College of Science
Department of Statistics & OR

OR 122
Introduction to Operations Research

Chapter 1:

Linear Programming

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Definition 1:

Linear programming is a mathematical technique for detecting an optimum solution of certain real problems.

- Real-life problems includes:

1) Transportation problems
2) Assignment problem
3) Network problem
4) Decision theory-based problems
5) Game theory-based problems. …, etc.
6) Inventory, …, etc.

- Basic Requirements:

a) Decision Variables is denoted by $x_1, x_2, \dots, x_n$.
   - They represent activity such as production, quantity.
   - Through the chapter of linear programing these variables restricted by the non-negative inequalities given

$$x_1 \geq 0, x_2 \geq 0, \dots, x_n \geq 0.$$

$$\text{or} \qquad x_i \geq 0, i = 1, 2, \dots, n.$$

b) The objective function is a function of the decision variables and in general it is denoted by

$$f(x_1, x_2, \dots, x_n)$$

   - Example: profit, cost, time, …, etc.
   - It should be maximized or minimized.

c) Constraints are mathematical expression which combine the decision variables in order to make limits on the possible solutions.

- Standard form of linear programming problems (LPP):

Optimize (Max or Min)

$$Z = f(x_1, x_2, \dots, x_n)$$

Subject to

$$g_j(x_1, x_2, \dots, x_n) \leq (= \text{or } \geq b_j)$$
$$j = 1, 2, \dots, m.$$
$$x_i \geq 0;\ i = 1, 2, \dots, n.$$

where,

$$Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n \qquad \text{objective function.}$$

and $g_j(x_1, x_2, \ldots, x_n)$ can be represented by

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &\le b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &\le b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &\le b_m \end{aligned} \qquad (= \text{or} \ge)$$

$x_1 \ge 0, x_2 \ge 0, \ldots, x_n \ge 0$ non-negative constraints.

$$\text{Constants} \begin{cases} c_i\,, & i = 1,2,\ldots,n. \\ a_{ji}\,, & i = 1,2,\ldots,n;\ j = 1,2,\ldots,m. \end{cases}$$

Notes: Z and $g_j(x_1, x_2, \ldots, x_n);\ j = 1,2,\ldots,m$, must be linear functions

Some Important Definitions

Solution:

Any vector $(x_1, x_2, \dots, x_n)$ satisfies the constraints of the LPP is called a solution.

Feasible Solution (FS):

Any solution satisfies the constraints. The non-negative constraints is called FS.

Basic Solution (BS):

For a set of *m* equations in *n* unknown variables $(n > m)$.

A solution that is obtained by setting $(n - m)$ of the variables equal to zero and solving the remaining *m* equation in *n* unknowns is called basic solution.

Example:

Let

$$x_1 + 2x_2 - x_3 = 1$$
$$2x_1 - x_2 + x_3 = -1$$

We have $m = 2$, and $n = 3$. So $n - m = 1$.

Then we set one of the variables by zero. For example, let $x_3 = 0$.

$$\Rightarrow \begin{aligned} x_1 + 2x_2 &= 1 \\ 2x_1 - x_2 &= -1 \end{aligned}$$

$$\Rightarrow \quad x_1 = -\frac{1}{5}, \qquad x_2 = \frac{3}{5}$$

Then, $\left(-\frac{1}{5}, \frac{3}{5}, 0\right)$ is called a basic solution.

Basic feasible solution:

For a LPP a basic feasible solution is any basic solution that satisfies the constraints and the non-negative constraints.

Optimum feasible solution:

Any basic feasible solution which optimizes (Maximize or minimize) the objective function is known as an optimum feasible solution for LPP.

Methods of Solving LPP

Through this chapter we use two different methods to solve LPP:

1) Graphical Method (only for two variables)
2) The simplex Algorithm

The Graphical Approach

This approach consists of many steps:

Step 1: Graph the constraints

Step 2: Identify the feasible region

Step 3: Locate the solution points

Step 4: Select one of the following two methods

i. The corner-point method, (better to use when the feasible area is bounded).
ii. The iso-profit or iso-cost method

Iso means the profit (or cost) anywhere on the line is the same.

The following examples show how to apply the graphical approach.

Example 1:

Solve using the graphical approach the following LPP

$$\text{Maximize} \quad Z = 20x_1 + 30x_2$$

$$\text{Subject to the constraints} \quad \begin{cases} 3x_1 + 3x_2 \leq 36 \\ 5x_1 + 2x_2 \leq 50 \\ 2x_1 + 6x_2 \leq 60 \end{cases}$$

$$x_1, \ x_2 \geq 0$$

Solution

Step 1: Graph the constraints.

$$3x_1 + 3x_2 = 36 \qquad (1)$$
$$5x_1 + 2x_2 = 50 \qquad (2)$$
$$2x_1 + 6x_2 = 60 \qquad (3)$$

Each constraint from the above is defined by two points in (x_1, x_2)-plane as follows:

In equation (1),

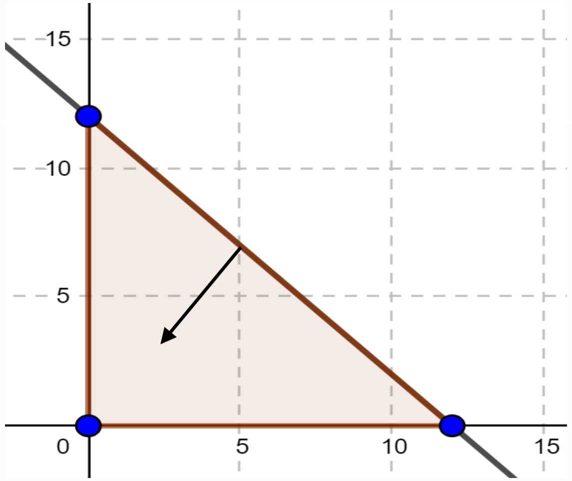
Let $x_1 = 0 \Rightarrow x_2 = 12 \Rightarrow$ The first point is $(0,12)$.

Let $x_2 = 0 \Rightarrow x_1 = 12 \Rightarrow$ The second point is $(12,0)$.

In equation (2) we have $(0,25)$ and $(10,0)$.

