Part 1: Laser Fundamentals

Physics 435: Laser Physics

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Version 3

Outline

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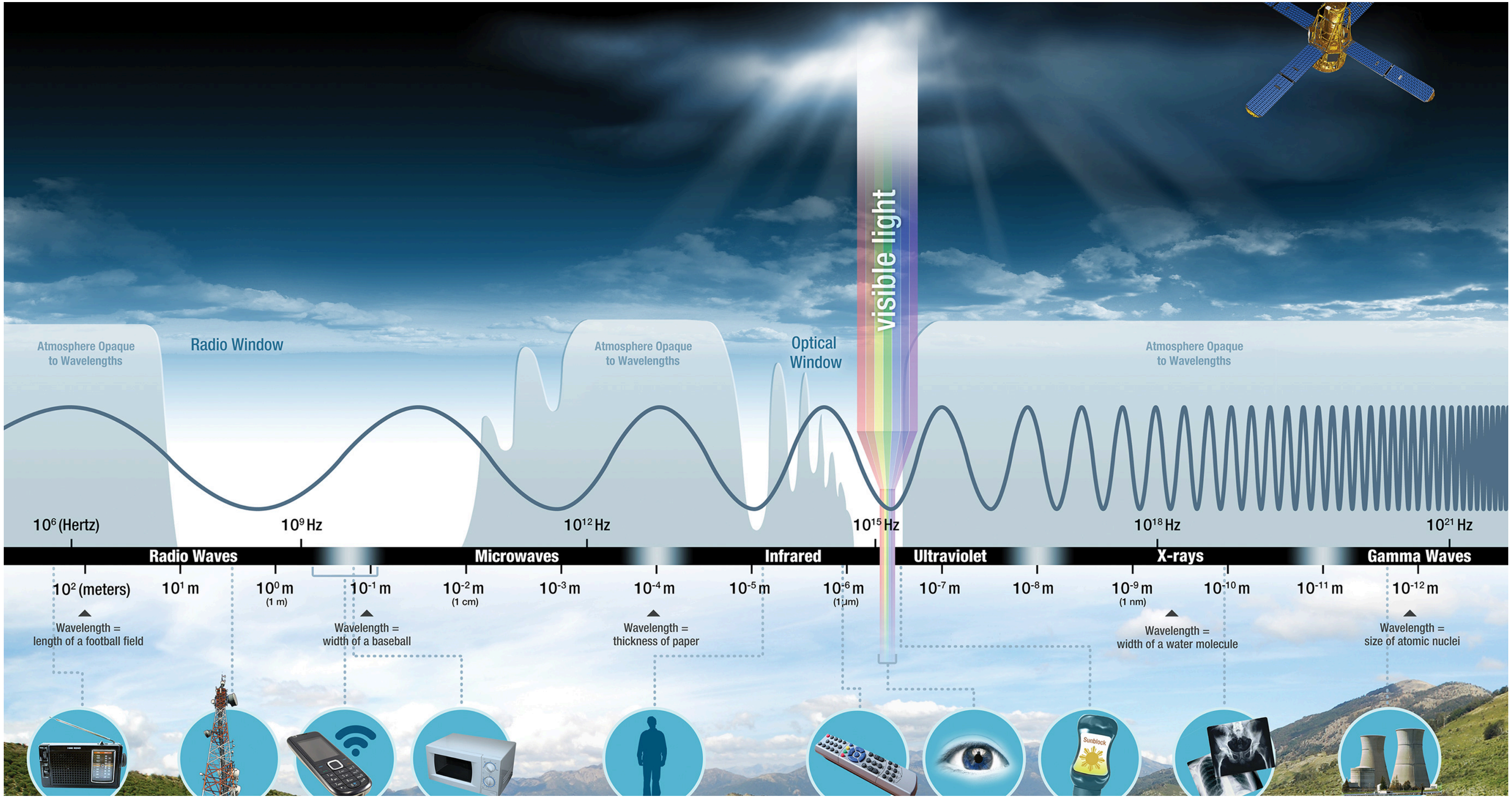
1.1 What is light?

- Light is an electromagnetic wave that exhibits both wave-like and particle-like properties, a concept known as wave-particle duality.
- Light normally is described by its **wavelength** (λ), **frequency** (ν), **energy** (E), **polarization**, and **intensity**.
- Light can be treated as discrete packets of energy called **photons**, which can interact with matter in **quantized** ways. Therefore, the total energy of light is the sum of the energies of its individual photons.
- Light spans a broad spectrum from radio waves to gamma rays.

1.1 What is light?



1.1 What is light?

Wavelength and Frequency

- The wavelength (λ) and frequency (ν) of light are inversely related through the speed of light (c):

$$\lambda\nu = c \tag{1}$$

- Wavelength is the distance between successive peaks of the wave, typically measured in meters (m)
- Frequency is the number of wave cycles that pass a given point per second, measured in hertz (Hz)
- The speed of light in a vacuum is approximately 3×10^8 m/s.

1.1 What is light?

Energy of a photon

$$E = h\nu = h\frac{c}{\lambda} \tag{2}$$

where:

- E is the energy of the photon (in joules, J)
- h is Planck's constant (6.626×10^{-34} J·s)

1.1 What is light?

Example 1.1

Calculate the energy and frequency of a photon with a wavelength of 500 nm (nanometers).

Solution 1.1

$$E = h\frac{c}{\lambda}$$

$$E = (6.626 \times 10^{-34}) \times \frac{3 \times 10^8}{500 \times 10^{-9}}$$

$$E = 3.9756 \times 10^{-19} J$$

$$\nu = \frac{c}{\lambda}$$

$$\nu = \frac{3 \times 10^8}{500 \times 10^{-9}}$$

$$\nu = 6 \times 10^{14}\ \text{Hz}$$

1.1 What is light?

Electron volt (eV)

- The electron volt (eV) is a unit of energy commonly used when the energy is very low, such as the energy of a photon.
- 1 eV is defined as the amount of kinetic energy gained or lost by an electron when it is accelerated through an electric potential difference of one volt.
- In terms of joules, 1 eV is equivalent to approximately 1.602×10^{-19} joules (J).

1.1 What is light?

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Example 1.2

Convert the photon energy of a wavelength of 500 nm from joules (J) to electron volts (eV).

Solution 1.2

$$1 \text{ eV} = 1.602 \times 10^{-19} J$$

Therefore,

$$E = \frac{3.9756 \times 10^{-19} J}{1.602 \times 10^{-19} J/ \text{ eV}} = 2.48 \text{ eV}$$

1.1 What is light?

Example 1.3

Derive an expression for the change in frequency ($\Delta\nu$) that corresponds to a change in wavelength ($\Delta\lambda$).

Solution 1.3

$$\nu = \frac{c}{\lambda}$$

Taking the derivative with respect to λ:

$$\Delta\nu = -\frac{c}{\lambda^2}\Delta\lambda$$

where λ is the center wavelength.
This formula is typically used to calculate the bandwidth of light sources.

1.1 What is light?

Refractive index

- The refractive index (n) of a medium is a dimensionless number calculating the ratio of the speed of light in a vacuum (c) to the speed of light in the medium (v):

$$n = \frac{c}{v} \tag{3}$$

- A refractive index greater than 1 indicates that light travels *slower* in the medium than in a vacuum.
- Different materials have different refractive indices, which affect how light bends (refracts) when entering or exiting the material.

1. Introduction

2. Absorption and stimulated emission of light

3. Einstein Relations

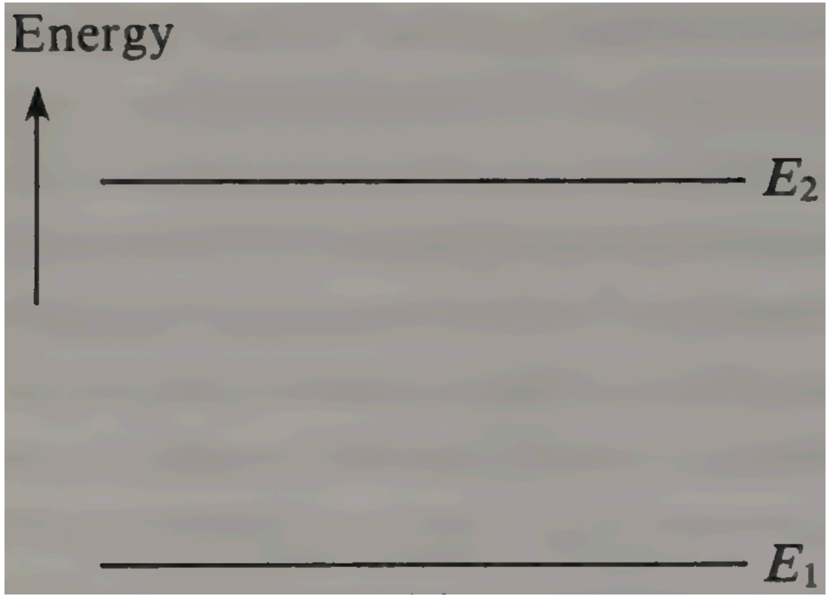
4. The Gain Coefficient

5. The Optical Resonator

6. Laser Modes

7. Problems

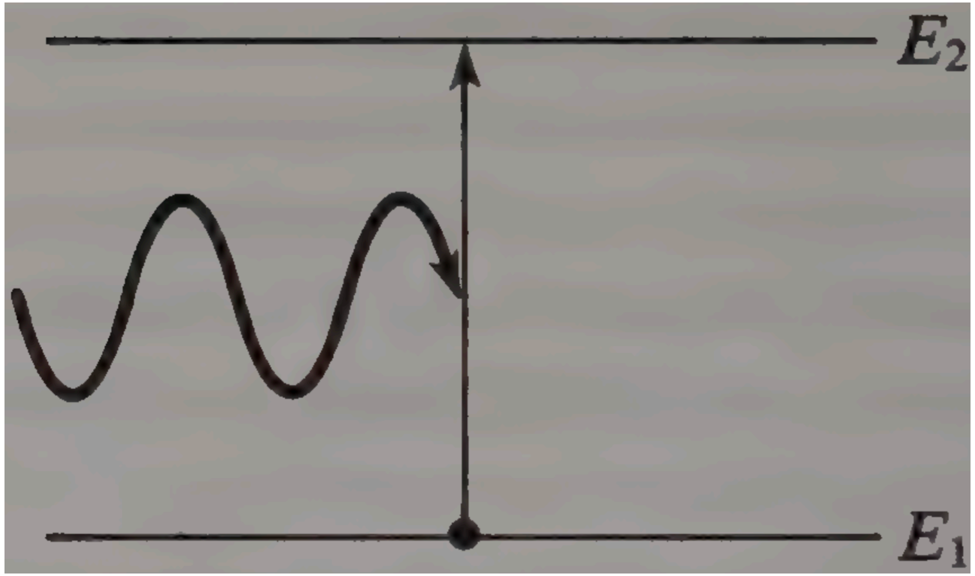
2.1 Energy Levels System



- Atoms and molecules have **discrete** energy levels, meaning that electrons can only occupy specific energy states.

