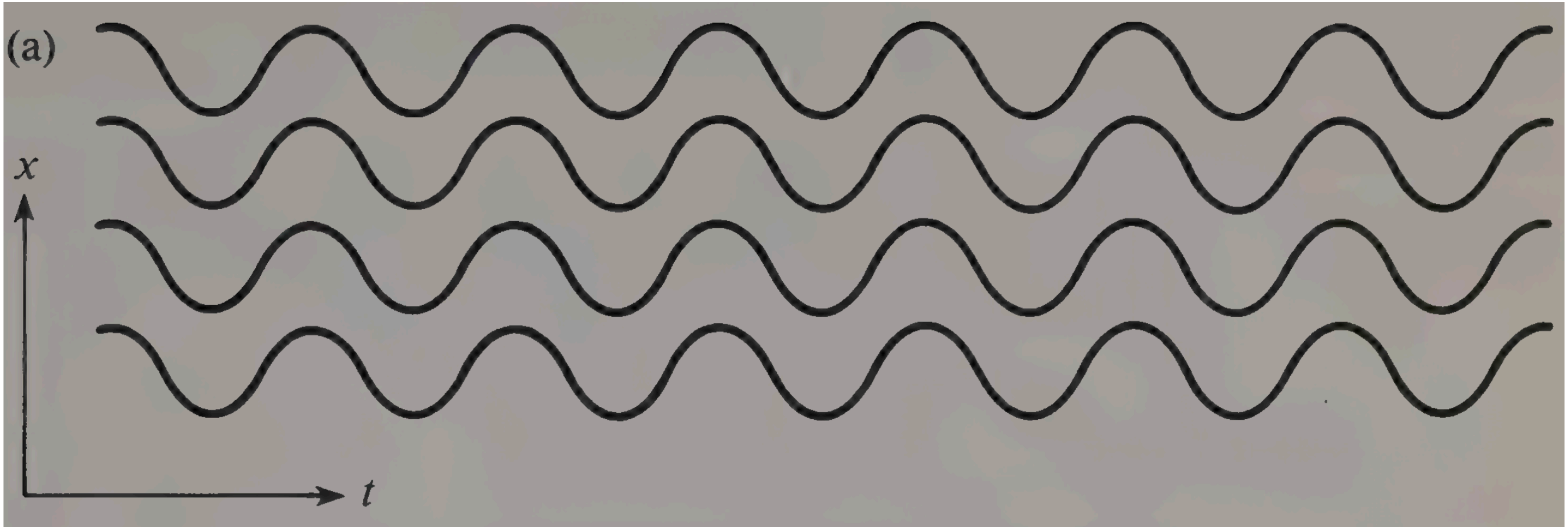
- In contrast, **laser light is coherent**, meaning that the emitted light waves have a fixed phase relationship, are emitted in the same direction, and have a narrow frequency spectrum.
- The coherence of laser light can be categorized into two types: **spatial coherence** and **temporal coherence**.

