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Academic Year 1442-1443 Semester

Exam Information معلومات الامتحان

Course name	Modeling and Simulation	النمذجة والمحاكاة	اسم المقرر
Course Code	OPER 441	441 بحث	رمز المقرر
Exam Date	2020-11-11	1442-03-25	تاريخ الامتحان
Exam Time	12:00 PM		وقت الامتحان
Exam Duration	2 hours	ساعتان	مدة الامتحان
Classroom No.			رقم قاعة الاختبار
Instructor Name			اسم استاذ المقرر

Student Information معلومات الطالب

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Section No.		رقم الشعبة
Serial Number	7	الرقم التسلسلي

General Instructions:

- Your Exam consists of PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.

- عدد صفحات الامتحان صفحة. (باستثناء هذه الورقة)
- يجب إبقاء الهواتف والساعات الذكية خارج قاعة الامتحان.

هذا الجزء خاص باستاذ المادة

This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1	Understanding the processes and steps for building a simulation model			
2	Implement an inverse cumulative distribution function based random variate generation algorithm			
3	Explain and implement the convolution algorithm for random variate generation			
4	Explain and implement the acceptance rejection algorithm for random variate generation			
5	Compute statistical quantities from simulation output			
6	Generate random numbers from any given distribution discrete or continuous			
7	Building simulation models from basic applications			
8				

EXAM COVER PAGE

Question #1: Simulations Modeling

Consider a bank with multiple number of servers. The manager is receiving many complains about the long waiting time in line. He decided to hire a simulation analyst to model this system and give his suggestions. The following steps and processes are done to model the system. To model the system the Analyst executed the following processes:

- ~~1.~~ checking if the system and the problem is understood correctly.
- ~~2.~~ Building the functions and logic between the behavior of customers in line.
- ~~3.~~ Gathering a sample of service time of each customer.
- ~~4.~~ Writing the functions and relations in Arena
- ~~5.~~ Writing the mathematical relations logic on Excel.
- ~~6.~~ Checking if the Arena program computes and give numbers when it runs
- ~~7.~~ Discovering that the results give departure value of 3rd arrival less than the departure time of the 2nd arrival.
- ~~8.~~ Determining the parameters needed for the model.
- ~~9.~~ Reviewing the data in the e-system of the bank
- ~~10.~~ Collecting a sample of the arrival time of each arrival.
- ~~11.~~ The output results of the simulation model match exactly the behavior of the customers in the system.
- ~~12.~~ Determining if the manager of the system wants to increase the efficiency and quality of service or to reduce cost.
- ~~13.~~ Determining the time that will be required, personnel that will be used, hardware and software requirements.
- ~~14.~~ Based on the analysis of runs that have been completed, the simulation analyst determines if additional runs are needed and if any additional scenarios need to be simulated.
- ~~15.~~ The result of all the analysis written in a report that is clear to help enable the management to review the final formulation and the alternatives.
- ~~16.~~ The simulation analyst acts as a reporter to present the best solution and how it will affect the performance of the system.
- ~~17.~~ meeting with the manager, the servers and the customers to fully understand the system and its details
- ~~18.~~ Deciding the arrival pattern of the customers to the bank and choosing the distribution of the service time.
- ~~19.~~ Experimenting with simulation model by trying different scenarios in the simulation model and choosing the best one.

20. Doing a long production runs for the best alternative and do data analysis to estimate final measures of performance for the scenarios that are being simulated.

Put the number of the processes above in the correct stage in simulation modeling methodology.
The number of the process should appear one time only.

	Stages of Simulation Model	The Process/Procedure Number
1.	Problem formulation	✓ 7, 9, 1
2.	Setting objectives and overall plan	✓ 13, 12
3.	Model conceptualization	✓ 2, 18, 8
4.	Data collection	✓ 3, 10
5.	Model translation	✓ 4, 5
6.	Verifying the Simulation Code	✓ 6
7.	Validating the Simulation model	✓ 11, 7
8.	Experimental design	✓ 19
9.	Production runs and analysis	✓ 20
10.	Performing More runs	✓ 14
11.	Documentation and reporting	✓ 15
12.	Implementation	✓ 16

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Excellent
in this question!