In equation (3) we have $(0,10)$ and $(30,0)$.

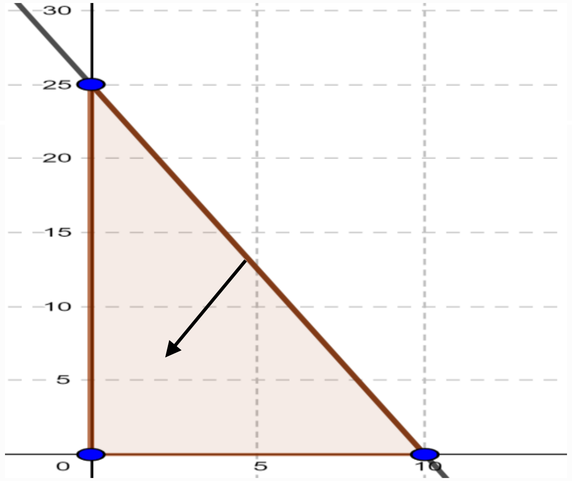
Consider one extra point for each constraint to identify the are that fulfils the constraint.



The shaded area satisfies

$3x_1 + 3x_2 \leq 36$

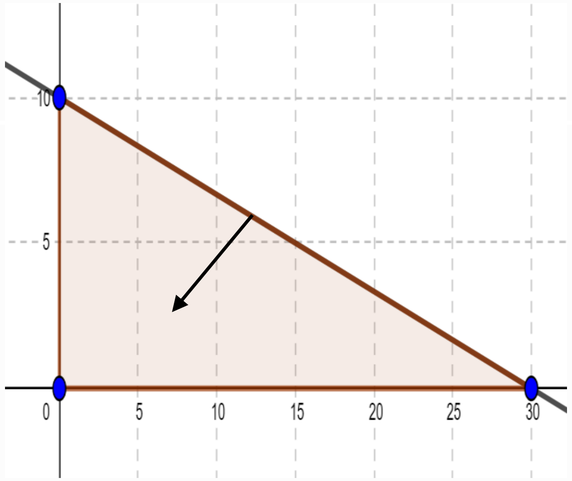
(The first constraint)



The shaded area satisfies

$5x_1 + 2x_2 \leq 50$

(The second constraint)



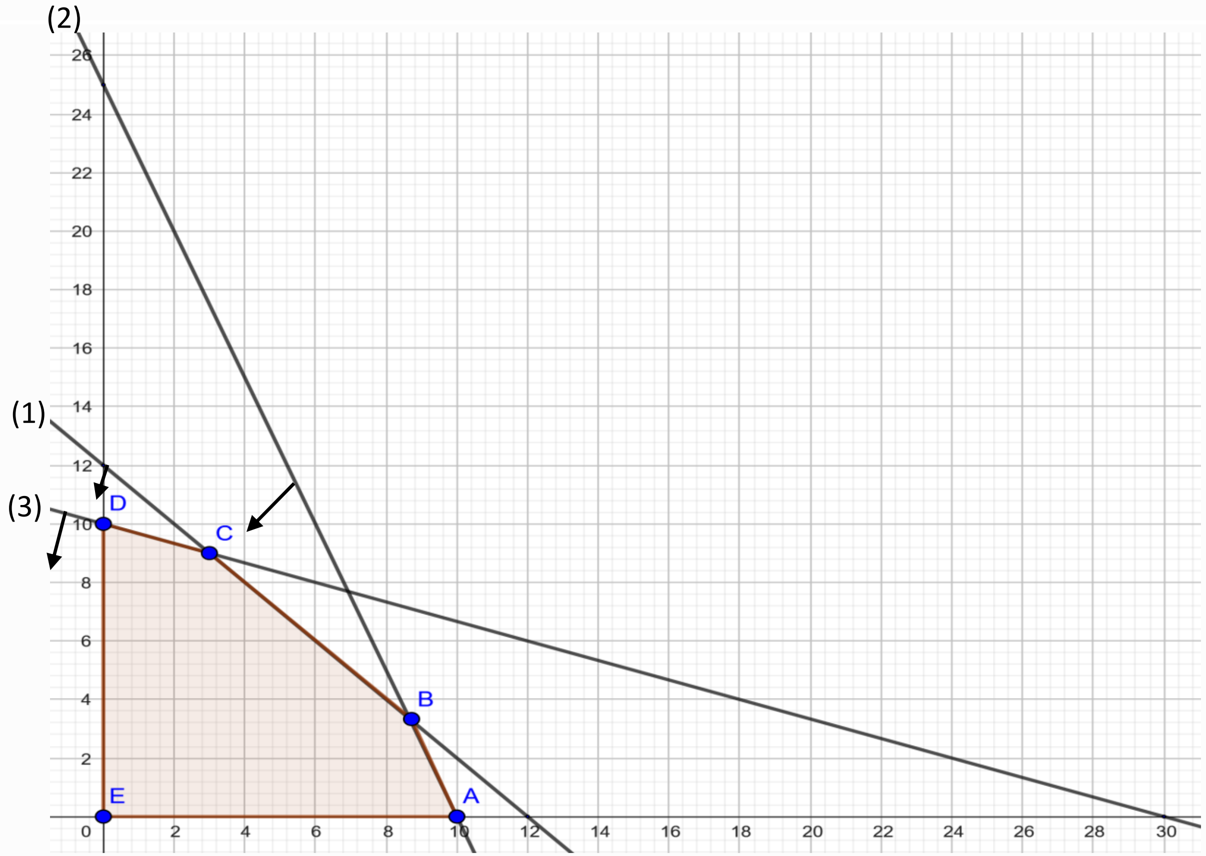
The shaded area satisfies

$2x_1 + 6x_2 \leq 60$

(The third constraint)

Step 2: Identify the feasible region:

The feasible region is represented by the shaded area defined by the shape (A B C D E).



Don't forget to draw the constraint $x_1,\ x_2 \geq 0$

Step 3: The feasible region:

Shaded area has an infinite number of solutions that would satisfy all constraints. Only we search for the points that makes Z maximum. Those points will be only among the points of the solution space (shaded area).

Theorem:

Let the solution space of an LPP be a compact region bounded by lines in plane. Then the objective attains its maxima (or minima) at vertices (corners of feasible region).

