King Saud University جامعة الملك سعود

College of Sciences
Department of Physics & Astronomy

كلية العلوم
قسم الفيزياء والفلك

Final Exam
Academic Year 1442 Hijri- Second Semester

Exam Information معلومات الامتحان			
Course name	**Classical Electrodynamics**		**اسم المقرر**
Course Code	**PHYS 507**		**رمز المقرر**
Exam Date	**Sunday 25th April 2020**	**13-09-1442**	**تاريخ الامتحان**
Exam Time	**09:00 – 12:00**		**وقت الامتحان**
Exam Duration	**180 minutes**	**180 دقيقة**	**مدة الامتحان**
Classroom No.			رقم قاعة الاختبار
Instructor Name	**Professor Vasileios Lempesis**		**اسم استاذ المقرر**

Student Information معلومات الطالب		
Student's Name		**اسم الطالب**
ID number		**الرقم الجامعي**
Section No.	**72551/48936**	**رقم الشعبة**
Serial Number		**الرقم التسلسلي**

تعليمات عامة:

1. الالتزام بوقت بداية الاختبار المدرجة في البوابة الاكاديمية ونهاية الاختبار التي ستحدد من قبل عضو هيئة التدريس.
2. تسليم اجابة الاختبار دون تأخير .ومن يتأخر عن موعد التسليم لن يقبل منه.
3. تسليم الإجابة عبر البلاك بورد أو البريد الإلكتروني أو كليهما(إما مستند pdf أو صورة ممسوحة ضوئيًا.)
4. يجب أن يتم إتمام الاختبار بشكل فردي .ويُحظر عرضه أو مناقشته مع أي شخص آخر، بما في ذلك (على سبيل المثال لا الحصر) الطلاب الآخرين في نفس المقرر.
5. يمكن للطالب استخدام أي مادة متاحة يريدها، بما في ذلك العروض التقديمية ومذكرة المحاضرات والكتب والإنترنت، ولا يجب نسخ المعلومة كما هي ولكن تكتب حسب فهم الطالب وإلا ستعتبر إقتباسا يؤثر على درجة الطالب.
6. سوف يتم النظر في جميع الحالات الطلابية التي لم يتمكنوا من أداء الاختبار المنزلي بعقد اختباربديل لها في بداية الفصل الدراسي الأول.

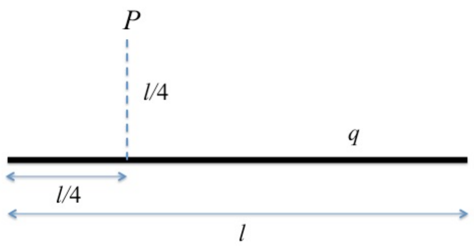
هذا الجزء خاص بأستاذ المادة
This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1	CLO 1.1:			40
2	CLO 1.2:			
3	CLO 1.3:			

Please answer all the following questions. The best four will contribute to your exam grade.

1. The rod in the figure has a positive charge q and a length l. The charge is uniformly distributed along the rod. Find the **electric potential** at the point P. You are given: $\int \frac{dx}{\left(x^2+k\right)^{1/2}} = \ln\left(x+\sqrt{x^2+k}\right)$

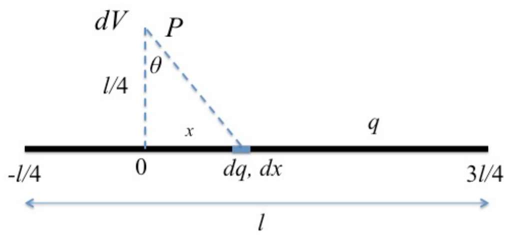
(5 marks)



Solution

Commented [VL1]: Some of you found the electric field Instead of electric potential. This is a completely wrong answer

The rod has a linear charge density $\lambda = q/l$



The elementary charge dq shown in the figure creates an elementary potential at P

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\left(x^2+l^2/16\right)^{1/2}} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{\left(x^2+l^2/16\right)^{1/2}}$$

To find the total potential we need to integrate from x = -l/4 to x = 3l/4.