Question #2:

Consider the continuous random Y with the following pdf:

$$f_Y(y) = \begin{cases} 0, & y < 0 \\ 0.2y, & 0 \leq y \leq 1 \\ 0.1 + 0.1y, & 1 < y \leq 2 \\ 0.25 + 0.025y, & 2 < y \leq 4 \\ 0, & y > 4. \end{cases}$$

- Write the cumulative distribution function of $f_Y(y)$ and compute the expected value of Y?
- Write the Inverse transform for $f_Y(y)$?
- Write the algorithm for generating 10 random numbers from $f_Y(y)$.
- Let Y be the time (in hourse) for surgery in an Operations Room (OR) in K.A.N Hospital. The hospital has one Operations Room. Patients are transferred to the OR according to a Poisson Process with average time between arrivals equals to 5 hours. The operations Room work 24 hours per day. Define the simulation model for the OR and apply it for 5 patients. Use the following U[0,1] numbers as needed. Starting simulations time is zero.

Move by

rows →	0.744	0.443	0.820	0.166
	0.256	0.542	0.844	0.936
	0.744	0.444	0.017	0.967

a)

$$F(y_1) = \int_0^y 0.2y dy = \frac{0.2}{2} y^2 \Big|_0^y = 0.1y^2$$

$$F(y_2) = F_1(1) + \int_1^y (0.1 + 0.1y) dy = 0.1(1)^2 + [0.1y + 0.05y^2]_1^y$$

$$= 0.1 + [0.1y + 0.05y^2 - 0.1 - 0.05]$$

$$F(y_3) = F_2(2) + \int_2^y (0.25 + 0.025y) dy = 0.1y + 0.05y^2 - 0.05$$

$$= [0.1(2) + 0.05(2)^2 - 0.05] + [0.25y + 0.0125y^2]_2^y$$

$$= 0.35 + [0.25y + 0.0125y^2 - 0.25(2) - 0.0125(2)^2]$$

$$= 0.25y + 0.0125y^2 - 0.2$$

$$F(y) = \begin{cases} 0 & y < 0 \\ 0.1y^2 & 0 \leq y < 1 \\ 0.1y + 0.05y^2 - 0.05 & 1 \leq y < 2 \\ 0.25y + 0.0125y^2 - 0.2 & 2 \leq y < 4 \\ 1 & y \geq 4 \end{cases}$$

$$E(y) = \int_0^1 y(0.2y) dy + \int_1^2 y(0.1 + 0.1y) dy + \int_2^4 y(0.25 + 0.025y) dy$$

$$=$$

12/15

Question #3:

The period of time (in months) between rainfalls in Abha city is modeled using the following pdf:

$$f(x) = 1.06 e^{-\frac{x}{2}} ; 1 \leq x \leq 4$$

Where random variable X is time between rainfalls in months.

- Write the inverse transform for measuring the time between rainfalls.
- Simulate the next 4 rainfalls (in months) in Abha city.
- If we want to simulate the rainfall that is at least two months from now. Write the inverse transform and give two simulated values.
- Write the algorithm for applying Acceptance/Rejection method for the pdf $f(x)$.
- Using the following $U[0,1]$ as needed, generate **three** random numbers from $f(x)$ using the acceptance/rejection method.

Move by				
rows →	0.744	0.443	0.820	0.166
	0.256	0.542	0.844	0.936
	0.744	0.444	0.017	0.967

a) $\int_1^x 1.06 e^{-\frac{x}{2}} dx$

$u = \frac{-x}{2} \quad du = -\frac{1}{2}$

$-2(1.06) \int e^u du = -2(1.06) e^{-\frac{x}{2}}]_1^x$
 $= -2.12 e^{-\frac{x}{2}} + 1.285$

b)

U	0.744	0.443	0.820	0.166
R _i	2.73	1.24	3.0343	1.278
RF	2.73	4.57	7.612	2.29

a) $u = -2.12 e^{-\frac{x}{2}} + 1.285$
 $\frac{u - 1.285}{-2.12} = e^{-\frac{x}{2}}$

$-2 \ln \left(\frac{u - 1.285}{-2.12} \right) = x$

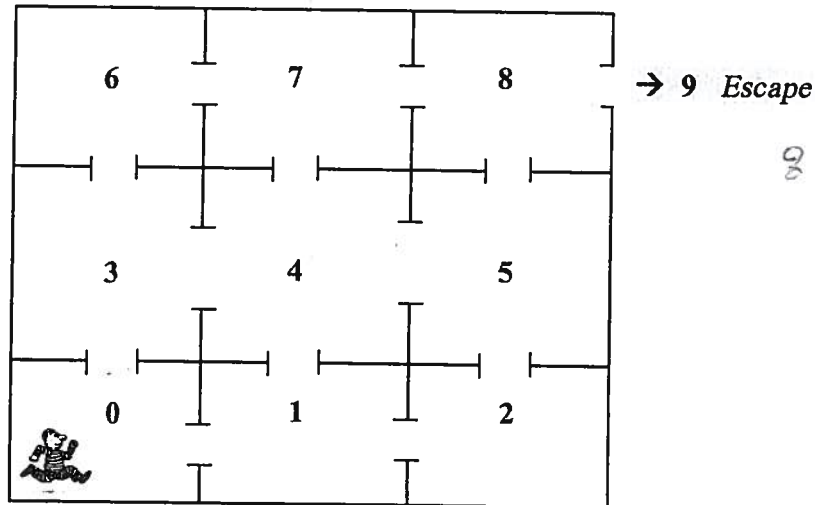
c) $= -2 \ln \left(\frac{u - 1.285}{-2.12} \right) + 2 = x$