In our example, the corner points are: A, B, C, D, E

where, $A = (10, 0)$, $B = (?,\ ?)$, $C = (?\,,?\,)$, $D = (0, 10)$, $E = (0, 0)$.

Calculating the point B:

It is obtained by solving equation (1) and (2) algebraically:

$$3x_1 + 3x_2 = 36 \qquad (1)$$

$$5x_1 + 2x_2 = 50 \qquad (2)$$

$$\Rightarrow B = \left(\frac{26}{3}, \frac{10}{3}\right)$$

Calculating the point C:

It is obtained by solving equation (1) and (3) algebraically:

$$3x_1 + 3x_2 = 36 \qquad (1)$$
$$2x_1 + 6x_2 = 60 \qquad (3)$$
$$\Rightarrow C = (3, 9)$$

Note:

The two equations can be solved using determinants as follows:

$$a_{11}x_1 + a_{12}x_2 = b_1$$
$$a_{21}x_1 + a_{22}x_2 = b_2$$

Solution:

$$x_1 = \frac{\triangle x_1}{\triangle} \quad \text{and} \quad x_2 = \frac{\triangle x_2}{\triangle}$$

where,

$$\triangle = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

$$\triangle x_1 = \begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix} = b_1 a_{22} - b_2 a_{21}$$

$$\triangle x_2 = \begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix} = a_{11}b_2 - b_1 a_{21}$$

Step 4 (i): Using the corner-point Method:

Evaluate the objective function at each corner point as follows:

$$Z(0,0) = 20(0) + 30(0) = 0$$

$$Z(A) = 20(10) + 30(0) = 200$$

$$Z(B) = 20\left(\frac{26}{3}\right) + 30\left(\frac{10}{3}\right) = \frac{820}{3} = 273.33$$

$$Z(C) = 20(3) + 30(9) = 330$$

$\text{Max } Z = 330$ at $C = (3, 9)$ then C is the optimum solution.

Step 4 (ii): Using the iso-profit method:

Assume any point that belongs to the shaded area, here we assume the point (1,1) that belongs to the shades.

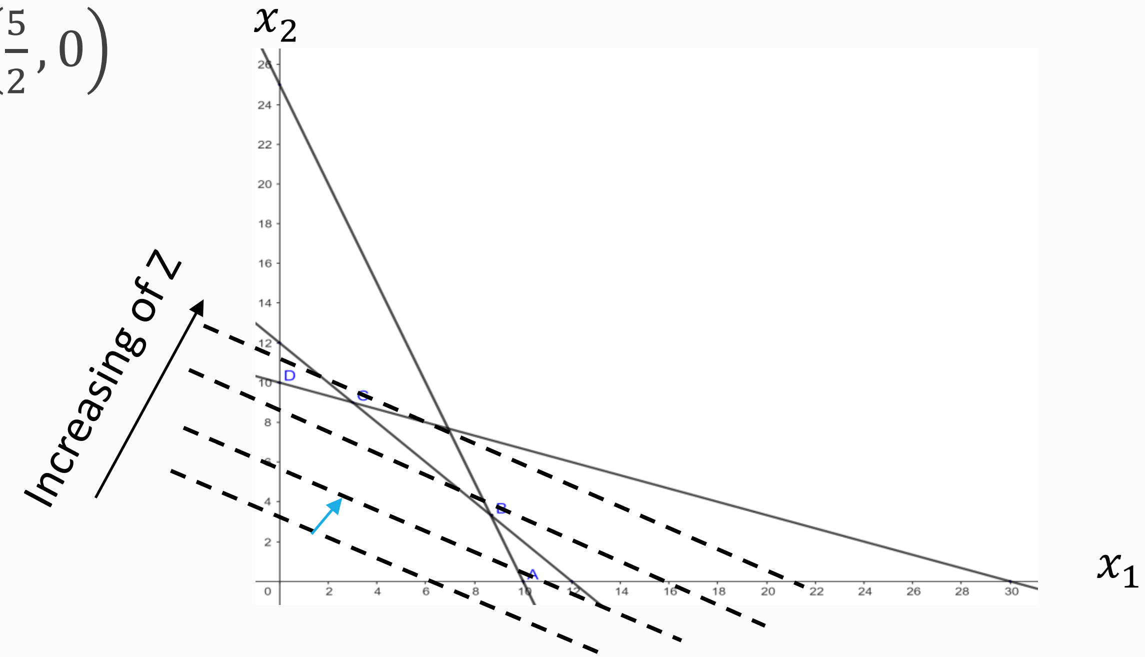
$$\Rightarrow Z(1,1) = 20 + 30 = 50$$

Then we have the iso-profit line as $20x_1 + 30x_2 = 50$, which can be plotted using two points

$\left(0, \frac{50}{30}\right)$ and $\left(\frac{50}{20}, 0\right)$ or $\left(0, \frac{5}{3}\right)$ and $\left(\frac{5}{2}, 0\right)$

- Move the iso-profit line parallel to the increasing direction (as we need to maximize the profit). Assume points under and above Z to determine Z direction.

- Identify the optimum solution as the point on where the height possible iso-profit is touched.



In the example, C is the optimum solution because highest iso-profit line touches it.

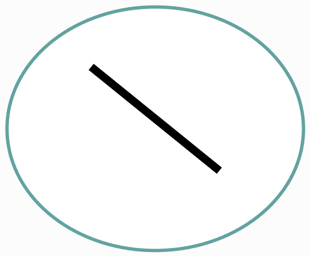
$\Rightarrow$ The optimal solution is $C = (3, 9)$ and $\max.\ Z = 20(3) + 30(9) = 330$.

From the above example we can conclude the following results:

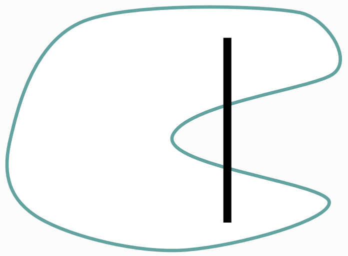
1) Any solution lies within (or on the border lines) the shaded region is called feasible solution.

2) The shaded region is convex. In linear programming, the feasible solution space forms a convex set if the line segment joining any two distinct feasible points also falls in the set.