Thus

$$V = \int_{x=-l/4}^{x=3l/4} dV = \frac{\lambda}{4\pi\varepsilon_0} \int_{x=-l/4}^{x=3l/4} \frac{dx}{\left(x^2 + l^2/16\right)^{1/2}} =$$

$$\frac{\lambda}{4\pi\varepsilon_0}\left[\ln\left(x + \sqrt{x^2 + l^2/16}\right)\right]_{x=-l/4}^{3l/4} =$$

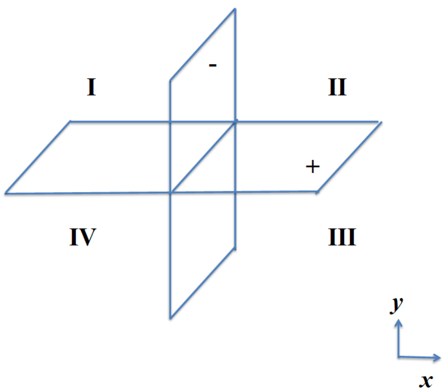
$$\frac{\lambda}{4\pi\varepsilon_0}\left[\ln\left(3l/4 + \sqrt{9l^2/16 + l^2/16}\right) - \ln\left(-l/4 + \sqrt{l^2/16 + l^2/16}\right)\right] =$$

$$\frac{\lambda}{4\pi\varepsilon_0}\left[\ln\left(3l/4 + \sqrt{10l^2/16}\right) - \ln\left(-l/4 + \sqrt{l^2/8}\right)\right] =$$

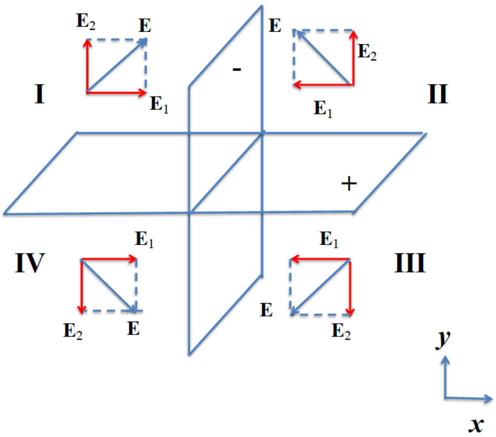
$$\frac{\lambda}{4\pi\varepsilon_0}\left[\ln\left(\frac{3l/4 + \sqrt{10l^2/16}}{-l/4 + \sqrt{l^2/8}}\right)\right] = \frac{q}{4\pi\varepsilon_0 l}\left[\ln\left(\frac{3l/4 + \sqrt{10l^2/16}}{-l/4 + \sqrt{l^2/8}}\right)\right]$$

2. Two infinite plane sheets with equal but opposite surface charge densities ($\pm\sigma$) divide the space in four regions (I, II, III and IV) as shown in figure. Find the electric field (magnitude and direction) at each region. The axes directions are shown in the figure.

(5 marks)



Solution:



The total electric field is the resultant of the field $\mathbf{E}_1$ due to the positively charged sheet and of the field $\mathbf{E}_2$ of the negatively charged sheet. Both of them are everywhere perpendicular to each other and have the same magnitude $\sigma/2\varepsilon_0$. Thus for the total field we have:

I. $\mathbf{E} = \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{x}} + \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{y}}, \quad \theta = 45^0$

II. $\mathbf{E} = -\frac{\sigma}{2\varepsilon_0}\hat{\mathbf{x}} + \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{y}}, \quad \theta = 135^0$

III. $\mathbf{E} = -\frac{\sigma}{2\varepsilon_0}\hat{\mathbf{x}} - \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{y}}, \quad \theta = 225^0$

IV. $\mathbf{E} = \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{x}} - \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{y}}, \quad \theta = 315^0$

$$E = \sqrt{E_1^2 + E_2^2} = \sqrt{\left(\frac{\sigma}{2\varepsilon_0}\right)^2 + \left(\frac{\sigma}{2\varepsilon_0}\right)^2} = \frac{\sigma\sqrt{2}}{2\varepsilon_0}$$

3. We have shown that the field created by a polarized object is made up by the field of a volume charge density and a field of a surface charge density. Assume that we have an electrically polarized sphere of polarization $\mathbf{P} = Ar\mathbf{r}$ (spherical coordinates), with A being a positive constant. The radius of the sphere is R. Find the electric field at a point outside the sphere ($r > R$).