- The lowest energy level is called the **ground state**, while higher energy levels are referred to as **excited states**.
- **Ground state**: is the only stable state where electrons normally live.
- **Excited state**: is any energy level above the ground state. Theses states are unstable, and the electrons will eventually return to the ground state.

2.2 Stimulated Absorption

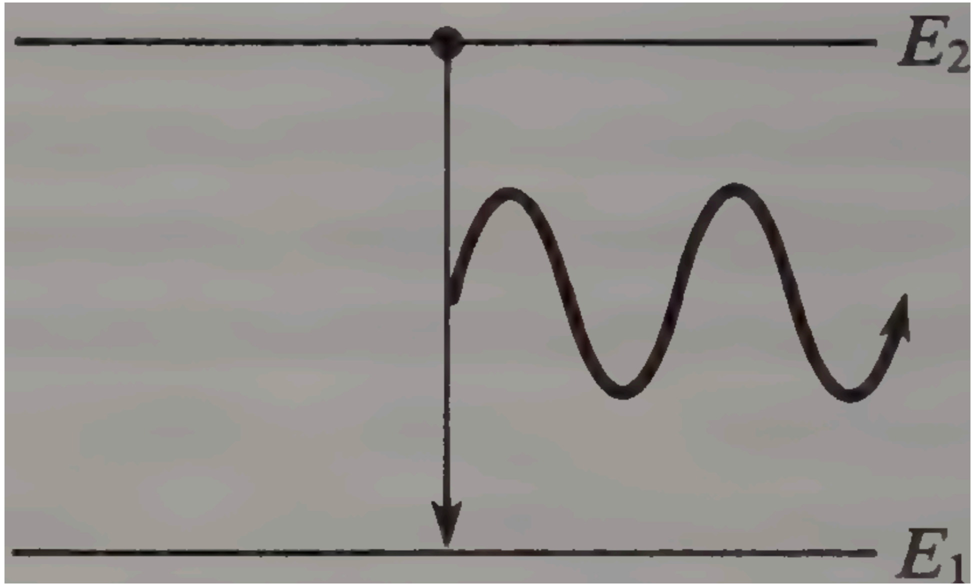


- Stimulated Absorption is the process by which an atom or molecule takes in a photon of light and transit from a lower energy state E_1 to a higher energy state E_2.
- The photon must have an energy equal to the energy difference between the two states:

$$\nu = \frac{E_2 - E_1}{h} \tag{4}$$

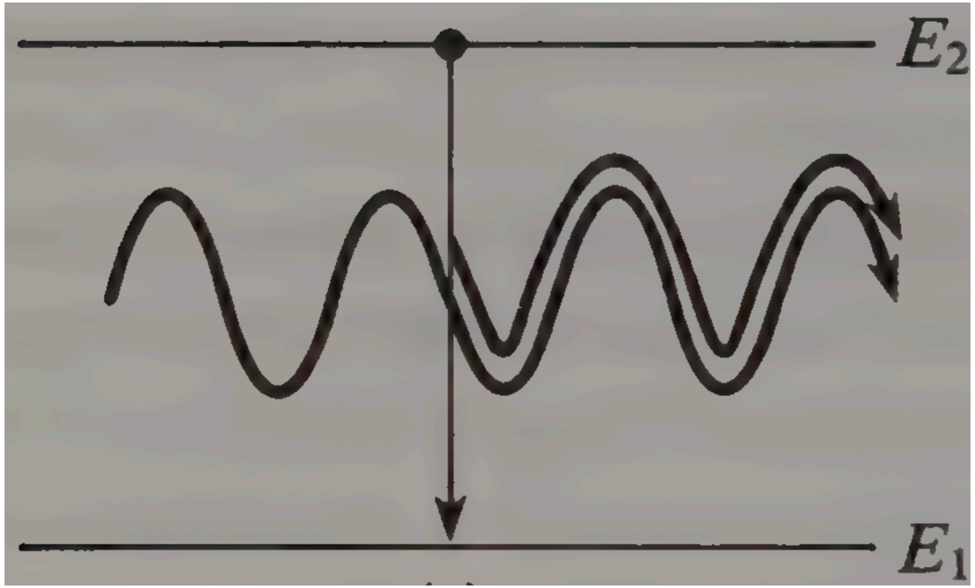
- If the incoming photon does not have the correct energy, it will not be absorbed, and the electron will remain in its current energy state.

2.3 Spontaneous Emission



- The excited electron will stay for a short time known as the **lifetime** of the excited state before it spontaneously (naturally) returns to the ground state.
- Spontaneous Emission is the process by which an electron in an excited state E_2 spontaneously decays to a lower energy state E_1, emitting a photon with a frequency given by Equation 4.

2.4 Stimulated Emission



- Stimulated Emission is the process by which an incoming photon of a specific frequency interacts with an excited electron, causing it to transition to a lower energy state and emit a second photon.
- The emitted photon has the same **frequency**, **phase**, **polarization**, and **direction** as the incoming photon, resulting in **coherent light**.
- This process is the fundamental principle behind the operation of **LASERs** (Light Amplification by Stimulated Emission of Radiation).

2.5 Comparison between Spontaneous and Stimulated Emission

	Spontaneous Emission	Stimulated Emission
Requires External Photon?	No	Yes
Emission Direction	Random	Same as Incoming Photon
photon Energy	Can be any energy that matches the energy difference between states	Equal to the incident photon energy
photon Phase	Random	Same as Incoming Photon
Photon Polarization	Random	Same as Incoming Photon
Example	Normal light lamp	Laser

3.1 Introduction

- Albert Einstein introduced a *theoretical framework* to describe the interaction between light and matter, specifically focusing on the processes of absorption, spontaneous emission, and stimulated emission.
- He proposed three coefficients, known as the **Einstein coefficients**, to quantify the probabilities of these processes occurring.
- The three processes of stimulated absorption, stimulated emission and spontaneous emission are related mathematically through the **Einstein relations**.

3.2 Two-Level System

- For simplicity, we will study Einstein relations in a **two-level** atomic system with energy levels E_1 (ground state) and E_2 (excited state). Also, we will assume thermal equilibrium conditions.
- Let N_1 and N_2 be the population (number of atoms) in the ground and excited states, respectively.
- In such system, the the upward (from E_1 to E_2) and downward (from E_2 to E_1) transition rates must be equal at thermal equilibrium.

3.3 Stimulated Absorption Rate

- Let ρ_ν be the energy density of the photons at frequency ν.

$$\rho_\nu = \eta_\nu h\nu \tag{5}$$

where η_ν is the number of photons per unit volume at frequency ν.

- If there are N_1 atoms in the ground state, the rate of stimulated absorption (transitions from E_1 to E_2) is proportional to both N_1 and ρ_ν:

$$\text{Stimulated Absorption Rate} = N_1 \rho_\nu B_{12} \tag{6}$$

where B_{12} is the Einstein coefficient *constant* for stimulated absorption.

- The "12" subscript indicates a transition from level 1 to level 2.

3.4 Stimulated Emission Rate

- Similarly, if there are N_2 atoms in the excited state, the rate of stimulated emission (transitions from E_2 to E_1) is also proportional to both N_2 and ρ_ν:

$$\text{Stimulated Emission Rate} = N_2 \rho_\nu B_{21} \tag{7}$$

where B_{21} is the Einstein coefficient *constant* for stimulated emission.

3.5 Spontaneous Emission Rate

- The rate of spontaneous emission (transitions from E_2 to E_1) depends on the number of atoms in the excited state:

$$\text{Spontaneous Emission Rate} = N_2 A_{21} \tag{8}$$

where A_{21} is the Einstein coefficient *constant* for spontaneous emission.

- A_{21} depends on the *average lifetime* of the excited state τ_{21},

$$A_{21} = \frac{1}{\tau_{21}}. \tag{9}$$

3.6 Einstein coefficients and relations

- The relation between the three Einstein coefficients (A_{21}, B_{12}, and B_{21}) can be established by considering an assembly of atoms to be in thermal equilibrium, therefore,

$$\begin{gathered}\text{Upward transition Rate} = \text{Downward transition Rate} \\ N_1 \rho_\nu B_{12} = N_2 \rho_\nu B_{21} + N_2 A_{21}\end{gathered} \tag{10}$$

- By rearranging Equation 10 and solving for ρ_ν, we get:

$$\rho_\nu = \frac{A_{21}/B_{21}}{\frac{B_{12}}{B_{21}} \frac{N_1}{N_2} - 1} \tag{11}$$

3.6 Einstein coefficients and relations

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- From Boltzmann statistics at thermal equilibrium, the ratio of population N_2 at energy level E_2 relative to N_1 at E_1 is given by:

$$\frac{N_2}{N_1} = \exp\left(-\frac{E_2 - E_1}{kT}\right) \tag{12}$$

where k is the Boltzmann constant and T is the absolute temperature.

