PHYS 301
HANDOUT 9

1. Discuss the singularities of the function $1/z$.

2. Discuss the singularities of the function $f(z) = (z+1)/z^3(z^2+1)$.

3. Discuss the singularities of the function Logz.

4. Discuss the singularities of the function $1/\sin(\pi/z)$.

5. Discuss the singularities of the function $(z^2-2z+1)/[z(z+1)^3]$.

6. You are given the function $f(z) = \dfrac{z^2+5z+3}{(z-1)(z+2)^2}$. Find its poles and the corresponding residues.

7. You are given the function $f(z) = \dfrac{1}{\cos z}$. Find its poles and the corresponding residues.

8. Discuss the singularities-poles of the function $\log(z+1)/(z-1)$.

9. Discuss the singularities-poles of the function $(z^{1/2}-1)/(z-1)$

10. Calculate the integral $\int_C \dfrac{e^{-z}}{(z-1)^2} dz$, where C is the positive circle $|z| = 2$.

11. Show that $\int_C \exp\left(\dfrac{1}{z^2}\right) dz = 0$ where C is the positive circle $|z| = 2$.

12. Apply the residue theorem in the case of the integral $\int_C \dfrac{5z-2}{z(z-1)} dz$, where C is the positive circle $|z| = 2$.

13. Identify the singular points of the function: $(z^2-2z+3)/(z-2)$.

14. Identify the singular points of $\sinh z / z^4$.

15. Identify the singular points $\exp(1/z)$.

16. Identify the singular points of the function $f(z)=\left(e^{z}-1\right)/z$.

17. Identify singular points, poles and find the residue of the function $f(z)=\left(z+1\right)/\left(z^{2}+9\right)$.

18. Identify singular points, poles and find the residue of the function $f(z)=\left(z^{3}+2z\right)/\left(z-i\right)^{3}$.

19. Identify singular points, poles and find the residue of the function $f(z)=\left(\sinh z\right)/z^{4}$.

20. Identify singular points, poles and find the residue of the function $f(z)=1/\left[z\left(e^{z}-1\right)\right]$.

21. Find the residues at the poles of the function $f(z)=z/\left[\left(z-1\right)\left(z+1\right)^{2}\right]$ (Sch. 172).

22. Find the residue of $f(z)=e^{-1/z}$. (Sch 173)

23. Find the residues at the poles of the function $f(z)=\left(z^{2}-2z\right)/\left[\left(z^{2}+4\right)\left(z+1\right)^{2}\right]$ (Sch. 177).

24. Calculate the integral $f(z)=\dfrac{1}{2\pi i}\oint_{C}\dfrac{e^{z}}{z^{2}\left(z^{2}+2z+2\right)}dz$, where C is the contour, with $|z|=3$. (Sch.179)

25. Calculate the integral $f(z)=\int_{0}^{\infty}\dfrac{dx}{x^{2}+1}$. (Sch.179)

26. Find the poles of the function $f(z)=1/\cos z$ as well as the residues which, corresponds to these. (Ver. 80)

27. Find the residue of $f(z)=z/\left(z^{4}+4\right)$ at the isolated singular point $z_{0}=e^{i\pi/4}\sqrt{2}=1+i$.

28. Discuss the singularities at $z=0$ of the function $\left(z+1\right)/\left(z\sin z\right)$.

29. Discuss the singularities of the function tanz.

30. If $f(z)=\dfrac{N(z)}{D(z)}$, with N, D analytic at z_0 and $D(z)$ has a root of order $m=1$ at $z=z_0$, then

$$\operatorname*{Res}_{z_0} f(z)=\frac{N(z_0)}{D'(z_0)}$$