OPER 122
Introduction to Operations Research

Introduction

Overview

- **Operations Research** (OR) is a science which deals with problem, formulation, solutions and finally appropriate decision making.
- It is a scientific approach to determine the optimum (best) solution to a decision problem under the restriction of limited resources, using the mathematical techniques to model, analyze, and solve the problem.
- OR is both an art and a science—the art of describing and modeling the problem and the science of solving the model using (precise) mathematical algorithms.
- Operations Research is a very important area of study, which tracks its roots to business applications. It combines the three broad disciplines of Mathematics, Computer Science, and Business Applications.

The Origins of Operations research

- World War II: British military leaders asked scientists and engineers to analyze several military problems
  - Deployment of radar
  - Management of convoy, bombing, antisubmarine, and mining operations.
- The result was called *Military Operations Research*, later Operations Research
- Following the end of the war, the ideas advanced in military operations were adapted to improve efficiency and productivity in the civilian sectors.

The Origins of Operations research

- Operations Research has been applied extensively in such diverse areas as manufacturing, transportation, construction, telecommunications, financial planning, health care, the military, and public services, ..., etc.
- Today, OR is a dominant ***decision-making tool, that seeks the optimum state in all conditions and thus provides optimum solution to organizational problems.***

Factors influencing growth of OR

- Improved techniques
  - Simplex method.
- Computer revolution
  - Fast solution of complex computational problems.

The Nature of Operations Research

- Research on how to conduct and coordinate an organization's activities
- OR
  - Is also called Management Science, Systems Engineering, Industrial Engineering, Operations Management.
    - Utilizes the scientific method
  - Must provide positive, understandable conclusions to the decision maker
- Process
  - Carefully observe and formulate the problem
  - Gather data
  - Construct a mathematical model
  - Test whether the model represents the actual situation
  - Modify the model as appropriate and validate again

The Nature of Operations Research

- Broad viewpoint
  - Considers what is best for organization as a whole
  - Attempts to identify best possible course of action
- Team approach
  - Consists of individuals with diverse backgrounds and skills
  - Necessary for considering ramifications of problem throughout organization

Analytics Together with Operations Research

- Categories of analytics applications:
  - Descriptive analytics
    - Identify interesting patterns to describe what is currently happening
  - Predictive analytics
    - Using data to predict what will happen in the future
  - Prescriptive analytics
    - Using data to prescribe what should be done in the future
- OR focuses on predictive and prescriptive analytics.