3.2 Spatial Coherence



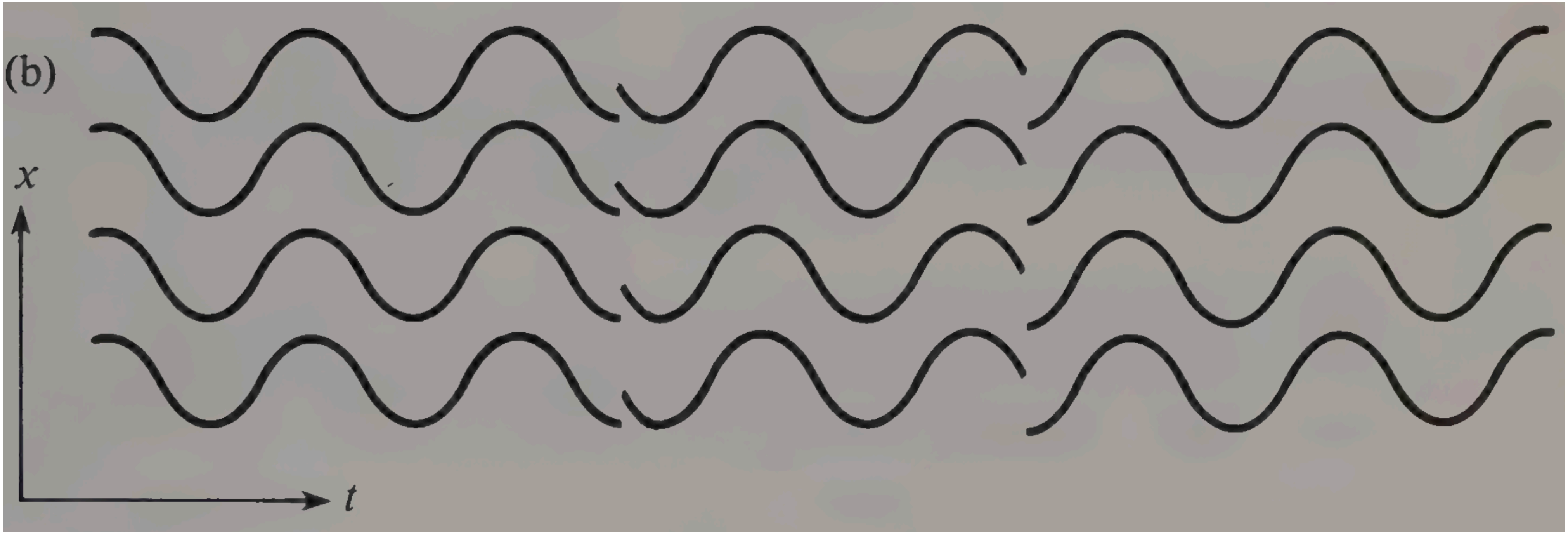
Spatial coherence refers to the correlation between the phases of light waves at different points in space. A laser beam with high spatial coherence will have identical phase across its entire cross-section in space.

3.3 Temporal Coherence



Temporal coherence refers to the relative phase relationship of the electric field at the same place as a function of time. If