Example:



Convex set



Nonconvex set

3) Any solution lies outside the shaded is infeasible solution.

Example 2:

$$\text{Minimize} \quad Z = 90x_1 + 135x_2$$

$$\text{Subject to} \quad \begin{cases} 2x_1 + 3x_2 \leq 80 \\ 4x_1 + 6x_2 \leq 150 \\ x_1 \leq 15 \\ x_2 \leq 10 \end{cases}$$

$$x_1,\ x_2 \geq 0$$

Solution

Step 1: Graph the constraints.

$$2x_1 + 3x_2 = 80 \qquad (1)$$

$$4x_1 + 6x_2 = 150 \qquad (2)$$

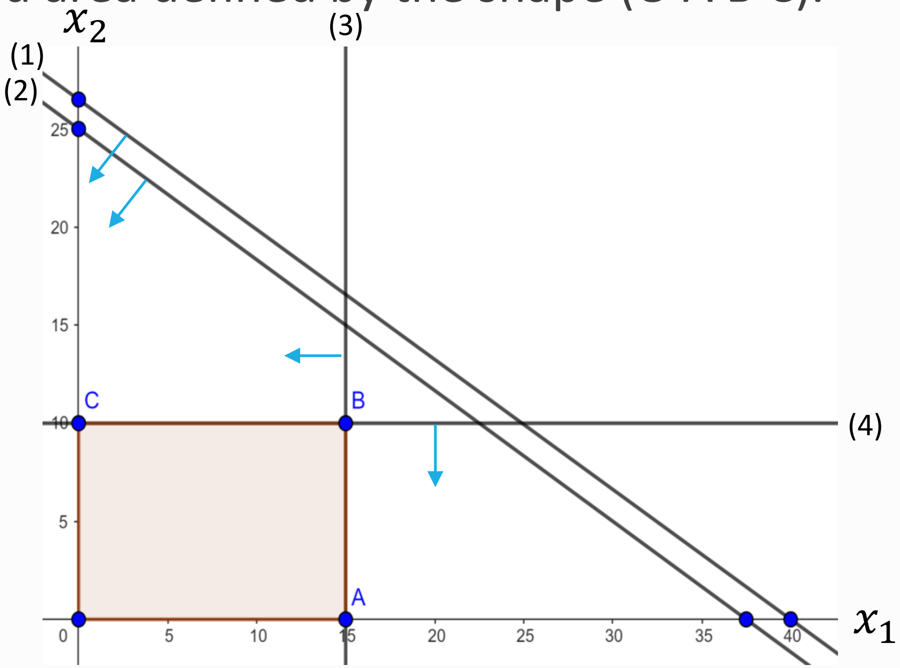
Each constraint from the above is defined by two points in (x_1, x_2)-plane as follows:

In equation (1) we have $\left(0, \frac{80}{3}\right)$ and $(40, 0)$.

In equation (2) we have $(0, 25)$ and $\left(\frac{150}{4}, 0\right)$.

Step 2: Identify the feasible region:

The feasible region is represented by the shaded area defined by the shape (O A B C).



Step 3: The feasible region:

where, $O = (0, 0)$, $A = (15, 0)$, $B = (15, 10)$, $C = (0, 10)$.

Step 4: Using the corner-point Method:

$Z(O) = 90(0) + 135(0) = 0$

$Z(A) = 90(15) + 135(0) = 1350$

$Z(B) = 90(15) + 135(10) = 2700$

$Z(C) = 90(0) + 135(10) = 1350$

$\Rightarrow \text{Min } Z = 0 \text{ at } O = (0,0).$

Example 3

$$\text{Minimize} \quad Z = 600x_1 + 400x_2$$

$$\text{Subject to} \quad \begin{cases} 3000x_1 + 1000x_2 \geq 24000 \\ 1000x_1 + 1000x_2 \geq 16000 \\ 2000x_1 + 6000x_2 \geq 48000 \end{cases}$$

$$x_1, \ x_2 \geq 0$$

Solution

Step 1: Graph the constraints.

$$3000x_1 + 1000x_2 = 24000 \qquad (1)$$

$$1000x_1 + 1000x_2 = 16000 \qquad (2)$$

$$2000x_1 + 6000x_2 = 48000 \qquad (3)$$

Each constraint from the above is defined by two points in (x_1, x_2)-plane as follows:

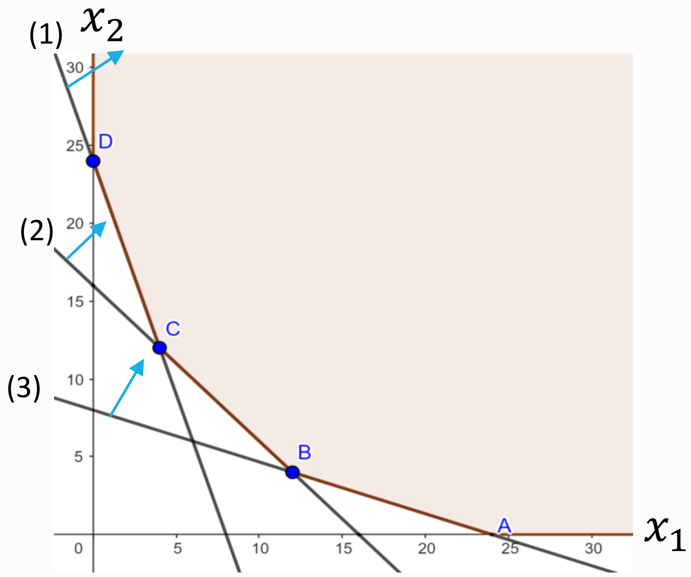
In equation (1) we have $(0, 24)$ & $(0, 8)$.

In equation (2) we have$(0, 16)$ &$(16, 0)$.

In equation (3) we have$(0, 8)$ &$(24, 0)$.

Step 2: Identify the feasible region:

The feasible region is unbounded. But we seek to minimize the function of Z. Thus, we have to focus on the boundary points only: A, B, C, D.



Step 3: The feasible region:

where, $A = (24, 0)$, $B = (12, 4)$, $C = (4, 12)$, $D = (0, 24)$.