(5 marks)

Solution:
Our first job is to find the volume charge density:

$$\rho_b = -\nabla\mathbf{P} = -\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 Ar) = -3A$$

Then we calculate the surface charge density:

$$\sigma_S = n \cdot \mathbf{P}|_{r=R} = \mathbf{r} \cdot \mathbf{P}|_{r=R} = \mathbf{r} \cdot (AR\mathbf{r}) = A$$

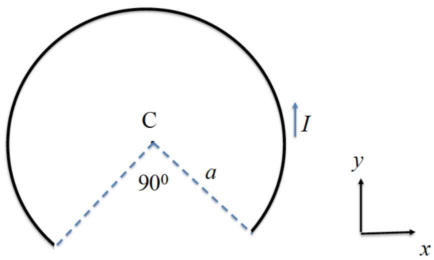
Commented [VL2]: Most of you had here *Ar*. Surface density means that we are **on** the surface Where *r* =*R*

The total electric field is the resultant of (a) field of a charged sphere of density $\rho_b = -3A$ and thus of total charge $Q_b = -3A\frac{4}{3}\pi R^3 = -4A\pi R^3$ and (b) of a charged spherical shell of surface charge density $\sigma_S = AR$ and thus of total charge $Q_S = AR4\pi R^2 = 4A\pi R^3$. The problem has a spherical symmetry and thus the electric field at a point outside the sphere will be:

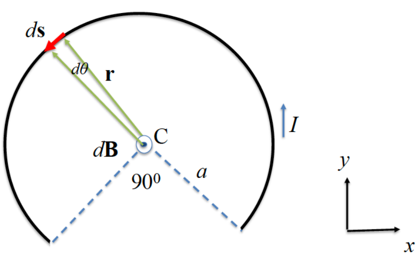
$$\mathbf{E} = -\frac{1}{4\pi\varepsilon_0}\frac{4A\pi R^3}{r^2}\mathbf{r} + \frac{1}{4\pi\varepsilon_0}\frac{4A\pi R^3}{r^2}\mathbf{r} = 0$$

4. An arc-like wire with radius a is shown in the figure. The wire carries a current *I*. (i) Find the magnetic field at the center C. (ii) Find the magnetic potential at the center C. You are given the Biot-Savart formula: $\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{Id\mathbf{s} \times \mathbf{r}}{r^3}$ and that $d\mathbf{A} = \frac{\mu_0 I}{4\pi}\frac{d\mathbf{s}}{r}$.

(5 marks)



Solutions:



(i) As we see in the figure an element $d\mathbf{s}$ of the wire creates an elementary magnetic field at the center of the wire given by:

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I d\mathbf{s}\times\mathbf{r}}{r^3} = \frac{\mu_0}{4\pi}\frac{a d\theta a}{a^3} = \frac{\mu_0}{4\pi}\frac{d\theta}{a}\mathbf{k}$$

Thus, the total magnetic field is given by:

$$\mathbf{B} = \mathbf{k}\frac{\boldsymbol{I}\mu_0}{4\pi a}\int_0^{3\pi/2} d\theta$$

$$\mathbf{B} = \mathbf{k}\frac{3\pi\mu_0 I}{4\pi a2} = \frac{3\mu_0 I}{8a}\mathbf{k}$$