- Substituting this expression into Equation 11 and simplifying, we get:

$$\rho_\nu = \frac{A_{21}/B_{21}}{\frac{B_{12}}{B_{21}} \exp(h\nu/kT) - 1} \tag{13}$$

3.6 Einstein coefficients and relations

- Since the system is in equilibrium, the radiation within the assembly of atoms must be identical to blackbody radiation, therefore,

$$\rho_\nu = \frac{8\pi h\nu^3}{c^3}\left(\frac{1}{\exp(h\nu/kT) - 1}\right) \tag{14}$$

- By comparing Equation 13 and Equation 14, we can derive the following relations:

$$B_{12} = B_{21} \qquad \text{Einstein relation 1} \tag{15}$$

$$A_{21} = B_{21}\frac{8\pi h\nu^3}{c^3} \qquad \text{Einstein relation 2} \tag{16}$$

3.7 Ratio of Spontaneous to Stimulated Emission

The ratio of spontaneous to stimulated emission rates is:

$$R = \frac{N_2 A_{21}}{N_2 B_{21} \rho_\nu} = \frac{8\pi h\nu^3}{\rho_\nu c^3}. \tag{17}$$

Substituting for ρ_ν from blackbody radiation formula (Equation 14), we get

$$R = \exp(h\nu/kT) - 1. \tag{18}$$

3.7 Ratio of Spontaneous to Stimulated Emission

Example 3.4

calculate R for the light emitted by an electrical discharge in a gas such as neon in the helium—neon (HeNe) laser. The discharge temperature may be taken as 370K and for the red line produced by this laser, which has a frequency of $\nu = 4.74 \times 10^{14}$ Hz

Solution 3.4

$$R = \exp(h\nu/kT) - 1 = \exp\left(\frac{6.626 \times 10^{-34} \times 4.74 \times 10^{14}}{1.38 \times 10^{-23} \times 370}\right) - 1$$

$$R \approx 5 \times 10^{26}$$

3.8 Remarks on Einstein Relations

- Under conditions of thermal equilibrium, stimulated emission is most *unlikely*.
- The process of stimulated emission competes with the process of stimulated absorption.
- If we wish to amplify a beam of light by stimulated emission we must increase the rate of this process relative to the other two.
- Stimulated emission will be *greater* if the population density of the upper level is greater than the lower level ($N_2 > N_1$ or **population inversion**) and the radiation density are increased.
- Because of the Einstein relation $B_{12} = B_{21}$, creating a population inversion in two-level system is *impossible*!

3.8 Remarks on Einstein Relations

- There are other Einstein coefficients A_{ij}, B_{ij} and B_{ji} and Einstein relations for systems with more than two energy levels.
- We usually reduce the total number of relevant energy levels to four or less for simplicity.

1. Introduction

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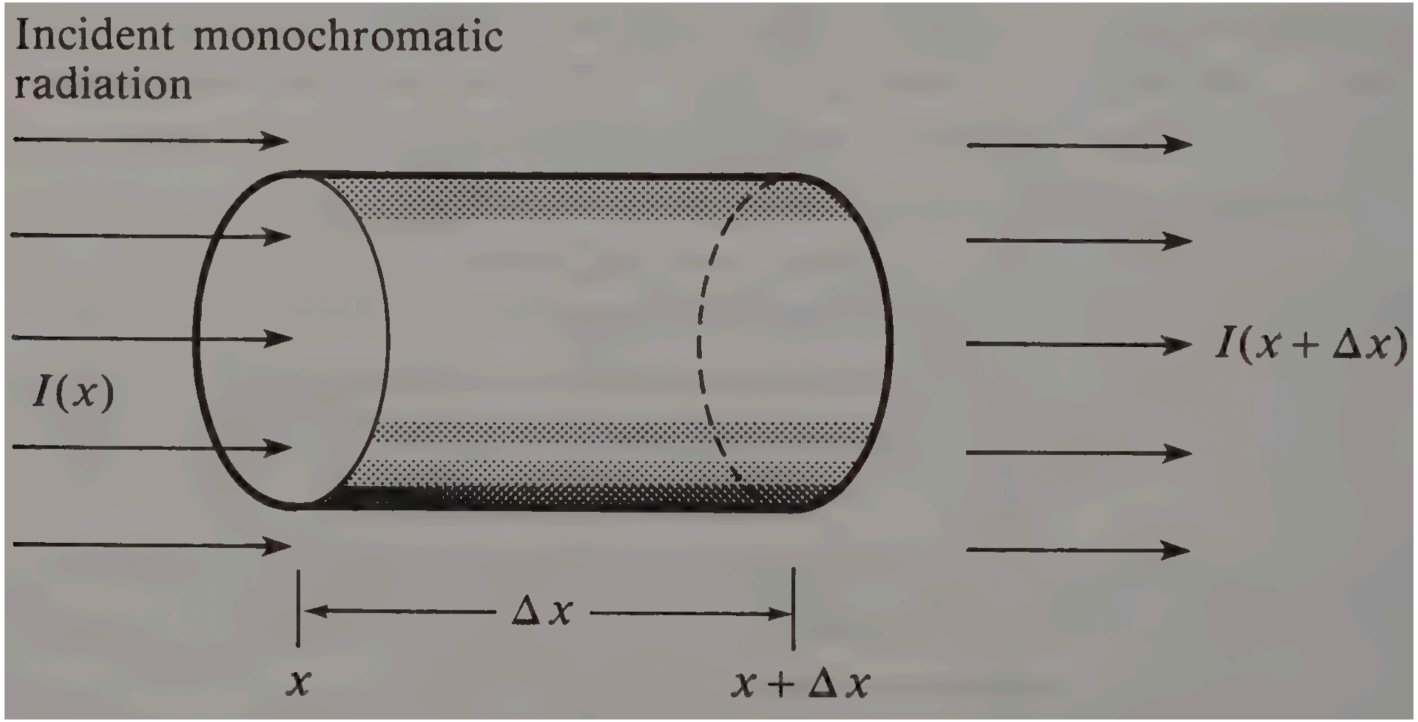
4.1 Absorption of a monochromatic beam of light



- When a collimated beam of monochromatic radiation of light of intensity $I(x)$ passes through a medium, its intensity decreases due to absorption.

4.1 Absorption of a monochromatic beam of light

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- The change in intensity $\mathrm{d}I$ is proportional to the intensity and the distance $\mathrm{d}x$ traveled in the medium:

$$\mathrm{d}I(x) = -\alpha I(x)\,\mathrm{d}x$$

where α is the **absorption coefficient** of the medium.

- Rearranging and integrating the above equation gives:

$$I(x) = I_0 e^{-\alpha x}$$

where I_0 is the initial intensity of light before entering the medium.

4.2 Absorption coefficient

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- For a two-level system, the absorption coefficient α depends on the population of the lower energy state N_1 relative to the upper energy state N_2:
- If $N_2 = 0$, then α is at its maximum value.
- When $N_2 - N_1$ increases, α decreases.

4.3 Population inversion

- When the population of the upper energy state exceeds that of the lower energy state

$$N_2 > N_1 \qquad \text{Population Inversion} \tag{19}$$

the system is said to have a **population inversion**.

- In this case, the absorption coefficient α becomes negative, indicating that the medium amplifies the light passing through it rather than absorbing it.

4.4 Small-Signal Gain Coefficient

- At population inversion, the intensity of light grows exponentially as it travels through the medium:

$$I(x) = I_0 e^{kx} \tag{20}$$

where k is called **the small-signal gain coefficient** of the medium.

- Is there a relationship between the gain coefficient k and Einstein coefficients?

4.5 Deriving the Gain Coefficient k

- From the discussion we had in previous sections, we can infer that the gain in the medium is proportional to $(N_2 - N_1)$ and the stimulated emission coefficient B_{21}:

$$K \propto (N_2 - N_1)B_{21} \tag{21}$$

- To derive the exact expression for k, we start by considering the rate of change in the density of photons $(\mathrm{d}\eta_\nu)/(\mathrm{d}t)$ as they travel a distance $\mathrm{d}x$ in the medium:

$$-\frac{\mathrm{d}\eta_\nu}{\mathrm{d}t} = N_1\rho_\nu B_{12} - N_2\rho_\nu B_{21} \tag{22}$$

since $B_{12} = B_{21}$ for a two-level system, we can write:

$$-\frac{\mathrm{d}\eta_\nu}{\mathrm{d}t} = (N_1 - N_2)\rho_\nu B_{21} \tag{23}$$

4.5 Deriving the Gain Coefficient k

- Next, we connect the change in photon density to the change in intensity of light.
- The intensity through a medium of a refractive index n is the energy crossing unit area per

second:

$$I = \rho_\nu \frac{c}{n} = \eta_\nu h\nu \frac{c}{n} \tag{24}$$

- The change in photon density is related to the change in intensity by:

$$\frac{\mathrm{d}\eta_\nu}{\mathrm{d}t} = \frac{\mathrm{d}I(x)}{\mathrm{d}x} \frac{1}{h\nu} \tag{25}$$

4.5 Deriving the Gain Coefficient k

- Using Equation 18 ($\mathrm{d}I(x)/\,\mathrm{d}x = -\alpha I(x)$), we get:

$$\frac{\mathrm{d}\eta_\nu}{\mathrm{d}t} = -\alpha I(x)\left(\frac{1}{h\nu}\right) = -\alpha\left(\rho_\nu \frac{c}{n}\right)\left(\frac{1}{h\nu}\right) \tag{26}$$

- Substituting this expression into Equation 23 gives:

$$\alpha\left(\rho_\nu \frac{c}{n}\right)\left(\frac{1}{h\nu}\right) = (N_1 - N_2)\rho_\nu B_{21} \tag{27}$$

4.5 Deriving the Gain Coefficient k

- Finally, rearranging the above equation and using $k = -\alpha$, gives the expression for the small-signal gain coefficient:

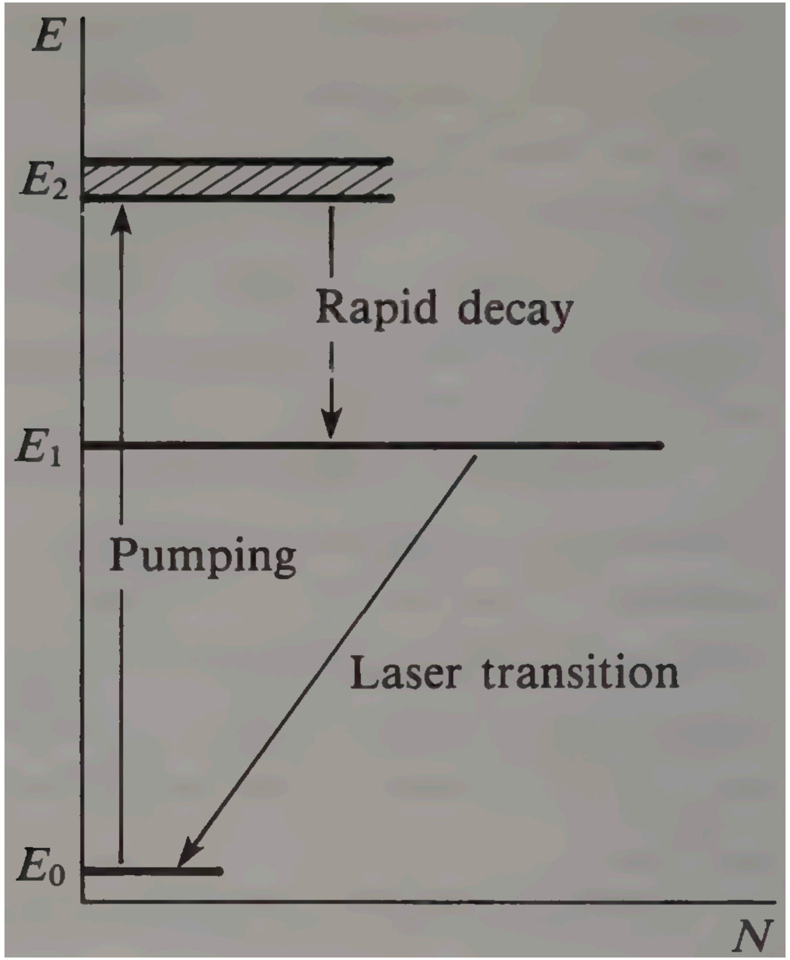
$$k = (N_2 - N_1)\frac{nh\nu_{21}B_{21}}{c} \tag{28}$$

4.6 Creating Population Inversion (Pumping)

- In a two-level system, achieving population inversion is impossible since $B_{12} = B_{21}$.
- Population inversion can be achieved in systems with three or more energy levels.
- To create population inversion, we need to use a process called **pumping** to excite electrons from the ground state to higher energy states.
- There are several methods of pumping, including

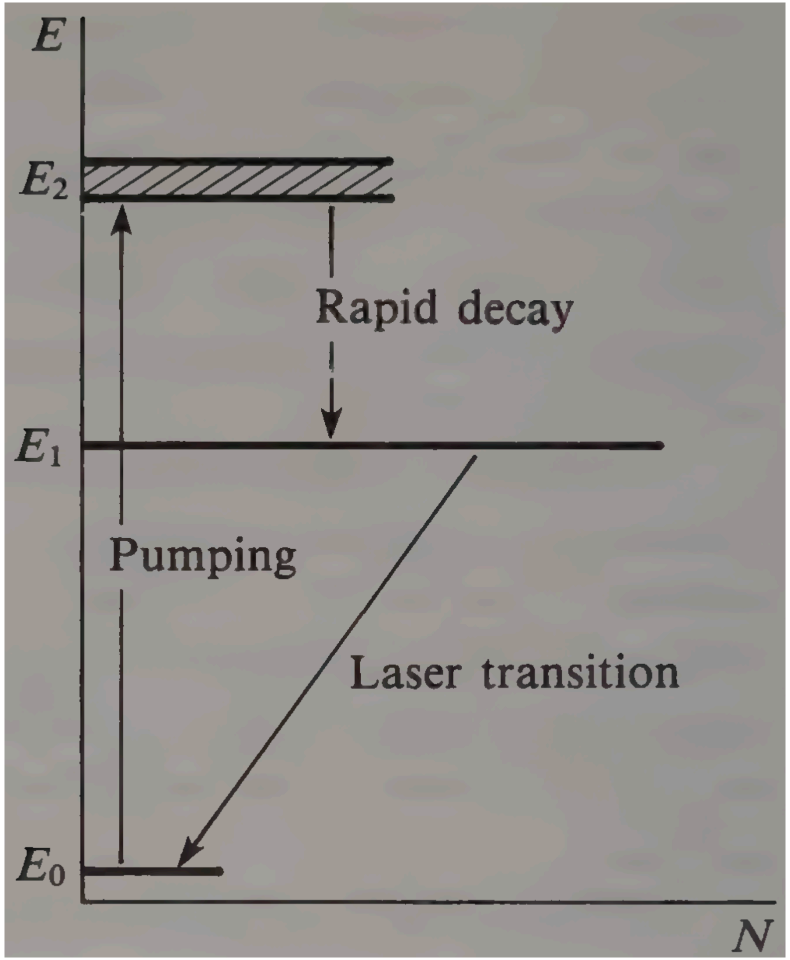
1. Optical pumping (using light).
2. Electrical pumping (collisions within an electric discharge or current).
3. Chemical pumping (through chemical reactions).

4.7 Optical Pumping of a Three-Level System



- The three-level system consists of a ground state E_0, an intermediate state E_1, and a higher energy state E_2.
- The laser medium is illuminated by intense radiation from a flash tube to pump atoms from the ground state E_0 to the higher energy state E_2 by absorption of photons with frequency v_{02}.
- The electrons in the excited state E_2 quickly decay to the intermediate state E_1, typically through non-radiative processes (without emitting photons).

4.7 Optical Pumping of a Three-Level System



- Over time, the population of the intermediate state E_1 builds up, while the population of the ground state E_0 decreases.
- If the pumping rate is sufficiently high, a population inversion can be achieved.
- The condition for population inversion in a three-level system is:

$$N_1 > N_0 \tag{29}$$

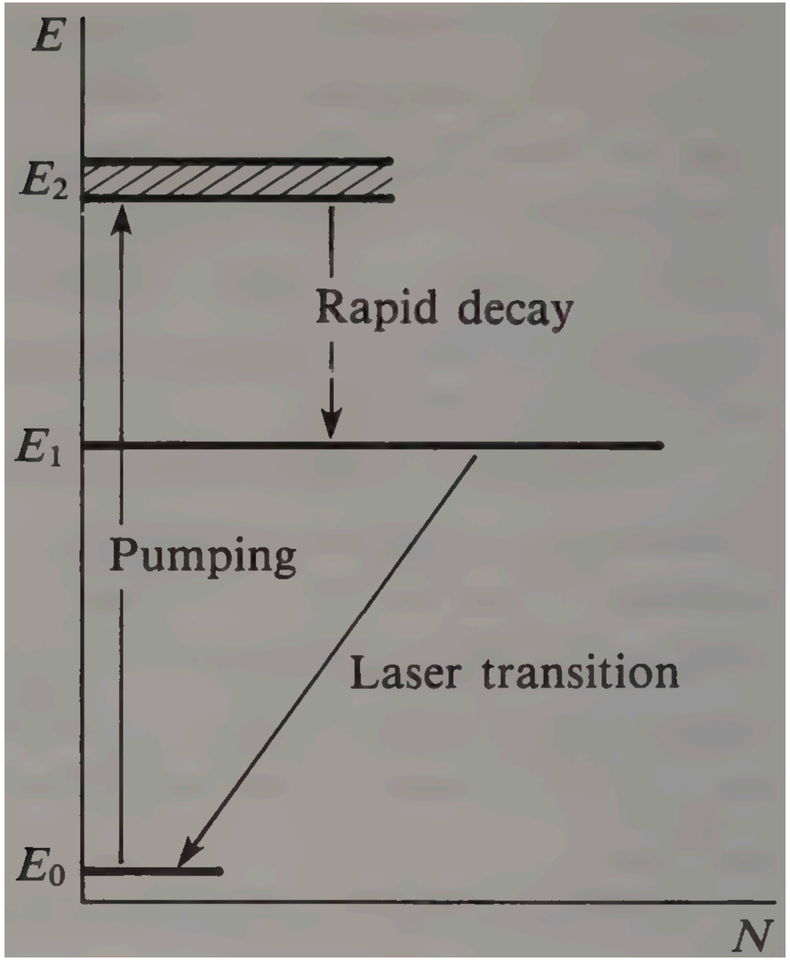
4.7 Optical Pumping of a Three-Level System

- Once population inversion is established, **Amplified Stimulated Emission (ASE)** can occur between E_1 and E_0, leading to laser action at frequency v_{10} given by:

$$v_{10} = \frac{E_1 - E_0}{h}, \tag{30}$$

the laser frequency for three-level system.

4.8 Conditions For Excellent Three-Level Laser Medium



- The lifetime of the upper laser level E_1 should be **long** enough (**metastable**) to allow for population inversion to build up.
- The lifetime of the higher energy state E_2 should be **short** to ensure rapid decay to the intermediate state E_1.
- Level E_2 should have wide energy band (δE) to increase the absorption of the pump radiation, that is

$$E_2 \rightarrow E_2 \pm \delta E = h(v_{02} \pm \delta v) \tag{31}$$

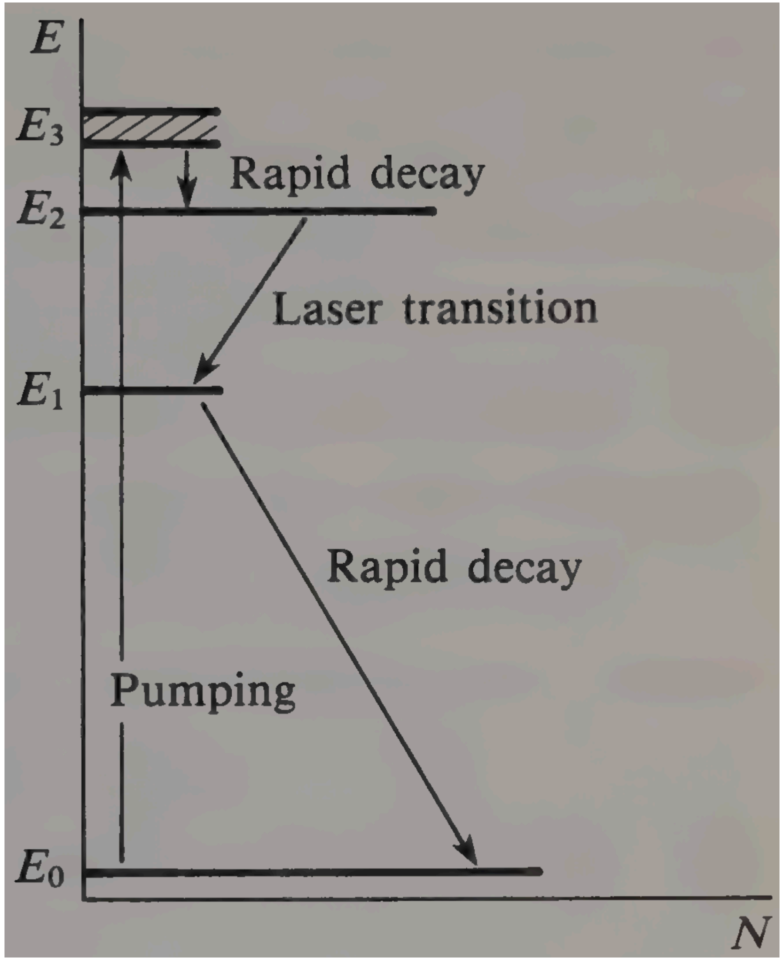
4.8 Conditions For Excellent Three-Level Laser Medium

- Typical lifetime are within the range of nanoseconds (10^{-9} s).
- Example for three-level laser is the Ruby laser, the first laser ever built.