Step 4: Using the corner-point Method:

$Z(A) = 600(24) + 400(0) = 14400$

$Z(B) = 600(12) + 400(4) = 8800$

$Z(C) = 600(4) + 400(12) = 7200$

$Z(D) = 600(0) + 400(24) = 9600$

$\Rightarrow \text{Min } Z = 7200 \text{ at the optimum solution} = \text{C}.$

It is better to check by iso-cost line. Assume the point (15,10) that belongs to the shades.

$$\Rightarrow Z(15,10) = 600(15) + 400(10) = 13000$$

Then we have the iso-cost line as $600x_1 + 400x_2 = 13000$, which can be plotted using two points$(0, 32.5)$ and $(21.6, 0)$

- Move the iso-cost line parallel to the decreasing direction (as we need to minimize the cost). Assume points under and above Z to determine Z direction.

- Identify the optimum solution as the point on where the lowest possible iso-cost is touched.

Example 4

A factory manufactures two articles A and B. To manufacture the article A, a certain machine has to be worked for 1.5 hours and in addition a craftsman has to work 2 hours. To manufacture the article B, the machine has to be worked for 2.5 hours and in addition the craftsman has to work 1.5 hours. In a week the factory can avoid of 80 hours of machine time and 70 hours of craftsman’s time. The profit on each article A is 5 SR and that on each article B is 4 SR. if all articles produced can be sold away. Find how many of each kind should be produced to earn the maximum profit per week.

Solution

	A	B	Hours
Machine	1.5	2.5	80
Craftsman	2	1.5	70
Profit	5	4	

Suppose that

$$\text{Decision Variables}\begin{cases} x_1 = \text{the number of units of article A produced} \\ x_2 = \text{the number of units of article B produced} \end{cases}$$

Then, the objective function is taken the following form:

$$\text{Maximize} \quad Z = 5x_1 + 4x_2 \;\Rightarrow Z \text{ is the profit}$$

The constraints:

The machine hours constraint $\Rightarrow 1.5x_1 + 2.5x_2 \leq 80$

The craftsman hours constraint $\Rightarrow 2x_1 + 1.5x_2 \leq 70$

The non-negative constraints are: $x_1 \geq 0, x_2 \geq 0$

Then the mathematical form of the application is:

$$\text{Maximize} \quad Z = 5x_1 + 4x_2$$

$$\text{Subject to} \quad \begin{cases} 1.5x_1 + 2.5x_2 \leq 80 \\ 2x_1 + 1.5x_2 \leq 70 \end{cases}$$

$$x_1,\ x_2 \geq 0$$

Solution

Step 1: Graph the constraints.

$1.5x_1 + 2.5x_2 = 80 \qquad (1)$

$2x_1 + 1.5x_2 = 70 \qquad (2)$

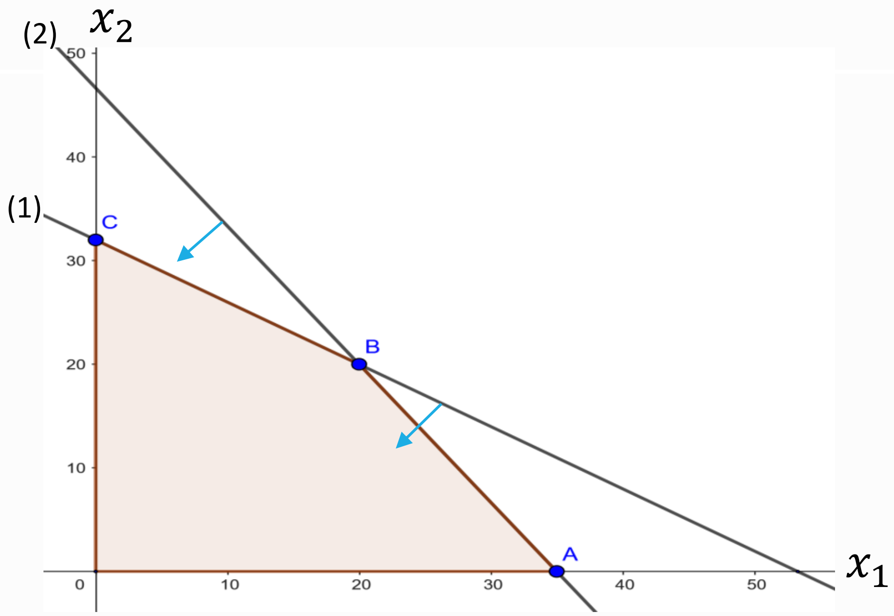
Each constraint from the above is defined by two points in (x_1, x_2)-plane as follows:

In equation (1) we have $(0, 32)$ & $(53.33, 0)$.

In equation (2) we have $(0, 46.67)$ & $(35, 0)$.

Step 2: Identify the feasible region:

The feasible region : O, A, B, C.



Step 3: The feasible region:

where, $\mathrm{O} = (0, 0)$, $\mathrm{A} = (35, 0)$, $\mathrm{B} = (20, 20)$, $\mathrm{C} = (0, 32)$.

Step 4: Using the corner-point Method:

$$Z(O) = 5(0) + 4(0) = 0$$

$$Z(A) = 5(35) + 4(0) = 175$$

$$Z(B) = 5(20) + 4(20) = 180$$

$$Z(C) = 5(0) + 4(32) = 128$$

$\Rightarrow \text{Max } Z = 180$ at the optimum solution $\Rightarrow \text{B} = (20,20)$. Hence, to maximize the profit Z the company should manufacture 20 units of article A and 20 units of article B per week.

Example 5

The manager of an oil refining must decide on the optimum mix of two possible blending processes of which the inputs and outputs per production ran are as follows:

	Inputs		Outputs	
Process	Crude A	Crude B	Gasoline X	Gasoline Y
1	5	3	5	8
2	4	5	4	4

The maximum amount available of crudes A and B is 200 units and 150 units respectively. Market requirements show that at least 100 units of gasoline X and 80 units of gasoline Y must be produced. The profit per production run from process 1 and process 2 are 300 SR and 400 SR respectively. Solve the LPP by graphical approach.

Suppose that

x_1= the number of production run from process 1.

x_2= the number of production run from process 2.