$u = 0.256 \rightarrow x = 3.4457$

$u = 0.542 \rightarrow x = 4.97$

Question #4:

Consider an escaped prisoner who entered in a maze. (The maze contains 8 chambers.) If the prisoner enters any chamber he is equally likely to choose any door in the chamber (including the door he entered through). The prisoner has no time to waste, he has only 5 moves to escape out starting from chamber 0 before he gets caught up and put back into the prison.



- Write the algorithm for generating the moves of the prisoner.
- Using your answer in (a), Simulate the path of the prisoner for 5 attempts in the following table

	Move-1	Move-2	Move-3	Move-4	Move-5	Move-6
Attept#1	0.3328	0.7665	0.9796	0.1070	0.1514	0.6884
Chambers	1	4	7	4	1	3
Attept#2	0.8479	0.1445	0.0851	0.3078	0.5483	0.9579
Chambers	3	0	1	0	3	6
Attept#3	0.7371	0.4837	0.3936	0.1464	0.9872	0.1820
Chambers	3	4	3	0	3	0

- From the simulation results in part (b), what is the estimate for probability of escape.

$$\begin{aligned} \text{The prop of escape is} &= \frac{\# \text{ of escape}}{\# \text{ of attempt}} \\ &= \frac{0}{3} = 0 \end{aligned}$$

Question #5:

Airplanes land on a small airport according to Poisson process with rate 5 airplanes per day. Also, the airplanes depart from the same airport at rate 4 airplanes per day according to a Poisson process. Assume that the airport works 18 hours.

1. Give a random number for total number of air planes landed in the airport on one working day using the following $U[0,1]$ numbers *as needed*. (Answer on the back of the page)

n	1	2	3	4	5	6	7	8	9	10
$U_n(0,1)$	0.171	0.023	0.879	0.305	0.696	0.415	0.721	0.901	0.344	0.051

2. Give a random generation for the time of the last airplane departed from the airport on one day using the following $U[0,1]$ numbers *as needed*.

n	1	2	3	4	5	6	7	8	9	10
$U_n(0,1)$	0.815	0.636	0.563	0.923	0.295	0.605	0.971	0.023	0.879	0.305

3. According to Poisson process the percentage of departing airplanes from the airport is 44.5%. Make a discrete event simulation run of the airport for 12 hours. Write the simulation algorithm for this system and use it with the following $U[0,1]$ as needed. (Answer on the back of the page)

Event	$U[0,1]$	Do or A?	$U[0,1]$	D_i	$U[0,1]$	T_i	$U[0,1]$	A_t / D_t
1	0.248	D	0.817	7.64	0.132	—	0.214	$Dt_1 = 7.64$
2	0.968	A	0.465	—	0.668	3.96	0.482	$A_t = 3.96$
4	0.876	A	0.860	—	0.694	4.26	0.732	$A_{t2} = 8.22$
5	0.639	A	0.002	—	0.546	2.84	0.695	$11.06 = A_{t3}$
6	0.035	D	0.243	1.25	0.321	—	0.328	$Dt_2 = 8.99$
7	0.174	D	0.416	2.42	0.923	—	0.455	$Dt_3 = 11.31$
8	0.439	D	0.280	1.47	0.432	—	0.255	$Dt_4 = 12.78$
9	0.815	A	0.522	—	0.104	0.3953	0.377	$A_{t4} = 11.435$
10	0.199	D	0.479	2.934	0.963	—	0.420	$Dt_5 = 15.714$

1) process (1) \rightarrow Do or A \rightarrow

D $0 \leq u \leq 0.445$
A $0.445 < u \leq 1$

2) process (2) $\rightarrow t_i \sim \text{EXP}(k=5)$

$$A_t = \sum_{i=1}^t t_i$$

3) Process (3) $\rightarrow D_i \sim \text{EXP}(4)$

$$D_T = \sum_{i=1}^T D_i$$