ODE

1-

Given $xy' + y = 1,\ y(1) = 0$, the approximate value of $y(2)$ using Euler's method when $n = 2$ is:

(a) 0.3333 **(b)** 0.6667 **(c)** 0.1667 **(d)** None of These

$$x_0 = 1,\ y_0 = 0$$

$$h = \frac{2-1}{n} = \frac{1}{2} = 0.5$$

$(n-1)$

$$y_{i+1} = y_i + h\, f(x_i, y_i)\ ;\ i = 0, 1$$

$$xy' + y = 1 \Rightarrow xy' = 1 - y \Rightarrow y' = \frac{1-y}{x}$$

$$\Rightarrow f(x,y) = \frac{1-y}{x}$$

$$\boxed{i=0} \longrightarrow y_1 = y_0 + h f(x_0, y_0)$$

$$= 0 + 0.5 \cdot \frac{1-y_0}{x_0} = 0.5\left(\frac{1}{1}\right) = 0.5$$

$$x_1 = x_0 + h = 1 + 0.5 = 1.5$$

$$\boxed{i=1} \longrightarrow y_2 = y_1 + h f(x_1, y_1)$$

$$= 0.5 + 0.5 \cdot \frac{1-y_1}{x_1}$$

$$= 0.5 + 0.5\, \frac{1-0.5}{1.5} = 0.5 + \frac{(0.5)^2}{1.5}$$

$$= 0.6667$$

$$y(2) \approx y_2 = 0.6667$$

$$\Rightarrow (b) \checkmark$$

2-

The absolute error by using the Taylor's method of order 2 of $y(1)$ where $4y' - y = 0,\ y(0) = 1,\ n = 2$, and exact solution $y(x) = e^{x/4}$, is:

(a) 0.0080 **(b)** 0.0008 **(c)** 0.1512 **(d)** None of These

هذا الهامش

$x_0 = 0,\ y_0 = 1$

$h = \dfrac{1-0}{\underset{n \to}{2}} = 0.5$

$y_{i+1} = y_i + h f(x_i, y_i) + \dfrac{h^2}{2} f'(x_i, y_i)\ ;\ i = 0, 1 \quad (n-1)$

$4y' - y = 0 \Rightarrow 4y' = y \Rightarrow y' = \frac{1}{4} y \Rightarrow f(x,y) = \frac{1}{4} y$

$f'(x,y) = \frac{1}{4} y' = \frac{1}{4}\left(\frac{1}{4} y\right) = \frac{1}{16} y$

$\boxed{i=0} \rightarrow y_1 = y_0 + h f(x_0, y_0) + \frac{h^2}{2} f'(x_0, y_0)$

$= 1 + 0.5 \cdot \frac{1}{4} y_0 + \frac{(0.5)^2}{2}\left(\frac{1}{16} y_0\right)$

$= 1 + \frac{0.5}{4}(1) + \frac{(0.5)^2}{2}\left(\frac{1}{16} \cdot 1\right) = 1.1328$

$x_1 = x_0 + h = 0 + 0.5 = 0.5$

$\boxed{i=1} \rightarrow y_2 = y_1 + h f(x_1, y_1) + \frac{h^2}{2} f'(x_1, y_1)$

$= 1.1328 + 0.5 \cdot \frac{1}{4} y_1 + \frac{(0.5)^2}{2} \cdot \frac{1}{16} y_1$

$= 1.1328 + \frac{0.5}{4}(1.1328) + \frac{(0.5)^2}{32}(1.1328)$

$= 1.28325$

Exact Solution is $y = e^{\frac{1}{4}} = 1.284025$ (بالحاسبة)

Absolute error $= |$Exact solution $-$ approximation$|$

بالحاسبة $= |1.284025 - 1.28325|$

$= 0.000775$

$\Rightarrow$ (b) ✓