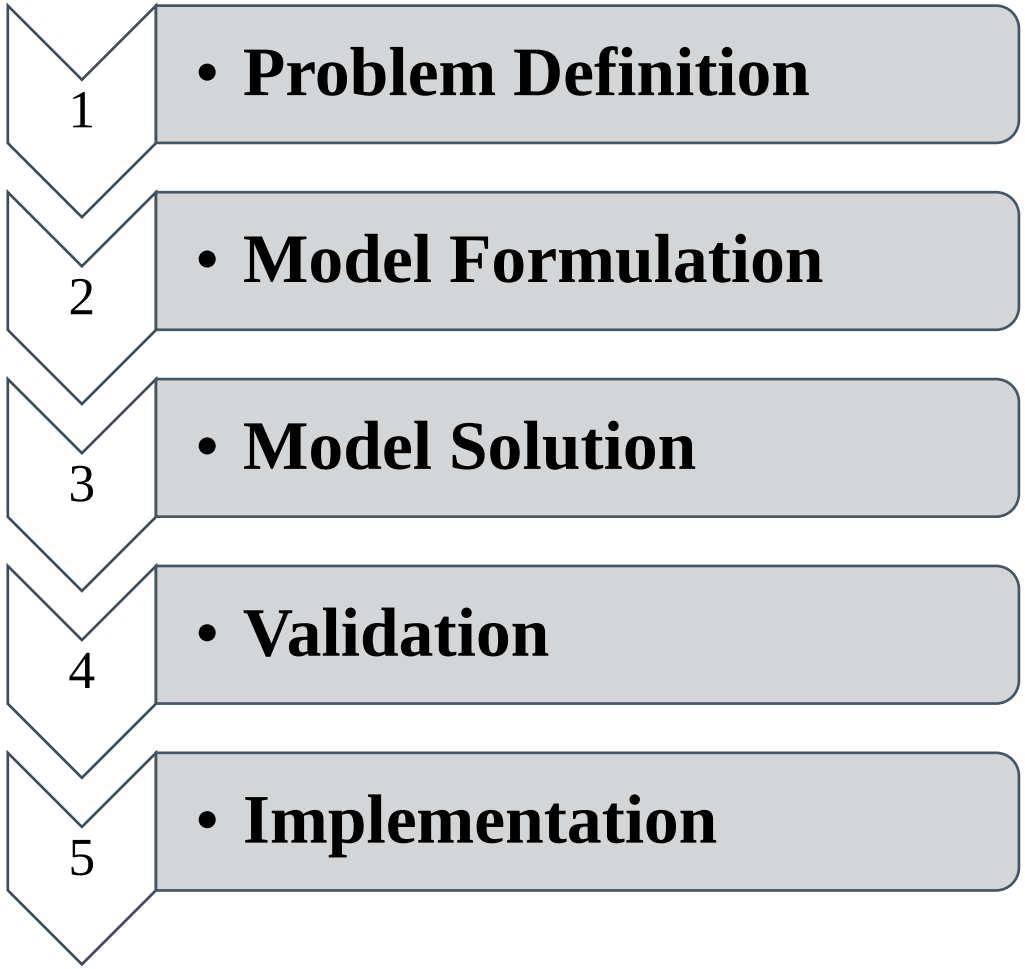
Operations Research Applications

- OR is regularly applied in areas such as:
  - supply chain management
  - marketing and revenue management systems
  - manufacturing plants
  - Scheduling (e.g. in what order should I schedule deliveries?)
  - Facility Location (e.g. where should I locate my factory?)
  - financial engineering
  - telecommunication networks
  - healthcare management (e.g. , how do I assign shifts to hospital staff?)
  - transportation networks
  - energy and the environment
  - service systems
  - web commerce
  - military defense

Operations Research in action:

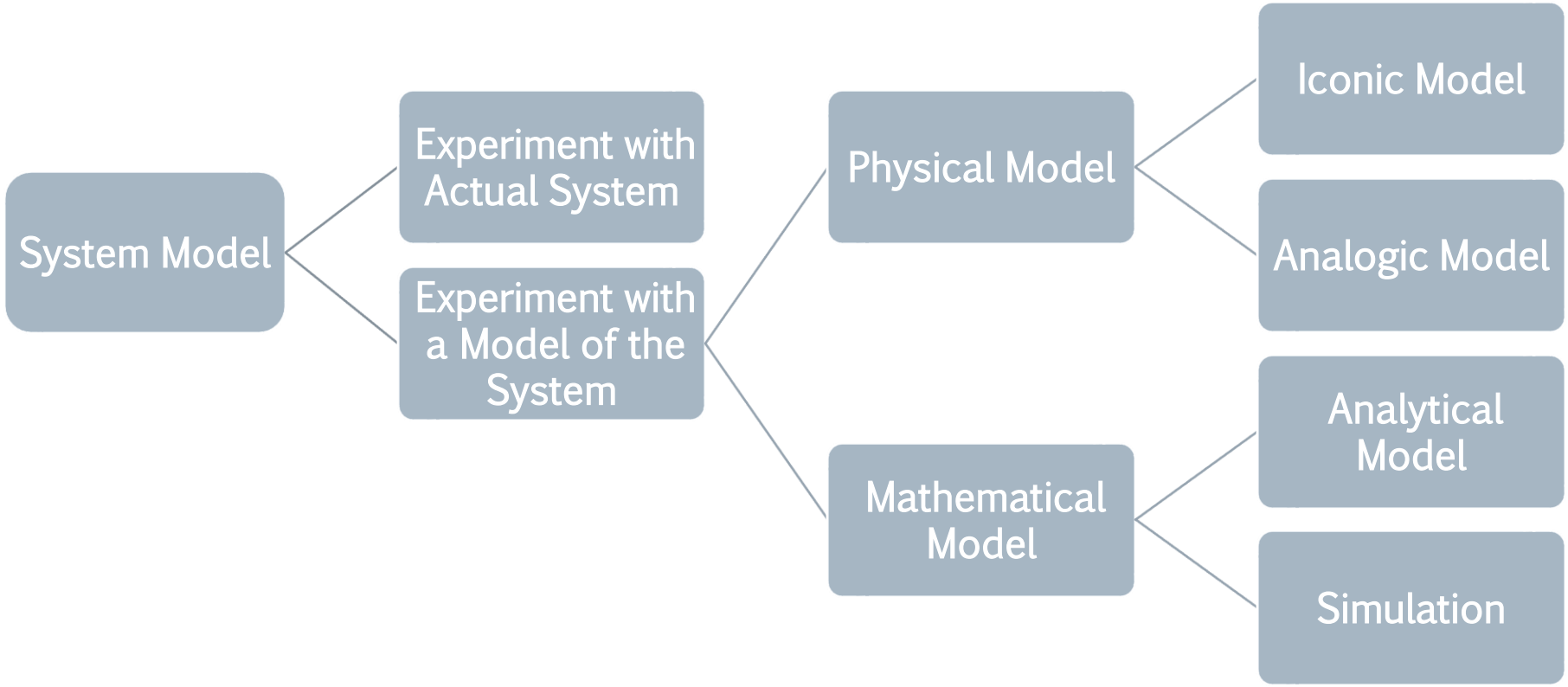
1. You have to drop off packages at 100 different locations in a city. How can you do it in the shortest amount of time? (UPS does this calculation thousands of times per day)
2. An airport hub shuts down due to a storm — all flights in and out are canceled. Rearrange the flights of all planes in your airline to get as many passengers as possible to their destinations. You have 20 minutes. (Continental Airlines built such a system)