Then, the total profit $\Rightarrow 300x_1 + 400x_2$

$\Rightarrow$ Maximize $Z = 300x_1 + 400x_2$

The constraints:

Cude A $\Rightarrow 5x_1 + 4x_2 \leq 200$

Crude B $\Rightarrow 3x_1 + 5x_2 \leq 150$

Gasoline X $\Rightarrow 5x_1 + 4x_2 \geq 100$

Gasoline Y $\Rightarrow 8x_1 + 4x_2 \geq 80$

Non-negative constraints : $x_1 \geq 0, x_2 \geq 0$

Then the mathematical formulation is:

$$\text{Maximize} \quad Z = 300x_1 + 400x_2$$

$$\text{Subject to} \quad \begin{cases} 5x_1 + 4x_2 \leq 200 \\ 3x_1 + 5x_2 \leq 150 \\ 5x_1 + 4x_2 \geq 100 \\ 8x_1 + 4x_2 \geq 80 \end{cases}$$

$$x_1, \ x_2 \geq 0$$

Solution

Step 1: Graph the constraints.

$5x_1 + 4x_2 = 200 \qquad (1)$

$3x_1 + 5x_2 = 150 \qquad (2)$

$5x_1 + 4x_2 = 100 \qquad (3)$

$8x_1 + 4x_2 = 80 \qquad (4)$

In equation (1) we have $(0, 50)$ & $(40, 0)$.

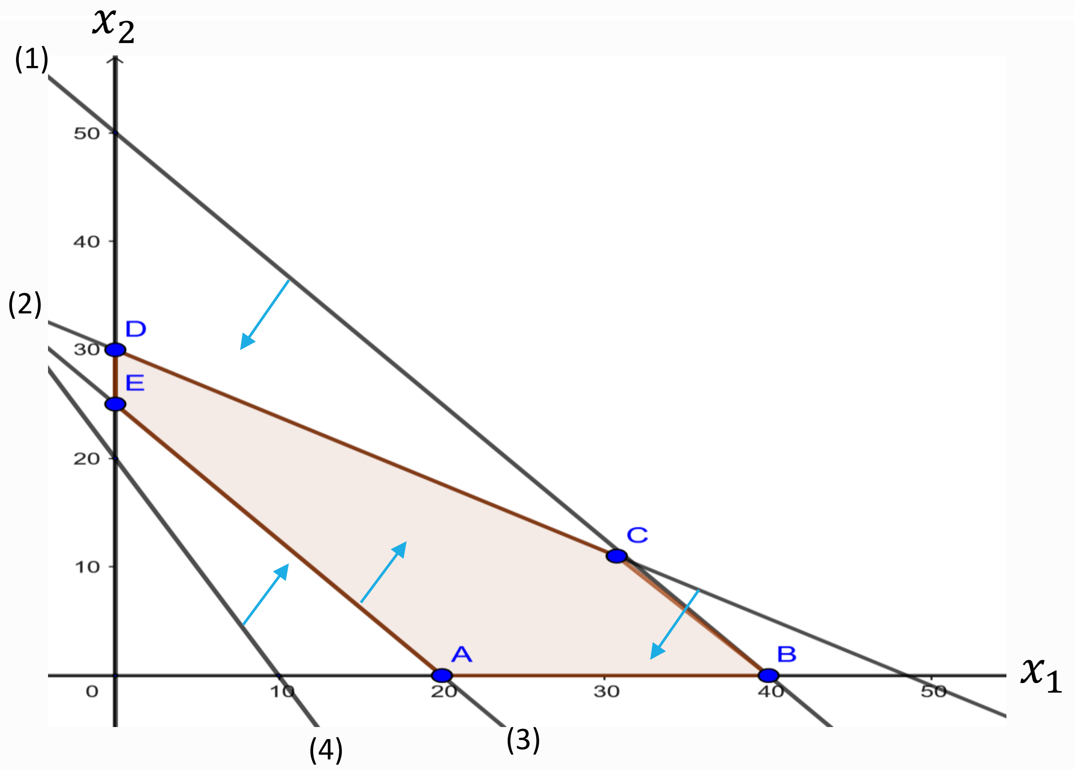
In equation (2) we have $(0, 30)$ &$(50\ 0)$.

In equation (3) we have $(0, 25)$ & $(20, 0)$.

In equation (4) we have $(0, 20)$ &$(10, 0)$.

Step 2: Identify the feasible region:

The feasible region is A, B, C, D, E.



Step 3: The feasible region:

where, $\mathrm{A} = (20, 0)$, $\mathrm{B} = (40, 0)$, $\mathrm{C} = \left(\frac{400}{13}, \frac{150}{13}\right)$, $\mathrm{D} = (0, 30)$, $\mathrm{E} = (0, 25)$.

Step 4: Using the corner-point Method:

$$Z(A) = 300(20) + 400(0) = 6000$$

$$Z(B) = 300(40) + 400(0) = 12000$$

$$Z(C) = 300\left(\frac{400}{13}\right) + 400\left(\frac{150}{13}\right) = \frac{180000}{13} = 13846.15$$

$$Z(D) = 300(0) + 400(30) = 12000$$

$$Z(E) = 300(0) + 400(25) = 10000$$

Since the maximum value of $Z = \frac{180000}{13} = 13846.15$ occurs at the optimum solution C. This means that the manger of the oil refinery should produce $x_1 = \frac{400}{13}$ units under process 1 and $x_2 = \frac{150}{13}$ units under process 2 to achieve the maximum profit of $\frac{180000}{13}$.

Example 6

Graphically solve the following LPP given by:

$$\text{Maximize} \quad Z = 2x_1 - x_2$$

$$\text{Subject to} \quad \begin{cases} x_1 - x_2 \leq 1 \\ 2x_1 + x_2 \geq 6 \end{cases}$$

$$x_1,\ x_2 \geq 0$$

Solution

Step 1: Graph the constraints.

