Second Mid-Term 481
243 Number Theory

Remark:

(1) Write clearly.

(2) Order your answers in the same order as the questions.

1. If the Mersenne number $2^n - 1$ is prime, then show that n is prime. Show that the converse is not true.

2. If $F_n = 2^{2^n} + 1$, show that $F_n \equiv 7 \pmod{10}$ for all $n \geq 2$.

3. Find all solutions of the system:

$$\begin{aligned} x &\equiv 3 \pmod{4} \quad (1) \\ x &\equiv 1 \pmod{6} \quad (2) \\ x &\equiv 2 \pmod{7} \quad (3) \end{aligned}$$

4. Show that a has an inverse $\pmod{n}$ if and only if $(a, n) = 1$. Find the inverse of $19 \pmod{37}$.

5. If T_1 and T_2 are reduced residue systems $\pmod{n}$, then show in details that $|T_1| = |T_2|$.