4.9 Limitations of Three-Level Lasers

- Three-level lasers require a high pumping power to achieve population inversion since the ground state is heavily populated and you need to excite more than half of the atoms to E_1 through E_2.
- The efficiency of three-level lasers is generally *lower* compared to four-level lasers.

4.10 Optical Pumping of a Four-Level System

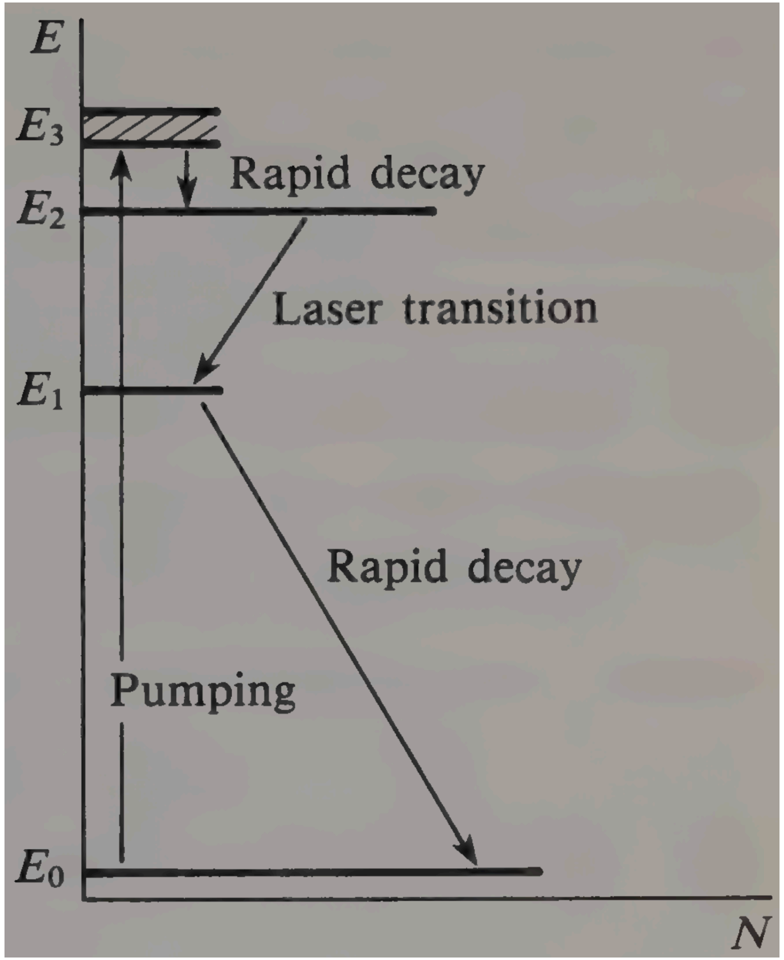


- The four-level system consists of a ground state E_0, two intermediate states E_1 and E_2, and a higher energy state E_3.

Pumping mechanism:

1. Atoms are excited from the ground state E_0 to the higher energy state E_3 by absorption of photons with frequency v_{03}.
2. The electrons in the excited state E_3 quickly decay to the intermediate state E_2.

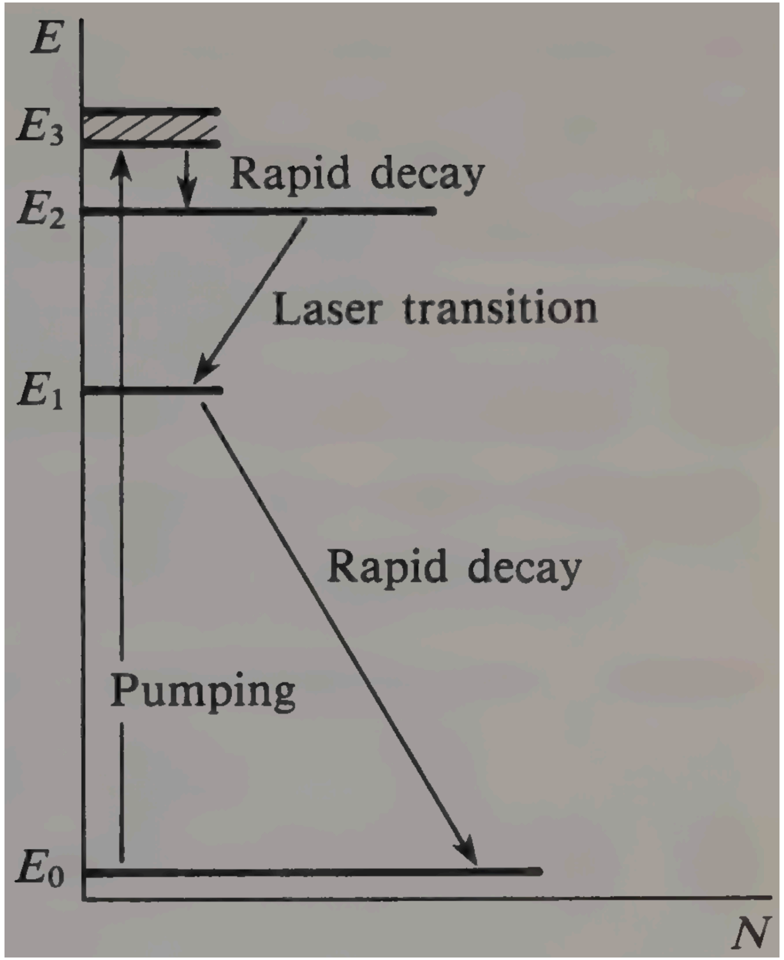
4.10 Optical Pumping of a Four-Level System



3. E_2 should have a relatively **long lifetime** (metastable state) to allow for population inversion to build up between E_2 and E_1.
4. The electrons decay to E_1 through stimulated (or spontaneous) emission, emitting photons with frequency v_{21}, which is the laser frequency for four-level system:

$$v_{21} = \frac{E_2 - E_1}{h}, \qquad (32)$$

4.10 Optical Pumping of a Four-Level System



5. Finally, the electrons in E_1 should quickly decay to the ground state E_0, where they can be pumped again.

- The condition for population inversion in a four-level system is:

$$N_2 > N_1 \tag{33}$$

- The population inversion is easier to achieve when the E_3 and E_1 states are short-lived compared to the E_2 state.

4.10 Optical Pumping of a Four-Level System

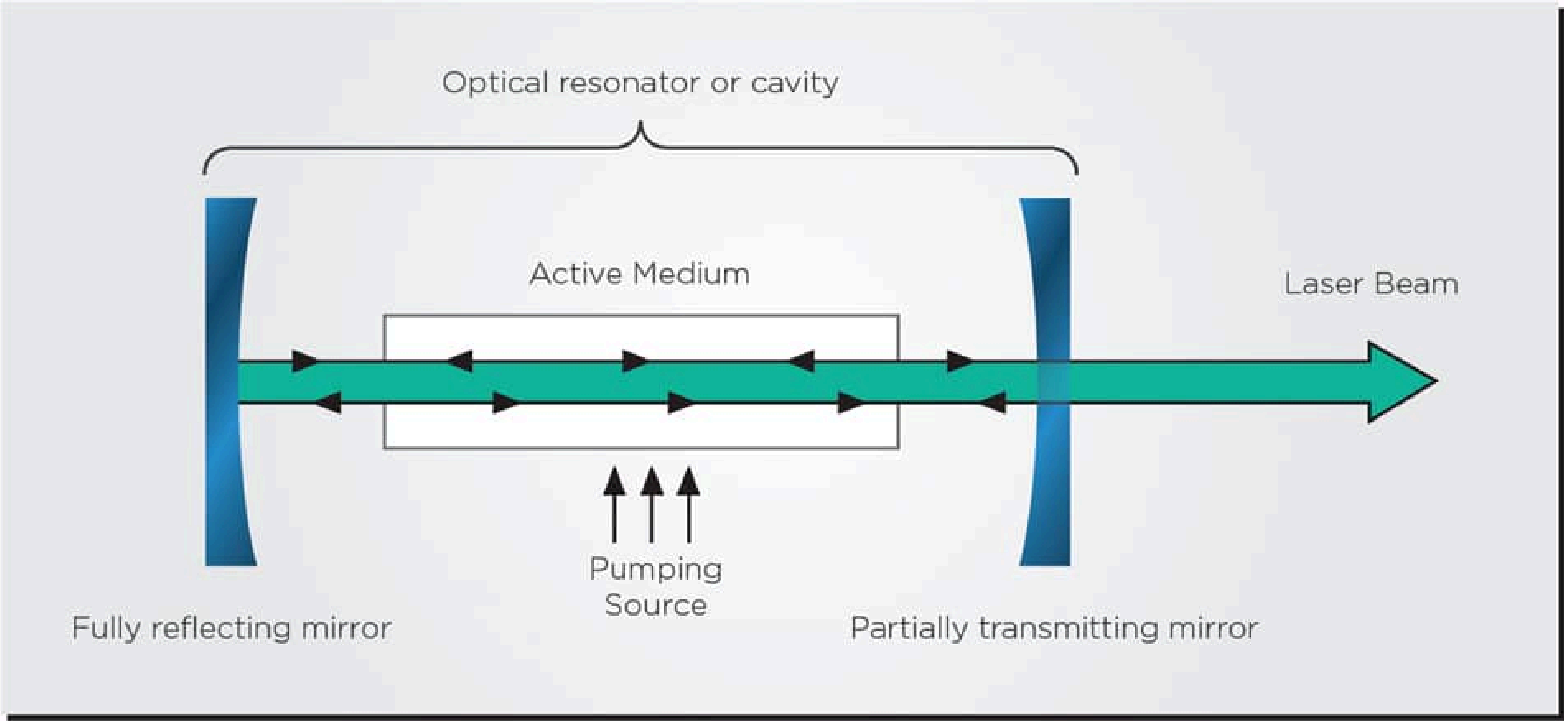
- One major advantage of four-level lasers is that they can achieve population inversion with much lower pumping power compared to three-level lasers, since the lower laser level E_1 can be kept nearly empty.
- An example of a four-level laser is the Nd:YAG laser.

