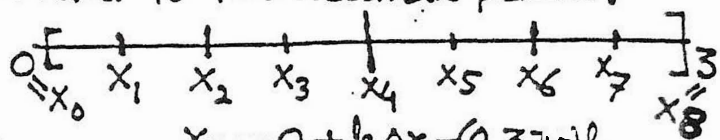
Problems on §5·7 In each of the following qns., use
(a) Trapezoidal Rule
(b) Simpson's Rule

to approximate the given integral for the given value of n: (use four decimal places ~~for approximation~~ of $f(x_k)$ and round off final answer to two decimal places.)

Q.6 $\boxed{\int_0^3 \frac{1}{1+x}\,dx\ ;\quad n=8}$



(a) $\Delta x = \frac{3-0}{8} = 0.375$ $\qquad f(x)=\frac{1}{x+1}$ $\qquad x_k = 0 + k\Delta x = (0.375)k$

TRAPEZOIDAL RULE

k	x_k	$f(x_k)=\frac{1}{x_k+1}$	m	$mf(x_k)$	The sum S of No.'s in previous column.
0	0.0000	1.0000	1	1.0000	
1	0.3750	0.7273	2	1.4546	
2	0.75	0.5714	2	1.1428	
3	1.125	0.4706	2	0.9412	
4	1.5	0.4000	2	0.8000	Sum $S=7.4514$
5	1.875	0.3478	2	0.6956	
6	2.25	0.3077	2	0.6154	
7	2.625	0.2759	2	0.5518	
8	3.000	0.2500	1	0.2500	

NOW $\int_0^3 \frac{dx}{1+x} \approx \left(\frac{3-0}{2(8)}\right)\left[1f(x_0)+2f(x_1)+2f(x_2)+\cdots+2f(x_7)+1f(x_8)\right] = \frac{3}{16}(S)$

$(0.1875)[7.4514] = 1.3971 \approx \boxed{1.40}$ ANS.

xxxxxx

(b) SIMPSON'S RULE

k	x_k	$f(x_k)=\frac{1}{x_k+1}$	m	$mf(x_k)$	The sum S of No.'s in prev. column
0	0.0000	1.0000	1	1.0000	
1	0.3750	0.7273	4	2.9092	
2	0.75	0.5714	2	1.1428	
3	1.125	0.4706	4	1.8824	
4	1.5	0.4000	2	0.8000	Sum $S=11.0946$
5	1.875	0.3478	4	1.3912	
6	2.25	0.3077	2	0.6154	
7	2.625	0.2759	4	1.1036	
8	3.000	0.2500	1	0.2500	

NOW $\int_0^3 \frac{dx}{1+x} \approx \left(\frac{3-0}{3(8)}\right)\cdot\left[1\cdot f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+2f(x_4)+\cdots+4f(x_7)+1\cdot f(x_8)\right]$

$\approx (0.125)\cdot[S] = (0.125)(11.0946)$

$\approx 1.3868 \approx 1.39$ ANS.

Q.8 $\boxed{\int_2^3 \sqrt{1+x^3}\,dx \;;\quad n=4}$

$\Delta x = \frac{3-2}{4} = 0.25$

$x_k = 2 + k(0.25)$

$2 = x_0$, $2.25 = x_1$, $2.5 = x_2$, $2.75 = x_3$, $3 = x_4$

$f(x) = \sqrt{1+x^3}$

(a) TRAPEZOIDAL Rule:

k	x_k	$f(x_k)=\sqrt{1+x_k^3}$	m	$mf(x_k)$	sum S
0	2.0	3.0000	1	3.0000	
1	2.25	3.5200	2	7.0400	
2	2.5	4.0774	2	8.1548	$S=32.8237$
3	2.75	4.6687	2	9.3374	
4	3.0	5.2915	1	5.2915	

$$\therefore \int_2^3 \sqrt{1+x^3}\,dx \approx \frac{3-2}{2(4)}\cdot\left[f(x_0)+2f(x_1)+2f(x_2)+2f(x_3)+f(x_4)\right]$$

$$\approx (0.125).(32.8237) = 4.1030 \approx 4.10 \text{ ANS.}$$

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(b) SIMPSON'S RULE:

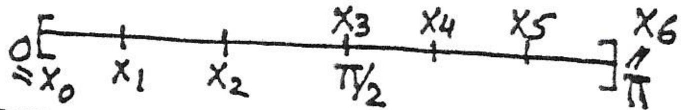
k	x_k	$f(x_k)=\sqrt{1+x_k^3}$	m	$mf(x_k)$	Sum S
0	2.0	3.0000	1	3.0000	
1	2.25	3.5200	4	14.0800	
2	2.5	4.0774	2	8.1548	$S=49.2011$
3	2.75	4.6687	4	18.6748	
4	3.0	5.2915	1	5.2915	

$$\therefore \int_2^3 \sqrt{1+x^3}\,dx \approx \frac{3-2}{3(4)}\cdot\left[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+f(x_4)\right]$$

$$\approx (0.0833)(49.2011) = 4.0985 \approx 4.10 \text{ ANS.}$$

Q.11 $\boxed{\int_0^{\pi} \sqrt{\sin x}\, dx \; ; \quad n=6}$ $\quad \Delta x = \frac{\pi - 0}{6} = \pi/6$

(a) TRAPEZOIDAL RULE $\quad x_k = 0 + k\Delta x = k\pi/6$



$f(x) = \sqrt{\sin x}$

k	x_k	$f(x_k) = \sqrt{\sin x_k}$	m	$m f(x_k)$	Sum S
0	0	$\sqrt{0} = 0.0000$	1	0.0000	
1	$\pi/6$	$\sqrt{1/2} = 0.7071$	2	1.4142	
2	$\pi/3$	$\sqrt{\sqrt{3}/2} = 0.9306$	2	1.8612	Sum
3	$\pi/2$	$\sqrt{1} = 1.0000$	2	2.0000	$S = 8.5508$
4	$2\pi/3$	$\sqrt{\sqrt{3}/2} = 0.9306$	2	1.8612	
5	$5\pi/6$	$\sqrt{1/2} = 0.7071$	2	1.4142	
6	π	$\sqrt{0} = 0.0000$	1	0.0000	

$$\therefore \int_0^{\pi} \sqrt{\sin x}\, dx \approx \left[\frac{\pi - 0}{2(6)}\right] \cdot \left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + 2f(x_4) + 2f(x_5) + f(x_6)\right] = \frac{\pi}{12}[S]$$

$$\approx \frac{\pi}{12} \cdot (8.5508)$$

$$\approx (0.2619)(8.5508) = 2.2395 \approx 2.24 \text{ Ans}$$

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(b) SIMPSON's RULE

k	x_k	$f(x_k)$	m	$m f(x_k)$	Sum S
0	0	$\sqrt{0} = 0.0000$	1	0.0000	
1	$\pi/6$	$\sqrt{1/2} = 0.7071$	4	2.8284	
2	$\pi/3$	$\sqrt{\frac{\sqrt{3}}{2}} = 0.9306$	2	1.8612	Sum
3	$\pi/2$	$\sqrt{1} = 1.0000$	4	4.0000	$S = 13.3792$
4	$2\pi/3$	$\sqrt{\frac{\sqrt{3}}{2}} = 0.9306$	2	1.8612	
5	$5\pi/6$	$\sqrt{1/2} = 0.7071$	4	2.8284	
6	π	$\sqrt{0} = 0.0000$	1	0.0000	

$$\therefore \int_0^{\pi} \sqrt{\sin x}\, dx \approx \left[\frac{\pi - 0}{3(6)}\right] \cdot \left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + 4f(x_5) + f(x_6)\right]$$

$$\approx \left(\frac{\pi}{18}\right) \cdot (S) = (0.1746) \cdot (13.3792) = 2.3360$$

$$\approx 2.34 \text{ Ans}$$