Phases of Operations Research



- **Problem Definition** - The problem must be clearly and consistently defined, showing its boundaries and interactions with the objectives of the organization - Identification of a problem that exists (or may occur soon) in a system or organization.
- **Model Formulation** - Development of the functional mathematical relationships that describe the decision variables, objective function and constraints of the problem.
- **Model Solution** - The model is solved using Operations Research techniques.
- **Validation -** does the solution make sense? Are the results intuitively acceptable? On the formal side, a common method for validating a model is to compare its output with historical output data.
- **Model Implementation** - Actual use of the model or its solution.

Types of Models



Mathematical Modelling

- Mathematical Modelling
  - A process that translates observed phenomena into mathematical expressions.
- Operations Research is Applied When –
  - Designing and implementing new operations
  - Evaluating ongoing Operations and Procedures.
  - Determining and recommending corrective actions.
- How to Start and How to Proceed
  - Identify the problem.
  - Observe the problem from various points of view.
  - Keep things simple.
  - Identify constraints.
  - Work with management, get feedback.

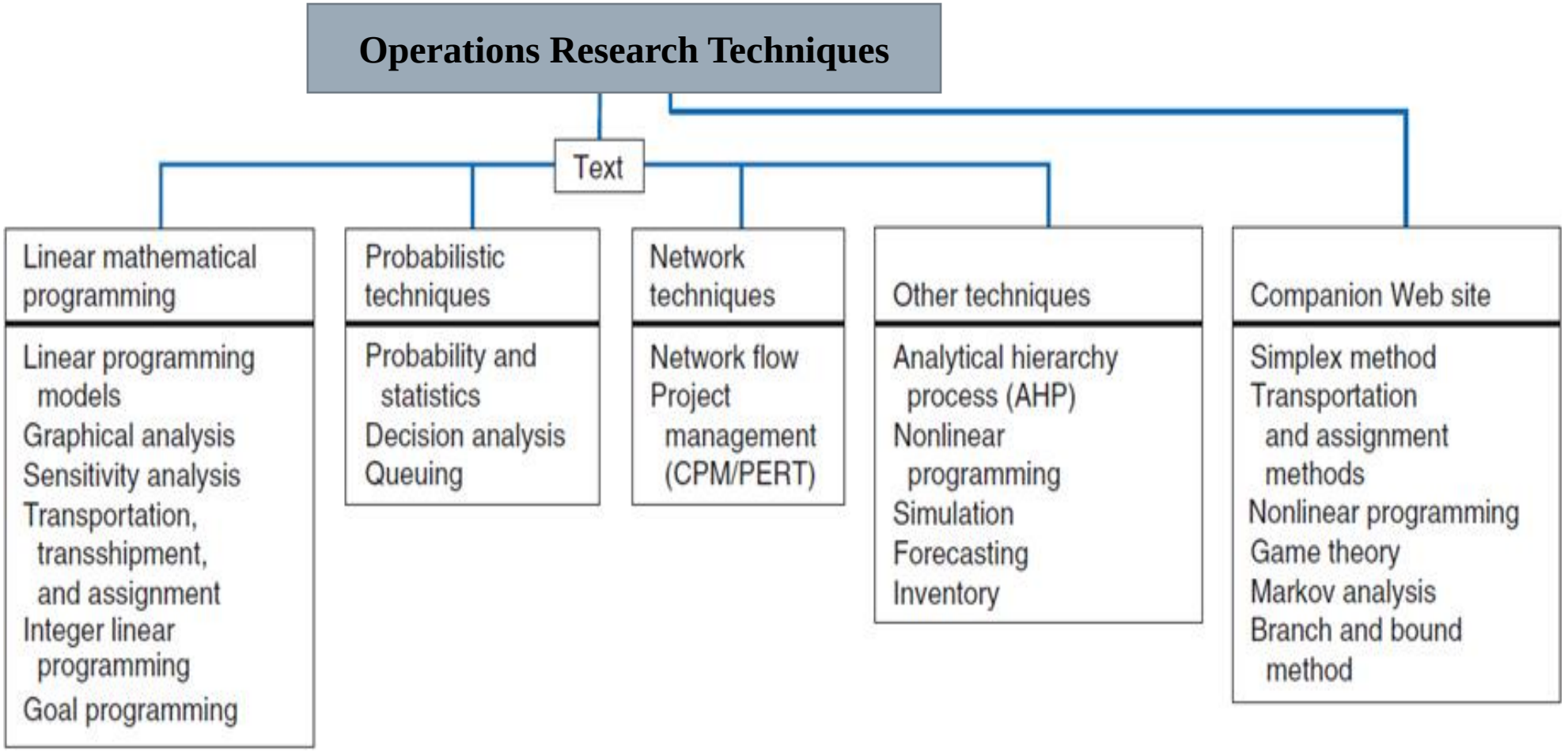
OR model Formulation

Model Formulation:

Maximize or minimize **Objective Function**
subject to
Constraints

- **feasible solution:** if it satisfies all the constraints.
- An **optimal solution:** if, in addition to being feasible, it yields the best (maximum or minimum) value of the objective function.

Operations Research Techniques



Model Building

Basic Components of The Model

1. Decision Variables

It is what does the model seeks to determine. It is one of the specific decisions made by a decision maker.

2. Objective Function

It is the goal desired to be achieved by the system. A common objective is to maximize profit or minimize cost. It is expressed as a mathematical function of the system decision variables.

3. Constraints

These are the limitations imposed on the variables to satisfy the restriction of the modeled system. They must be expressed as mathematical functions of the system decision variables.

OR model Formulation

Model Formulation:

Maximize or minimize **Objective Function**

subject to

Constraints

Linear Programming

- Linear programming
  - Programming means planning
  - Model contains linear mathematical functions