5.1 Is the Amplification from small-signal gain enough?

NO!

- The small-signal gain coefficient k provides only a modest amplification of the light intensity as it passes through the laser medium.
- The small-signal gain is typically only 10% per meter of gain medium length.
- The level of amplification that we want is over millions of times $> 10^6$!
- Therefore, we need to find a way to increase the effective path length of light within the gain medium.

5.2 Optical Resonator

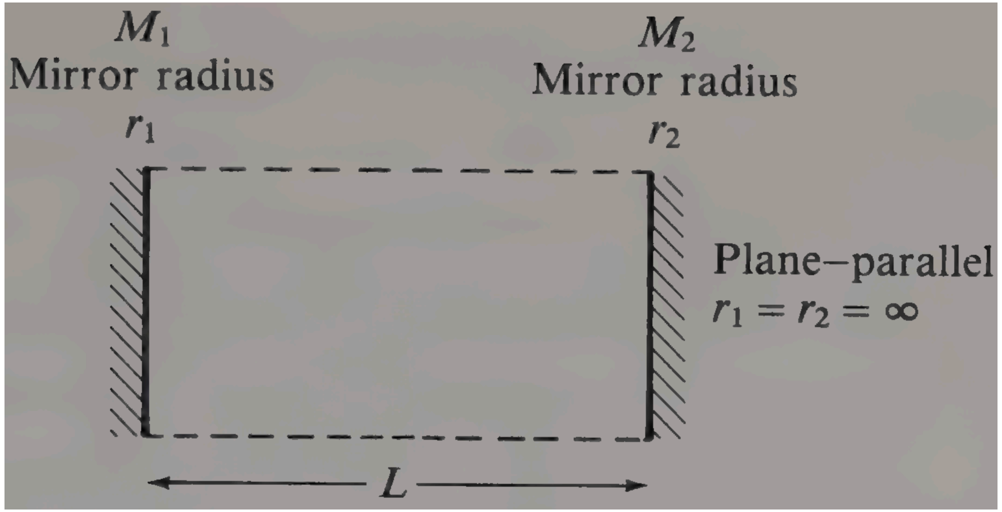


Ref.: https://effectphotonics.com/insights/shining-a-light-on-four-tunable-lasers/

5.2 Optical Resonator

- An **optical resonator** (or optical cavity) is a set of mirrors that form a closed loop, allowing light to bounce back and forth through the gain medium multiple times.
- The repeated passes through the gain medium increase the effective path length of the light, leading to a *significant* amplification.
- Each time the light passes through the gain medium, it experiences additional amplification according to the small-signal gain coefficient k.
- *Remember*, the light travels a distance of 3×10^8 m in one second.
- Some of the light is allowed to exit the resonator through a partially reflective mirror, forming the laser beam.
- Other names for optical resonator are **optical cavity** or **optical oscillator**.

5.3 Laser Cavity Mirror Configurations

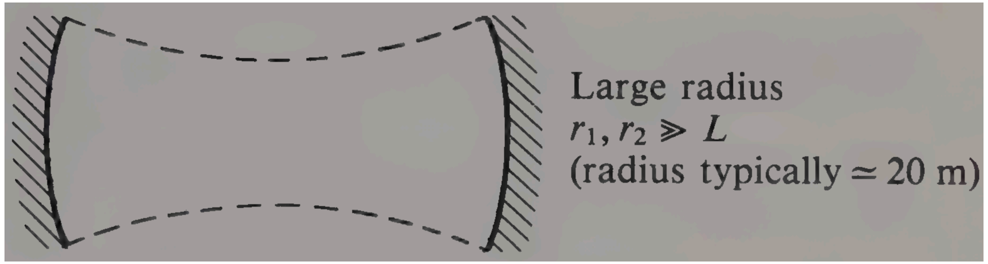


- **plane-parallel mirrors** are the simplest configuration, consisting of two flat mirrors facing each other.
- This configuration is very sensitive and *cannot* be perfectly maintained aligned.

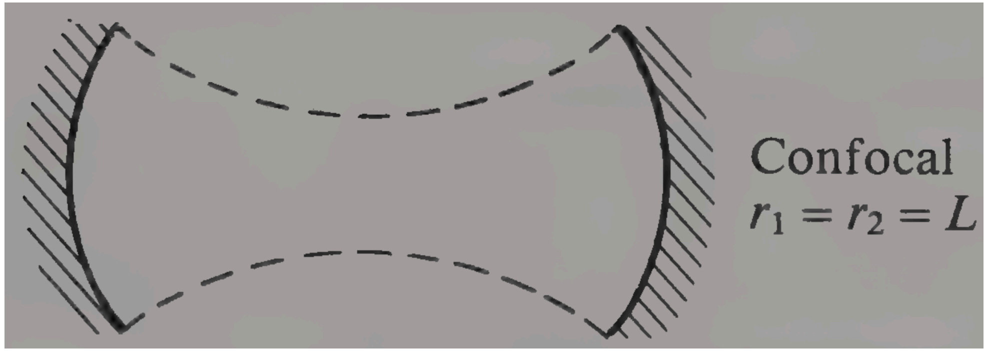
Some of the radiation will spread out beyond the edges of the mirrors.

- However, the laser beam makes maximum use of the gain medium volume.
- There are many different configurations of optical resonators, that provide different advantages and use cases.

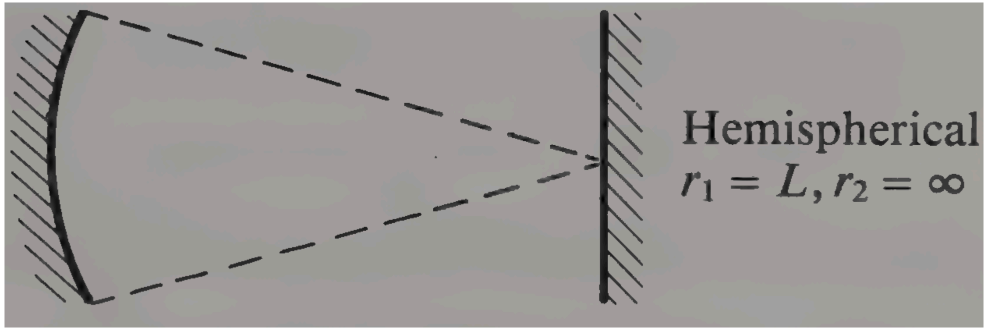
5.3 Laser Cavity Mirror Configurations



- **concave mirrors** are curved inward, focusing the light back into the gain medium.



- **Confocal mirrors** have both mirrors curved with the same radius of curvature and separated by a distance equal to that radius.



- **Hemispherical mirrors** have one flat mirror and one curved mirror with $r = L$.

5.4 Threshold Gain Coefficient

The gain coefficient k must be large enough to compensate for the losses in the system,

- The sources of losses include:
  1. Transmission at the mirrors, that is, the useful output (one of the mirrors is usually made as reflective as possible while the other, the output mirror, may have a reflectance of about 90%).
  2. Absorption and scattering by the mirrors.
  3. Diffraction around the boundary of the mirrors.
  4. Absorption in the laser medium due to transitions other than the desired one.
  5. Scattering at optical inhomogeneities in the laser medium.

5.4 Threshold Gain Coefficient

- Its convenient to express the losses other than the transmission at the mirrors as an effective loss coefficient γ (per unit volume).
- Therefore, the effective gain coefficient is $(k - \gamma)$.

5.4 Threshold Gain Coefficient

The Gain of ONE Round Trip in the Resonator

- A round trip in the resonator consists of the light traveling from one mirror to the other and back, passing through the gain medium twice.
- The intensity of the light before the round trip is I_0.
- The intensity of the light after passing the gain medium once and reflecting off the first mirror is I_1:

$$I_1 = I_0 R_1 e^{(k-\gamma)L} \tag{34}$$

where R_1 is the reflectivity of the first mirror and L is the length of the gain medium.

5.4 Threshold Gain Coefficient

- After reflecting off the second mirror and passing through the gain medium again, the intensity of the light after one round trip is I_2:

$$I_2 = I_1 R_2 e^{(k-\gamma)L} = I_0 R_1 R_2 e^{2(k-\gamma)L} \quad (35)$$

where R_2 is the reflectivity of the second mirror.

- The **round trip gain (G)** is the ratio of the final intensity to the initial intensity:

$$G = \frac{\text{Final Intensity}}{\text{Initial Intensity}} = R_1 R_2 e^{2(k-\gamma)L} \quad (36)$$

5.4 Threshold Gain Coefficient

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- Therefore, to keep the laser oscillating, the round trip gain must be greater than or equal to 1.
- The **threshold gain coefficient** k_{th} is the minimum gain coefficient required to achieve lasing, which occurs when the round trip gain is exactly 1:

$$R_1 R_2 e^{2(k_{\text{th}}-\gamma)L} = 1 \tag{37}$$

- Solving for k_{th} gives the expression for the threshold gain coefficient:

$$k_{\text{th}} = \gamma + \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right) \tag{38}$$