the phase changes uniformly with time then the beam is said to show perfect temporal coherence.

3.4 Coherence Length and Time



The **coherence length** (L_c) is the distance over which the light waves maintain a fixed phase relationship.

3.4 Coherence Length and Time

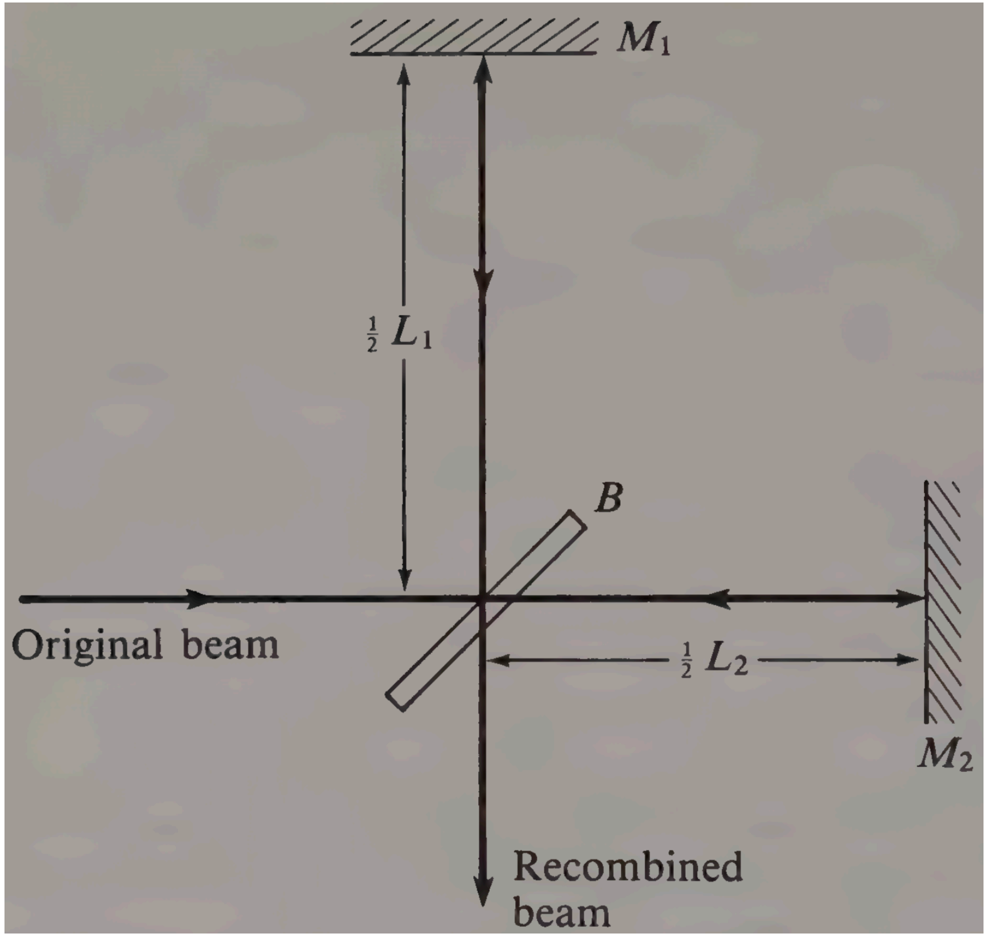
- **Coherence time** (t_c) is the time duration over which the light waves maintain a fixed phase relationship, give by:

$$t_c = \frac{L_c}{c}. \tag{15}$$

- The coherence length and time are inversely related to the linewidth of the laser ($\Delta\nu$), meaning that a narrower linewidth corresponds to a longer coherence length and time,

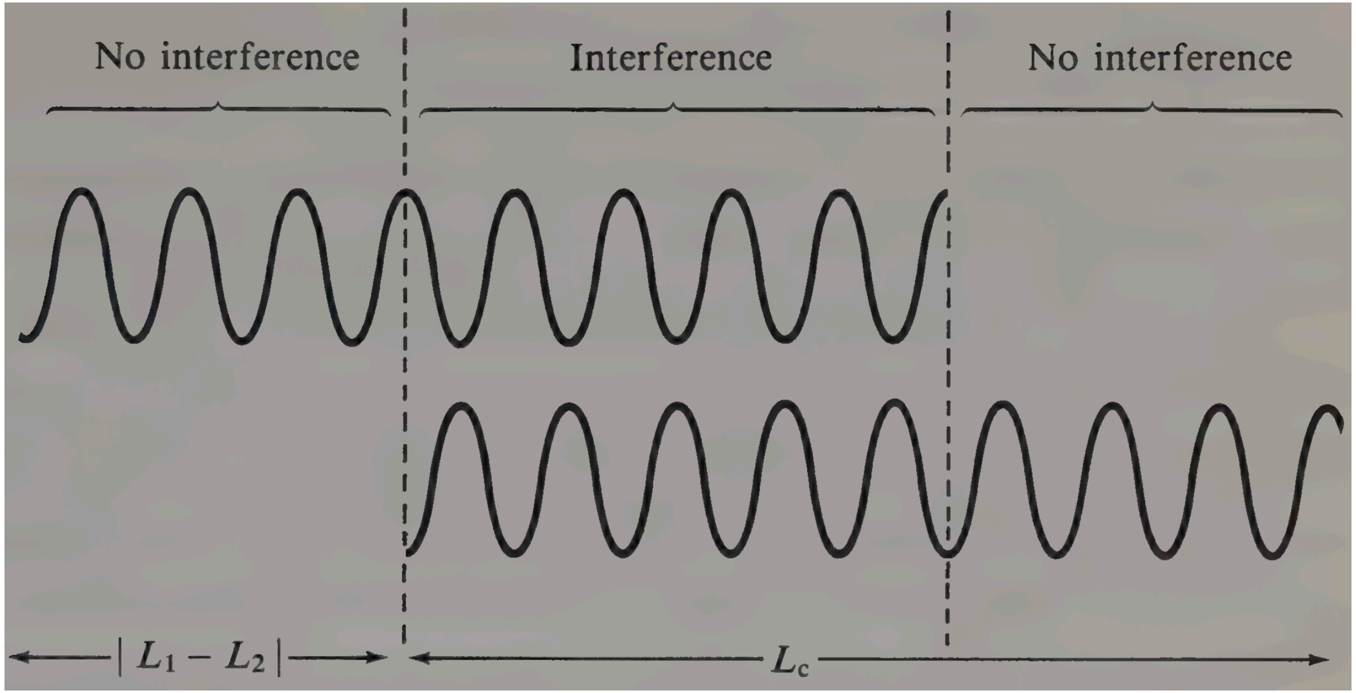
$$t_c = \frac{1}{\Delta\nu}. \tag{16}$$

3.5 How to measure the coherence length and time?



The coherence length and time can be measured using interferometric techniques, such as the Michelson interferometer, which allows for the observation of interference patterns that arise from the superposition of light waves with different path lengths or time delays.

3.5 How to measure the coherence length and time?



By analyzing the visibility of the interference fringes as a function of the path length difference or time delay, one can *experimentally* determine the coherence length and time of the laser light.

3.5 How to measure the coherence length and time?

Example 3.8

Find the coherence length and time of the following sources: (A) A conventional light source from low-pressure sodium lamp with a linewidth of 5.1×10^{11}Hz at $\lambda = 589\ \mu$m.

$$t_c = \frac{1}{\Delta\nu} = \frac{1}{5.1 \times 10^{11}} = 1.96 \text{ ps}$$

$$L_c = c\ t_c = 3 \times 10^8 \times \left(\frac{1}{5.1 \times 10^{11}}\right) = 0.6 \text{ mm}$$

3.5 How to measure the coherence length and time?

(B) A HeNe laser of multi-mode with a linewidth of 1500 MHz at $\lambda = 633$nm.

$$t_c = \frac{1}{\Delta\nu} = \frac{1}{1500 \times 10^6} = 0.67 \text{ ns}$$

$$L_c = c\, t_c = 3 \times 10^8 \times \left(\frac{1}{1500 \times 10^6}\right) = 20 \text{ cm}$$

(C) A HeNe laser of single-mode with a linewidth of 1 MHz.

$$t_c = \frac{1}{\Delta\nu} = \frac{1}{1 \times 10^6} = 1\ \mu s$$

$$L_c = c\, t_c = 3 \times 10^8 \times \left(\frac{1}{1 \times 10^6}\right) = 300 \text{ m}$$

3.6 Coherence of Some Common lasers

- Coherence is important in any applications where the laser beam is split into parts that traverse different distances. These include the interferometric measurement of distance.
- Coherence make focusing of the laser beam to a small spot with high intensity possible, which is important for applications requiring high power density such as laser cutting and laser surgery.

3.6 Coherence of Some Common lasers

Table 3.1 Summary of coherence lengths of some common lasers

Laser	*Typical coherence length*
HeNe single transverse, single longitudinal mode	up to 1000 m
He-Ne multimode	0.1 to 0.2 m
Argon multimode	0.02 m
Nd : YAG	10^{-2} m
Nd : glass	2×10^{-4} m
GaAs	1×10^{-3} m
Ruby: for whole output pulse	10^{-2} m
within a spike forming part of the pulse	$\lesssim c$ times spike length, i.e. $\lesssim 30$ m

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4.1 Definition of Brightness

- Brightness, also known as radiance, is a measure of the intensity of light emitted by a source per unit area per unit solid angle Ω,

$$\text{Brightness} = \frac{p}{A\Omega}, \tag{17}$$

where Ω can be calculated as

$$\Omega = \frac{A}{r^2} \approx \pi\theta^2. \tag{18}$$

- The unit of brightness is watts per square meter per steradian ($\text{W m}^{-2}\text{sr}^{-1}$).