- An application of linear programming
  - Allocating limited resources among competing activities in the best possible way
  - Applies to wide variety of situations

- Not all problems can be formulated to fit a linear programming model
  Alternatives: integer programming, nonlinear programming models, .. etc.

Common Rules

Objective Function	Constraints	Decision Variables
Maximum	$\leq$ type inequality	Non-negative
Minimum	$\geq$ type inequality	Unrestricted in sign
	= (equality)	

Types of Solution

Feasible solution

- Solution for which all constraints are satisfied
- Might not exist for a given problem

Infeasible solution

- Solution for which at least one constraint is violated

Optimal solution

- if, in addition to being feasible, it yields the best (maximum or minimum) value of the objective function
- Might not exist for a given problem

Assumptions of Linear Programming

Proportionality

- ✓ The contribution of any decision variable to the objective function is proportional to its value.
- ✓ For example in the diet problem, the contribution to the cost of the diet from one pound of apples is $0.75, from two pounds of apples its $1.50 and from four pound the contribution is $3.00. For four hundred pounds, the contribution would be $300.00.
- ✓ In many situations, you might get a volume discount such that the price per pound goes down if you purchase more apples. These discounts are often nonlinear, which that a linear programming model is either inappropriate or is really an approximation of the real world problem.

Additivity

- ✓ Every function in a linear programming model is the sum of the individual contributions of the activities
- ✓ This requirement rules out the possibility that interaction or multiplicative terms appear in the objective function or the constraints.

Assumptions of Linear Programming

Divisibility

- ✓ The values of decision variables can be fractions. Sometimes these values only make sense if they are integers; then we need an extension of linear programming called integer programming.

Certainty

- ✓ Value assigned to each parameter of a linear programming model is assumed to be a known constant.

Model Building Steps

Step 1
- Study the given situation
- Find the key decision to be made
- Identify the decision variables of the problem

Step 2
Formulate the objective function to be optimized

Step 3
Formulate the constraints of the problem

Step 4
- Add non-negativity restrictions or constraints

The objective function , the set of constraints and the non-negativity restrictions together form an **LP model.**