$x_1 - x_2 = 1 \qquad (1)$

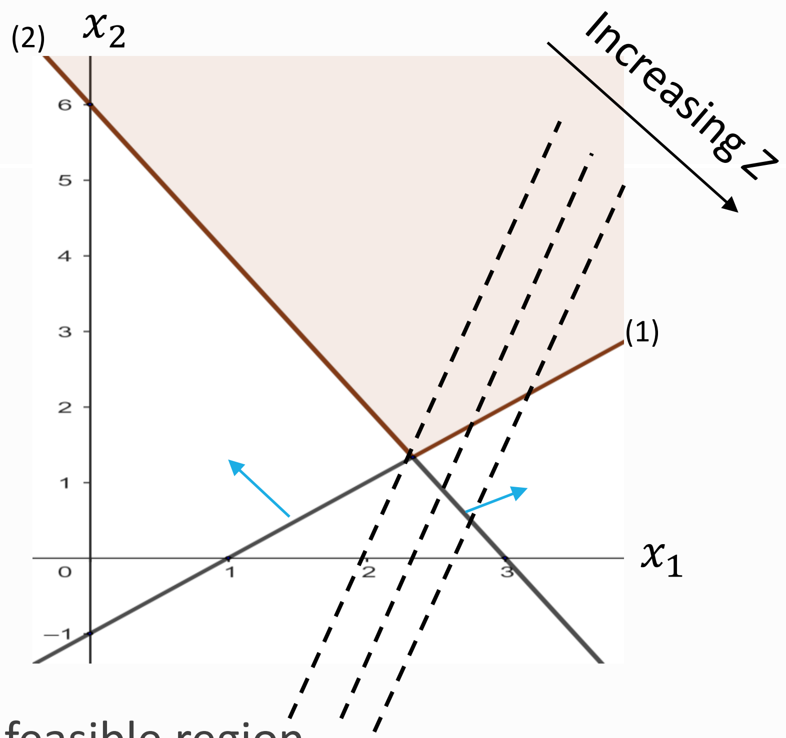
$2x_1 + x_2 = 6 \qquad (2)$

In equation (1) we have $(0, -1)$ & $(1, 0)$.

In equation (2) we have$(0, 6)$ &$(3\ 0)$.

Step 2: Identify the feasible region:

The feasible region is unbounded and convex too.



Identify the optimal solution:

Using the iso-profit line, suppose the point $(3, 3)$ in the feasible region

$\Rightarrow\ Z(3,3) = 2(3) - (3) = 3$

Then the iso-profit line is $2x_1 - x_2 = 3$.

It is obvious that as x_1 increase the value of Z increases. But the feasible region is unbonded and so in this case we have only unbounded solution.

Example 7

Solve the following LPP using graphical approach:

$$\text{Maximize} \quad Z = 3x_1 + 2x_2$$

$$\text{Subject to} \quad \begin{cases} \frac{1}{40}x_1 + \frac{1}{60}x_2 \leq 1 \\ \frac{1}{50}x_1 + \frac{1}{50}x_2 \leq 1 \end{cases}$$

$$x_1,\ x_2 \geq 0$$

Solution

Step 1: Graph the constraints.

$$\frac{1}{40}x_1 + \frac{1}{60}x_2 = 1 \qquad (1)$$

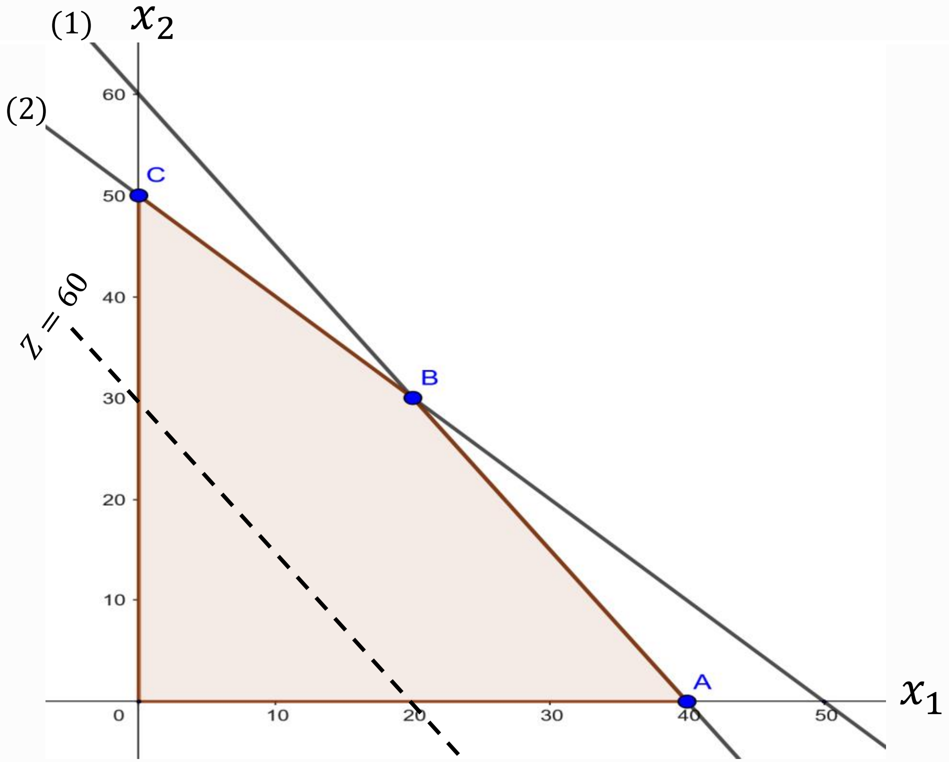
$$\frac{1}{50}x_1 + \frac{1}{50}x_2 = 1 \qquad (2)$$

In equation (1) we have $(0,60)$ & $(40,0)$.

In equation (2) we have$(0,50)$ &$(50\ 0)$.

Step 2: Identify the feasible region:

The feasible region is O, A, B, C.



Step 3: The feasible region:

where, $\text{O} = (0, 0)$, $\text{A} = (40, 0)$, $\text{B} = (20, 30)$, $\text{C} = (0, 50)$.

Step 4: Using the corner-point Method:

$Z(O) = 3(0) + 2(0) = 0$

$Z(A) = 3(40) + 2(0) = 120$

$Z(B) = 3(20) + 2(30) = 120$

$Z(C) = 3(0) + 2(50) = 100$

It is obvious that Max $Z = 120$ and it occurred at A, and B. Using the iso-profit line at (20,0)

$$\Rightarrow \; Z(20{,}0) = 60 \Rightarrow 3x_1 + 2x_2 = 60$$

If we make lines parallel to the iso-profit line, we see that one of these lines will pass through the line between A and B. This means that any points on the line $\overline{AB}$ will be an optimum solution and hence we have infinite number of optimal solutions.