4.2 Example

Example 4.9

Consider a HeNe laser with a power output of 3 mW, a beam diameter of 0.6 mm, and a divergence angle of 52μrad. Calculate the brightness of the laser beam.

$$\Omega = \pi\theta^2 = \pi \times \left(52 \times 10^{-6}\right)^2 = 8.5 \times 10^{-9}\text{sr}$$

$$A = \pi\left(\frac{D}{2}\right)^2 = \pi \times \left(\frac{0.6 \times 10^{-3}}{2}\right)^2 = 2.8 \times 10^{-7}\text{m}^2$$

$$\text{Brightness} = \frac{p}{A\Omega} = 1.2 \times 10^{12}\ \text{W m}^{-2}\text{sr}^{-1}$$

For comparison, the brightness of the sun is $\sim 1.3 \times 10^{6}\ \text{W m}^{-2}\text{sr}^{-1}$.

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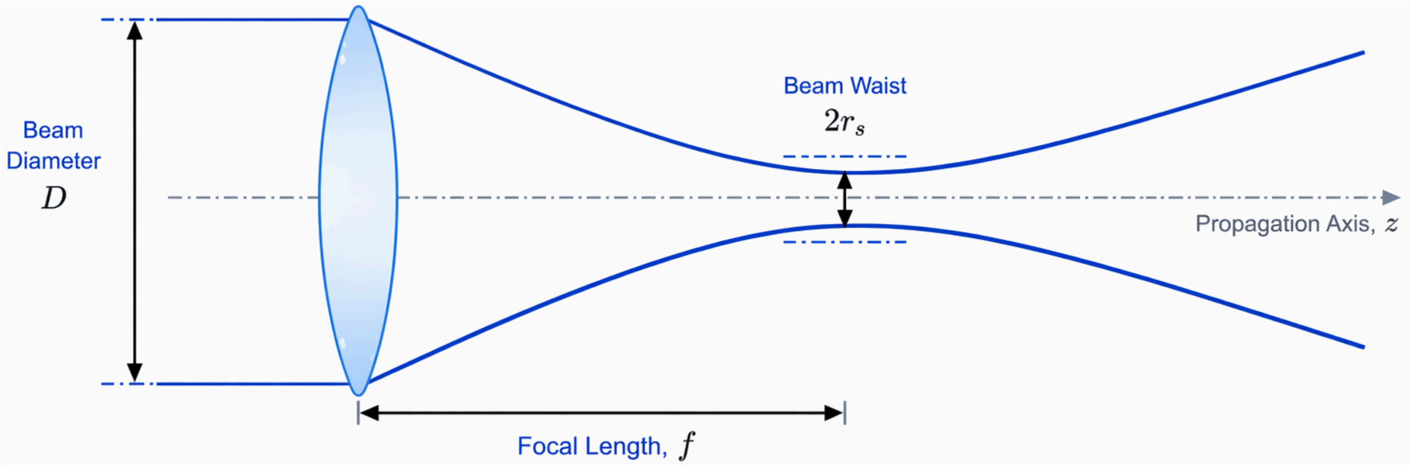
5.1 How tight can we focus a laser beam?



- Since laser beams diverge, and for many applications we need to focus the beam to the smallest spot possible of radius r_s, using a focusing lens with focal length f. The minimum spot size that can be achieved is:

$$r_s = \frac{2}{\pi}\lambda\frac{f}{D}, \qquad (19)$$

where D is the diameter of the laser beam before the lens.

5.1 How tight can we focus a laser beam?

- $F = f/D$ is known as the f-number of the focusing system, and it is a measure of how tightly the beam can be focused.
- Since $F < 1$ is not practical, the minimum spot size is typically on the order of the wavelength of the laser light, which is a fundamental limit known as the **diffraction limit**.

5.1 How tight can we focus a laser beam?

Example 5.10

Consider a 1 mW HeNe laser with a wavelength of 633 nm and an F number of 1. Calculate the intensity of the focused beam at the diffraction limit.

$$r_s = \frac{2}{\pi}\lambda F = \frac{2}{\pi} \times \left(633 \times 10^{-9}\right) \times 1 = 403 \text{ nm}$$

$$\text{Intensity} = \frac{p}{A} = \frac{p}{\pi r_s^2} = \frac{10^{-3}}{\pi \times \left(403 \times 10^{-9}\right)^2} = 2 \times 10^9 \text{ W m}^{-2}$$

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