5.4 Threshold Gain Coefficient

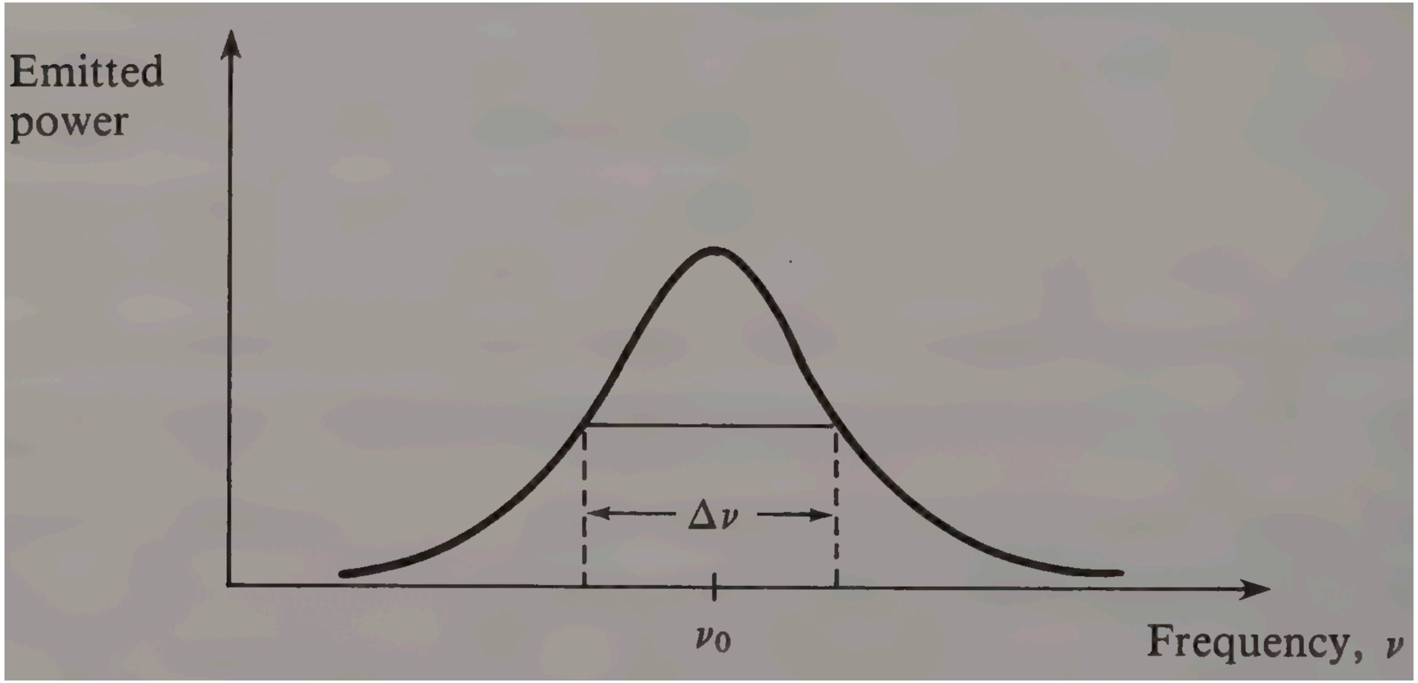
- For lasers designed for continuous output (CW lasers), the laser has to eventually run at $k = k_{\text{th}}$ to maintain a steady-state operation.

5.5 The Lineshape Function

- The energy levels in atoms and molecules are not perfectly equal to a single value, but rather have a certain width (range of energies) around the central energy level, known as the **linewidth** or **line broadening**.
- This is due to various factors, including collisions, natural damping and doppler broadening.
- Therefore, the emitted photons from the laser have a range of frequencies around the central frequency $\nu_0 \pm \Delta\nu$.
- The **lineshape function** $g(\nu)$ describes the frequency distribution of the emitted photons, and it is typically normalized such that the area under the curve is equal to 1.

5.5 The Lineshape Function

- $g(\nu)$ is a **gaussian** distribution for **doppler** broadening and a lorentzian distribution for natural damping.



5.5 The Lineshape Function

Doppler effects

- The motion of the atoms or molecules in the gain medium causes a shift in the frequency of the emitted photons known as the Doppler effect.
- An atom moving with a velocity v relative to the observer will emit photons with a frequency shifted by:

$$\nu = \nu_0 \left(1 \pm \frac{v}{c}\right) \tag{39}$$

- The plus sign corresponds to atoms moving towards the observer (blue shift), while the minus sign corresponds to atoms moving away from the observer (red shift).

5.5 The Lineshape Function

- The spectral half-width of the doppler broadened line is given by:

$$\Delta\nu = \frac{2\nu_0 v}{c} \tag{40}$$

- Therefore, the lineshape function for doppler broadening is a gaussian distribution centered around ν_0 with a width determined by the temperature and mass of the atoms in the gain medium.

5.5 The Lineshape Function

Example 5.5

Calculate the spectral broadening due to the Doppler effect in the carbon dioxide laser ($\lambda = 10.6\mu$ m) assuming that its temperature is 400K. The relative atomic masses of carbon and oxygen are 12 and 16.

5.5 The Lineshape Function

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Solution 5.5

$$\frac{1}{2}Mv^2 = \frac{1}{2}kT$$

$$\implies \quad v = \sqrt{kT/M}$$

The spectral broadening is:

$$\Delta\nu = 2\nu v/c = 2v/\lambda = \frac{2}{\lambda}\sqrt{kT/M}$$

$$M = \frac{12 + 2 * 16}{6.022 \times 10^{26}} = 7.31 \times 10^{-26}\ \text{Kg}$$

$$\implies \quad \Delta\nu = 51.9\ \text{MHz}.$$

5.6 Including the Linewidth function to the small signal gain

$$k = (N_2 - N_1)\frac{nh\nu_{21}B_{21}}{c}g(\nu) \tag{41}$$

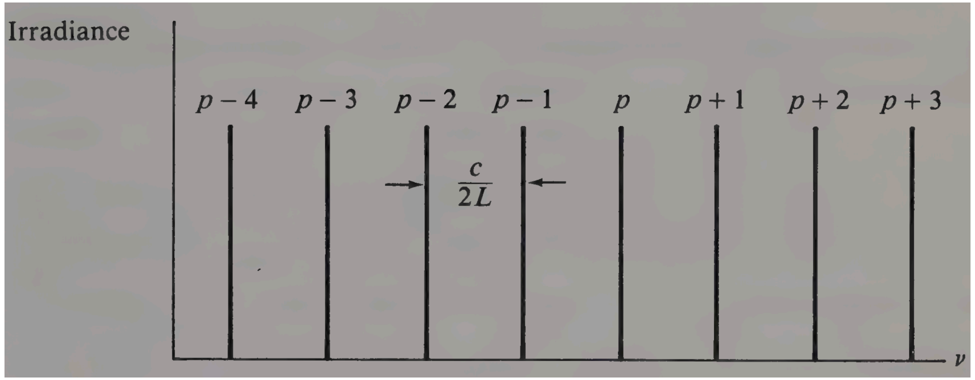
since $g(\nu_0)$, k at the central frequency ν_0 becomes:

$$k = (N_2 - N_1)\frac{nh\nu_{21}B_{21}}{c\Delta\nu} \tag{42}$$

Notice that it is harder to achieve population inversion and lasing at large $\Delta\nu$. Therefore, gain media with narrow linewidths are generally preferred for laser applications.

6.1 Axial Modes

- The optical resonator supports a set of discrete frequencies, known as **modes**, that can be amplified by the gain medium.



- The simplest modes are the **axial modes**, which correspond to standing waves along the axis of the resonator.
- The frequencies of the axial modes are determined by the length of the resonator and the speed of light, and are given by:

$$\nu_p = p\frac{c}{2L} \qquad (43)$$

where p is an integer (1, 2, 3, ...), and L is the length of the resonator.

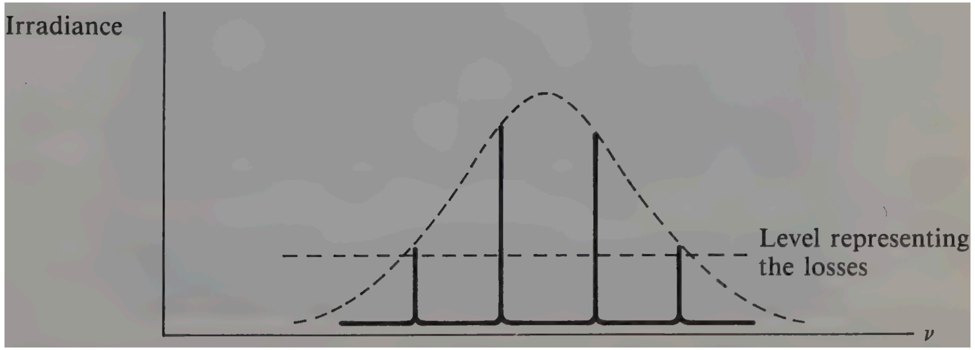
6.1 Axial Modes

- The axial modes are equally spaced in frequency, with a spacing of

$$\Delta\nu_{\text{spacing}} = \frac{c}{2L} \qquad (44)$$

- The number of axial modes that can be supported by the resonator depends on the gain bandwidth of the laser medium and the length of the resonator.

- In many lasers, only a few axial modes are amplified, leading to a narrow linewidth of the laser output.



6.1 Axial Modes

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Example 6.6

(a) Calculate the frequency spacing between axial modes for a HeNe laser (λ632.8 nm) with a resonator length of 0.5 m. (b) Calculate the number of axial modes (p) that can be supported by the resonator. (c) If the gain bandwidth of the laser medium is 1.5 GHz, how many axial modes can be amplified?