Example 8

Solve the following LPP:

$$\text{Maximize} \quad Z = 3x_1 + 2x_2$$

$$\text{Subject to} \quad \begin{cases} \frac{1}{40}x_1 + \frac{1}{60}x_2 \leq 1 \\ \frac{1}{50}x_1 + \frac{1}{50}x_2 \leq 1 \\ x_1 \geq 30 \\ x_2 \geq 20 \end{cases}$$

$$x_1,\ x_2 \geq 0$$

Solution

Step 1: Graph the constraints.

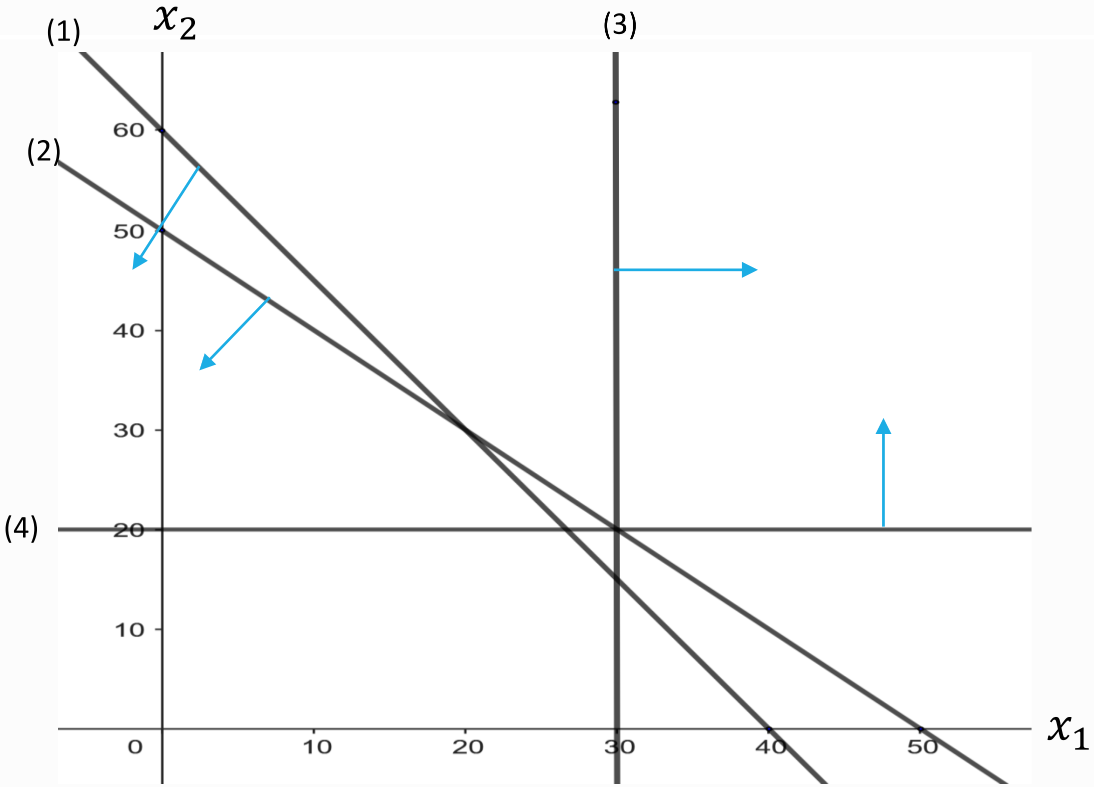
$$\frac{1}{40}x_1 + \frac{1}{60}x_2 = 1 \qquad (1)$$

$$\frac{1}{50}x_1 + \frac{1}{50}x_2 = 1 \qquad (2)$$

In equation (1) we have $(0, 60)$ & $(40, 0)$.

In equation (2) we have$(0, 50)$ &$(50\ 0)$.

From the figure we see that there is no feasible solution and hence we have an empty feasible region.



From the above examples and previous lectures, for a LPP of two variables it may have:

1) Unique optimal solution
2) No solution
3) Alternative or multiple optimal solutions.
4) Unbounded solution.

Exercises

1) Minimize $Z = 3x_1 + 2x_2$

$$\text{Subject to} \quad \begin{cases} x_1 + x_2 = 5 \\ x_1 \leq 4 \\ x_2 \geq 2 \end{cases}$$

$$x_1,\ x_2 \geq 0$$

2) An electric company produce two productsP_1 and P_2. Products are produced and sold on a weekly basis. The weekly production cannot exceed 25 for product $P_1$and 35 for product P_2 because of limited available facilities. The company employs total of 60 workers. Product P_1 requires 2 man-weeks of labor, which P_2 requires one man-week of labor. Profit margin on P_1 is 60 SR and on P_2 is 40 SR. Formulate it as a LPP and solve for maximizing the profit.

Exercises

3) Using graphic Method, find the maximum value of $Z = 7x_1 + 10x_2$

$$\text{Subject to} \quad \begin{cases} x_1 + x_2 \leq 30000 \\ x_1 \geq 6000 \\ x_2 \leq 12000 \\ x_1 \geq x_2 \end{cases}$$

$$x_1,\ x_2 \geq 0$$

4) Minimize $Z = 200x_1 + 400x_2$

$$\text{Subject to} \quad \begin{cases} x_1 + x_2 \geq 200 \\ \frac{1}{4}x_1 + \frac{3}{4}x_2 \geq 100 \\ \frac{1}{10}x_1 + \frac{1}{5}x_2 \leq 35 \end{cases}$$

$$x_1,\ x_2 \geq 0$$

Exercises

5) Minimize $Z = -x_1 + 2x_2$

$$\text{Subject to} \quad \begin{cases} 5x_1 - 2x_2 \leq 3 \\ x_1 + x_2 \geq 1 \\ -3x_1 + x_2 \leq 3 \\ -3x_1 - 3x_2 \leq 2 \end{cases}$$

$$x_1,\ x_2 \geq 0$$