(ii) For the magnetic potential we have

$$d\mathbf{A} = \frac{\mu_0 I}{4\pi}\frac{d\mathbf{s}}{r} \Rightarrow d\mathbf{A} = \frac{\mu_0 I}{4\pi}\frac{a d\theta}{a}\boldsymbol{\theta}$$

$$\mathbf{A} = \boldsymbol{\theta}\frac{\mu_0 I}{4\pi}\int_0^{3\pi/2} d\theta$$

$$\mathbf{A} = \boldsymbol{\theta}\frac{\mu_0 I}{4\pi}\frac{3\pi}{2}$$

$$\mathbf{A} = \frac{3\mu_0 I}{8}\boldsymbol{\theta}$$

5. A long circular cylinder of radius R carries a magnetization $\mathbf{M} = kr^2\boldsymbol{\theta}$ (cylindrical coordinates) where k is a constant and r is the distance from the axis and $\boldsymbol{\theta}$ is the usual azimuthal unit vector. Find (i) the bound current density $\mathbf{J}_b$, (ii) the surface current density $\mathbf{K}_b$, (iii) the total current, (iv) magnetic field due to $\mathbf{M}$, for points inside and outside the cylinder.

(5 marks)

Solutions:

(i) $\mathbf{J}_b = \nabla\times\mathbf{M} = \left(\frac{1}{r}\frac{\partial M_z}{\partial\theta} - \frac{\partial M_\theta}{\partial z}\right)\mathbf{r} + \left(\frac{\partial M_r}{\partial z} - \frac{\partial M_z}{\partial r}\right)\boldsymbol{\theta} + \frac{1}{r}\left(\frac{\partial}{\partial r}(rM_\theta) - \frac{\partial M_r}{\partial\theta}\right)\mathbf{z}$

$$\mathbf{J}_b = \left(0 - \frac{\partial kr^2}{\partial z}\right)\mathbf{r} + (0-0)\boldsymbol{\theta} + \frac{1}{r}\left(\frac{\partial}{\partial r}(rkr^2) - 0\right)\mathbf{z}$$

$$\mathbf{J}_b = 3kr\mathbf{z}$$

(iii) $\mathbf{K}_b = \mathbf{M}\times\mathbf{n} = kr^2(\boldsymbol{\theta}\times\mathbf{r})\big|_{r=R} = -kR^2\mathbf{z}$

Commented [VL3]: Most of you had here $-kr^2$. Surface density means that we are **on** the surface Where $r = R$.

(iv) To find the total current we need to calculate the current due to the bound current density and the current due to the surface current density.

$$\int \mathbf{J}_b \cdot d\mathbf{a} + \int K_b dl = \int_0^R 3kr2\pi r dr - \int_0^R kR^2 2\pi dr =$$

$$= 6\pi k \int_0^R r^2 dr - 2\pi k R^2 \int_0^R dr =$$

$$= 6\pi k \frac{R^3}{3} - 2\pi k R^3 = 0$$

(v) To find the magnetic field we need to apply the Ampere's law (we have a cylindrically symmetric problem thus the magnetic field has an azimuthal direction):

For $r < R$ the magnetic field is due to bound current thus

$$B2\pi r = \mu_0 I_{enc} \int \mathbf{J}_b \cdot d\mathbf{a}$$

$$B2\pi r = \mu_0 I_{enc} \int_0^r 3kr' 2\pi r' dr'$$

$$B2\pi r = 6k\pi \mu_0 I_{enc} \frac{r^3}{3}$$

$$B2\pi r = 6k\pi \mu_0 I_{enc} \frac{r^3}{3}$$

$$B = \mu_0 k r^2$$

$$\mathbf{B} = \mu_0 k r^2 \boldsymbol{\theta}$$

$$\mathbf{B} = \mu_0 \mathbf{M}$$

For the case outside the cylinder, where $r > R$ the magnetic field is zero because the enclosed current is zero.

MATHEMATICAL SUPPLEMENT

- CYLINDRICAL COORDINATES

  Surface element on x-y plane: $dA = \rho d\rho d\varphi$

  Elementary volume: $d\tau = \rho d\rho d\varphi dz$

  $0 \le \varphi \le 2\pi$

Unit vectors $\left\{\hat{\mathbf{r}}, \hat{\boldsymbol{\theta}}, \hat{\mathbf{z}}\right\}$, $\hat{\mathbf{r}} \times \hat{\boldsymbol{\theta}} = \hat{\mathbf{z}}$, $\mathbf{A} = A_r\,\hat{\mathbf{r}} + A_\theta \hat{\boldsymbol{\theta}} + A_z\,\hat{\mathbf{z}}$.