6.1 Axial Modes

Solution 6.6

a) The frequency spacing between axial modes is given by:

$$\Delta\nu_{\text{spacing}} = \frac{c}{2L} = \frac{3 \times 10^8}{2 \times 0.5} = 300 \ \text{MHz}.$$

b) The number of axial modes that can be supported by the resonator is given by:

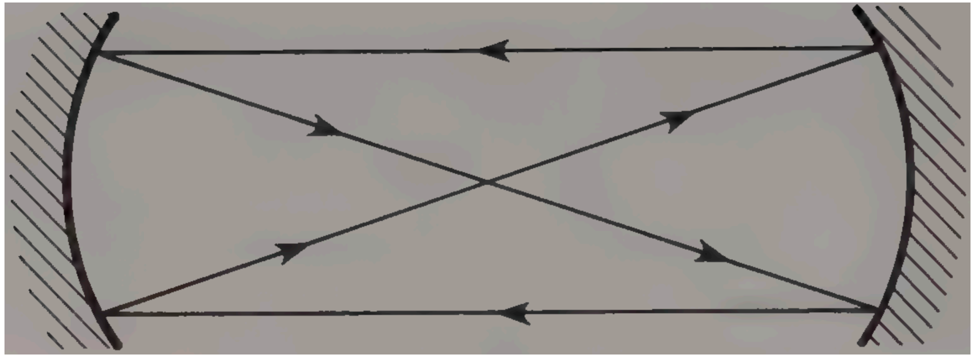
$$p = \frac{2L}{\lambda} = \frac{2 \times 0.5}{632.8 \times 10^{-9}} = 1.58 \times 10^6 \quad \text{Modes}.$$

c) The number of axial modes that can be amplified is given by:

$$N = \frac{\text{Gain Bandwidth}}{\text{Frequency Spacing}} = \frac{\Delta\nu}{\Delta\nu_{\text{spacing}}} = \frac{1.5 \times 10^9}{300 \times 10^6} = 5 \quad \text{Modes}.$$

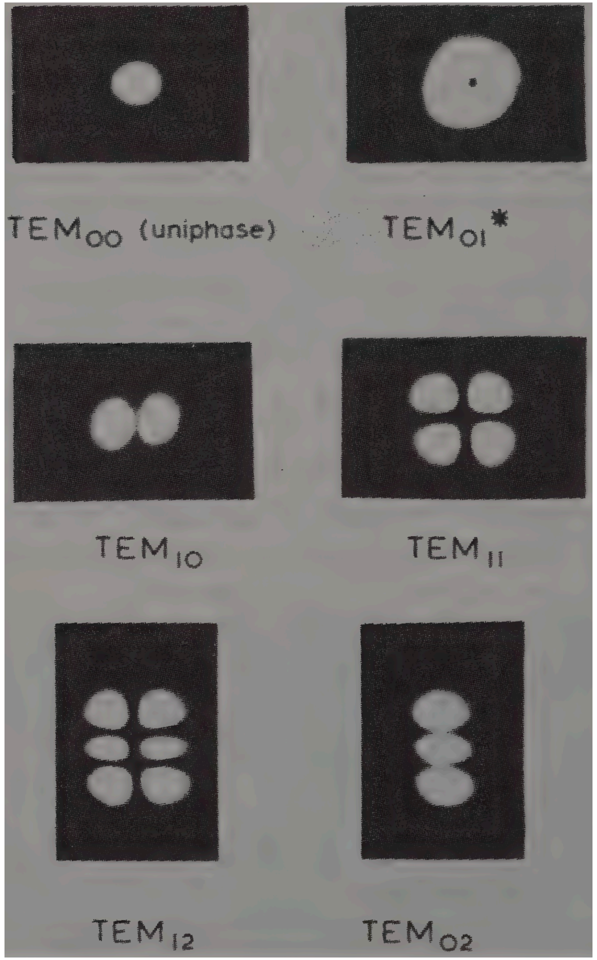
6.2 Transverse Modes

- In addition to axial modes, the optical resonator can also support **transverse modes**, which correspond to variations in the intensity distribution of the laser beam across its cross-section.
- The transverse modes arise due to the non-axial beam paths that leads to interference patterns in the transverse plane of the beam.



- The transverse modes are typically denoted by two integers (q,r), which represent the number of minima in the horizontal and vertical directions, respectively.

6.2 Transverse Modes



- The fundamental transverse mode is the TEM00 mode, which has a Gaussian intensity distribution and no minima in either direction.
- Higher-order transverse modes (e.g., TEM01, TEM10, TEM11) have more complex intensity distributions with minima in one or both directions.
- The transverse modes can be selectively excited or suppressed by adjusting the alignment of the optical resonator and the properties of the gain medium.
- The presence of multiple transverse modes can lead to a broader beam profile and reduced beam quality, which may be undesirable for certain applications.

6.2 Transverse Modes

- Therefore, many lasers are designed to operate in the fundamental transverse mode (TEM00) to achieve a high-quality, well-collimated beam.

جامعة الملك سعود
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1. Introduction

2. Absorption and stimulated emission of light

3. Einstein Relations

4. The Gain Coefficient

5. The Optical Resonator

6. Laser Modes

7. Problems

7.1 Problems

Problem 7.1

For a system in thermal equilibrium calculate the temperature at which the rates of spontaneous and stimulated emission are equal for a wavelength of 10 μm, and the wavelength at which these rates are equal at a temperature of 4000 K.

7.1 Problems

Answer 7.1

$$R = \exp(hc/\lambda kT) - 1 = 1$$

$$\frac{hc}{\lambda kT} = \ln(2)$$

$$\Longrightarrow \quad T = \frac{hc}{\lambda k \ln(2)} = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{10 \times 10^{-6} \times 1.38 \times 10^{-23} \times \ln(2)} = 2078K$$

$$\lambda = \frac{hc}{kT \ln(2)} = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{1.38 \times 10^{-23} \times 4000 \times \ln(2)} = 5.2\mu\text{m}$$

7.1 Problems

Problem 7.2

If 0.5% of the light incident onto a medium is absorbed in 1 mm, what fraction is transmitted by the medium if it is 0.1 m long? Calculate the absorption coefficient of the medium.

7.1 Problems

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Answer 7.2

$$I(x) = I_0 e^{-\alpha x}$$

$$\frac{I(x)}{I_0} = e^{-\alpha(0.001)} = \frac{99.5}{100}$$

$$\implies \quad \alpha = -\frac{\ln\left(\frac{99.5}{100}\right)}{0.001} = 5.01 m^{-1}$$

Therefore, the transmission fraction over 0.1 m is:

$$\frac{I(0.1)}{I_0} = e^{-\alpha(0.1)} = 60.59\%$$

7.1 Problems

Problem 7.3

If the irradiance of a beam of light increases by 25% after a round trip through a gain medium which is 0.3 m long, calculate the small-signal gain coefficient k assuming no losses.

7.1 Problems

Answer 7.3

$$I(x) = I_0 e^{kx}$$

$$\frac{I(x) - I_0}{I_0} = \frac{I(x)}{I_0} - 1 = \frac{25}{100}$$

$$\Rightarrow \quad e^{k(2L)} - 1 = \frac{25}{100}$$

$$k(2L) = \ln(1.25)$$

$$\implies \quad k = \left(\frac{1}{2(0.3)}\right) \ln(1.25) = 0.37m^{-1}$$

7.1 Problems

Problem 7.4

Given that the lifetime of the upper level of the 632.8 nm transition in the HeNe laser is 1×10^{-7} s, calculate the degree of population inversion required to give a gain coefficient of $0.07\ \mathrm{m}^{-1}$ ignoring line-broadening effects.

7.1 Problems

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Answer 7.4

$$K = (N_2 - N_1)\frac{nh\nu_{21}B_{21}}{c}$$

$$A_{21} = B_{21}\frac{8\pi h\nu^3}{c^3} = \frac{1}{\tau_{21}}$$

$$\Rightarrow \quad B_{21} = \frac{c^3}{8\pi h\nu^3\tau_{21}}$$

$$\Rightarrow K = (N_2 - N_1)\left(\frac{nh\nu_{21}}{c}\right)\left(\frac{c^3}{8\pi h\nu^3\tau_{21}}\right) = (N_2 - N_1)\frac{nc^2}{8\pi\nu^2\tau_{21}}$$

7.1 Problems

$$(N_2 - N_1) = \frac{8\pi\nu^2\tau_{21}K}{nc^2} = \frac{8\pi\tau_{21}K}{n\lambda^2}$$

$$(N_2 - N_1) = \frac{8\pi(10^{-7})(0.07)}{(1)(632.8\times10^{-9})^2} = 4.39\times10^5\ \text{m}^{-3}$$

7.1 Problems

Problem 7.5

Calculate the Doppler broadened line width in the argon ion laser for $\lambda = 488$ nm transition, given that the temperature of the discharge is 6000 K and the relative atomic mass of argon is 38.95. Repeat the calculation for the 632.8 nm line of the HeNe laser, where the temperature of the discharge is about 400 K. The relative atomic mass of neon is 20.2.

7.1 Problems

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Answer 7.5

$$\Delta\nu = \frac{2\nu_0 v}{c} = \frac{2v}{\lambda}$$

$$\frac{1}{2}Mv^2 = \frac{1}{2}kT \quad \Longrightarrow \quad v = \sqrt{kT/M}$$

$$M = \frac{38.95}{6.022 \times 10^{26}} = 6.47 \times 10^{-26} \ \text{ kg}$$

$$\Longrightarrow \Delta\nu = \left(\frac{2}{\lambda}\right)\sqrt{kT/M} = \left(\frac{2}{488 \times 10^{-9}}\right)\sqrt{\frac{(1.38 \times 10^{-23}) * 6000}{6.47 \times 10^{-26}}}$$

$$\Delta\nu = 4.64 \ \text{ GHz} \qquad \text{for argon ion laser}$$

$$\Delta\nu = 1.22 \ \text{ GHz} \qquad \text{for helium-neon laser}$$

7.1 Problems

Problem 7.6

Using the result of Problem 1.9, estimate the threshold gain coefficient for the 632.8 nm line of the HeNe laser, given that the threshold population inversion is $1 \times 10^{15}\ \mathrm{m}^{-3}$ and the spontaneous lifetime of the upper laser level is 1×10^{-7} s.

7.1 Problems

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Answer 7.6

$$K = (N_2 - N_1)\frac{nh\nu_{21}B_{21}}{c\Delta\nu}$$

$$B_{21} = \frac{c^3}{8\pi h\nu^3\tau_{21}}$$

$$K = (N_2 - N_1)\left(\frac{nh\nu_{21}}{c\Delta\nu}\right)\left(\frac{c^3}{8\pi h\nu^3\tau_{21}}\right) = (N_2 - N_1)\frac{n\lambda^2}{8\pi\tau_{21}\Delta\nu}$$

$$K = (10^{15})\frac{(632.8 \times 10^{-9})^2}{8\pi(10^{-7})(1.22 \times 10^9)} = 0.13\ \text{m}^{-1}$$

7.1 Problems

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Problem 7.7

Calculate the mode number nearest the line center of a carbon dioxide laser ($\lambda = 10.6\ \mu m$) which is 2 m long. Calculate the frequency separation of modes and estimate how many modes lie within the spectral linewidth (see example 1.3).

7.1 Problems

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Answer 7.7

The frequency separation of modes is:

$$\Delta\nu_{\text{sep}} = \frac{c}{2L} = \frac{3 \times 10^8}{2 * 2} = 75 \text{ MHz}$$

From example 1.3, the spectral broadening due to the Doppler effect is $\Delta\nu = 51.9$ MHz, which is much smaller than the frequency separation of modes $\Delta\nu_{\text{sep}}$.

Therefore, Only one mode will be amplified, and the laser will operate in a single longitudinal mode.