

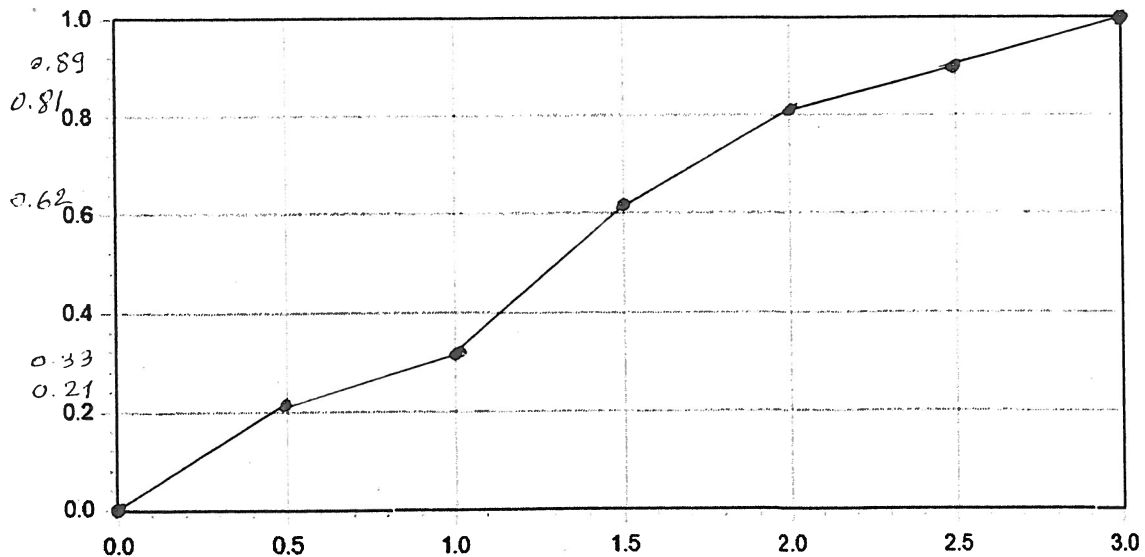


Problem 1 : Suppose that 100 machine repair times have been collected. The data are summarized in Table below in terms of the number of observations in various intervals.

1. Complete the following table (أكمل الجدول التالي)

i	Interval (Hours)	Frequency	Relative Frequency	Cumulative Frequency
1	$0.0 \leq x \leq 0.5$	21	0.21	0.21
2	$0.5 \leq x \leq 1.0$	12	0.12	0.33
3	$1.0 \leq x \leq 1.5$	29	0.29	0.62
4	$1.5 \leq x \leq 2.0$	19	0.19	0.81
5	$2.0 \leq x \leq 2.5$	8	0.08	0.89
6	$2.5 \leq x \leq 3.0$	11	0.11	1

2. Draw the graph of the empirical CDF $\hat{F}(x)$ ارسم دالة التوزيع التراكمي التجريبي



3. Generate the values of X using the following sequence of (0,1) random numbers.

u	0.3708	0.8635	0.9038	0.4503	0.25
$x = \bar{F}^{-1}(u)$	1.07	2.33	2.56	1.2	0.66

Problem 2: Consider the following probability density function :

$$f(x) = \begin{cases} \frac{2}{x^3}, & x \geq 1 \\ 0, & \text{otherwise} \end{cases}$$

Determine the CDF $F(x)$ of this distribution.

$$F(x) = \begin{cases} 0, & x < 1 \\ 1 - \frac{1}{x^2}, & x \geq 1 \end{cases}$$

Problem 3: Consider the following cumulative density function (CDF):

$$G(x) = \begin{cases} 0, & x \leq -3 \\ \frac{1}{2} + \frac{x}{6}, & -3 \leq x \leq 0 \\ \frac{1}{2} + \frac{x^2}{32}, & 0 < x \leq 4 \\ 1, & x > 4 \end{cases}$$

1. Determine the inverse transform $x = G^{-1}(u)$ of $G(x)$ when

- a) $-3 \leq x \leq 0$: أوجد الدالة العكسية على الفترة :
 b) $0 < x \leq 4$: أوجد الدالة العكسية على الفترة :

$$G^{-1}(x) = \begin{cases} 6u - 3, & 0 \leq u \leq \frac{1}{2} \\ 4\sqrt{2u - 1}, & \frac{1}{2} \leq u \leq 1 \end{cases}$$

2. Generate the values of X using the following sequence of (0,1) random numbers.

u	0.5108	0.3915	0.9804	0.2371	0.7311
$x = G^{-1}(u)$	0.587	-0.651	3.921	-1.577	2.72

Problem 4: Consider the random variable X following the Binomial distribution with parameters

$$p = 0.35 \text{ and } n = 4. \left(P(X = x) = \binom{n}{x} p^x (1-p)^{n-x} \right).$$

1. Fill the table (إملا الجدول)

x	0	1	2	3	4
$P(X = x)$	0.1785	0.3844	0.3105	0.1114	0.015
$F(x)$	0.1785	0.5629	0.8735	0.9849	1.00

2. Find the inverse transform function $F^{-1}(u)$ to generate observations for the random variable X .

$$F^{-1}(u) = \begin{cases} 0 & , 0 \leq u \leq 0.1785 \\ 1 & , 0.1785 < u \leq 0.563 \\ 2 & , 0.563 < u < 0.8735 \\ 3 & , 0.8735 < u < 0.985 \\ 4 & , 0.985 < u \leq 1 \end{cases}$$

3. Generate the values of X using the following sequence of (0,1) random numbers.

u	0.3237	0.6723	0.5649	0.9804	0.0356
$x = F^{-1}(u)$	1	2	2	3	0