$$\begin{aligned} x &= r\,\cos\theta & r &= \sqrt{x^2+y^2} \\ y &= r\,\sin\theta & \tan\theta &= \frac{y}{x} \\ z &= z & z &= z \end{aligned}$$

$$\boldsymbol{\nabla}\psi = \frac{\partial\psi}{\partial r}\,\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial\psi}{\partial\theta}\hat{\boldsymbol{\theta}} + \frac{\partial\psi}{\partial z}\,\hat{\mathbf{z}}$$

$$\boldsymbol{\nabla}\cdot\mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(r\,A_r) + \frac{1}{r}\frac{\partial A_\theta}{\partial\theta} + \frac{\partial A_z}{\partial z}$$

$$\boldsymbol{\nabla}\times\mathbf{A} = \left(\frac{1}{r}\frac{\partial A_z}{\partial\theta} - \frac{\partial A_\theta}{\partial z}\right)\hat{\mathbf{r}} + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\hat{\boldsymbol{\theta}} + \frac{1}{r}\left(\frac{\partial}{\partial r}(r\,A_\theta) - \frac{\partial A_r}{\partial\theta}\right)\hat{\mathbf{z}}$$

Note that

$$\boldsymbol{\nabla}\times\mathbf{A} = \frac{1}{r}\begin{vmatrix} \hat{\mathbf{r}} & r\hat{\boldsymbol{\theta}} & \hat{\mathbf{z}} \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial\theta} & \dfrac{\partial}{\partial z} \\ A_r & r\,A_\theta & A_z \end{vmatrix}.$$

$$\nabla^2\psi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\,\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2} + \frac{\partial^2\psi}{\partial z^2}$$

- SPHERICAL COORDINATES

  Elementary volume: $d\tau = r^2 dr \sin\theta d\theta d\varphi$

Elementary surface on a sphere of radius r: $dA = r^2 \sin\theta d\theta d\varphi$

$0 \le \theta \le \pi, \quad 0 \le \varphi \le 2\pi$

Unit vectors $\left\{\hat{\mathbf{r}}, \hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\phi}}\right\}$, $\hat{\mathbf{r}} \times \hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}$, $\mathbf{A} = A_r\,\hat{\mathbf{r}} + A_\theta \hat{\boldsymbol{\theta}} + A_\phi \hat{\boldsymbol{\phi}}$.

$$x = r\,\sin\theta\,\cos\phi \qquad r = \sqrt{x^2+y^2+z^2}$$

$$y = r\,\sin\theta\,\sin\phi \qquad \cos\theta = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

$$z = r\,\cos\theta \qquad \tan\phi = \frac{y}{x}$$

$$0 \le \theta \le \pi, \qquad 0 \le \phi \le 2\pi$$

.

$$\boldsymbol{\nabla}\,\psi = \frac{\partial \psi}{\partial r}\,\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial \psi}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\,\sin\theta}\frac{\partial \psi}{\partial \phi}\hat{\boldsymbol{\phi}}$$

$$\boldsymbol{\nabla}\cdot\mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\,A_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\,A_\theta\right) + \frac{1}{r\,\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\boldsymbol{\nabla}\times\mathbf{A} = \frac{1}{r\,\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta\,A_\phi\right) - \frac{\partial A_\theta}{\partial\phi}\right]\hat{\mathbf{r}} + \left[\frac{1}{r\,\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{1}{r}\frac{\partial}{\partial r}\left(r\,A_\phi\right)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\,A_\theta\right) - \frac{\partial A_r}{\partial\theta}\right]\hat{\boldsymbol{\phi}}$$

Note that

$$\boldsymbol{\nabla}\times\mathbf{A} = \frac{1}{r^2\,\sin\theta}\begin{vmatrix} \hat{\mathbf{r}} & r\hat{\boldsymbol{\theta}} & r\,\sin\theta\hat{\boldsymbol{\phi}} \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial\theta} & \dfrac{\partial}{\partial\phi} \\ A_r & r\,A_\theta & r\,\sin\theta\,A_\phi \end{vmatrix}.$$

$$\nabla^2\psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\,\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}$$