

Chapter 1, Slide page 10:

$$y = (x-1)^2$$

$$\begin{aligned} E(y) &= E((x-1)^2) = \\ &= E[x^2 - 2x + 1^2] \\ &= E(x^2) - 2E(x) + 1 \rightarrow (*) \end{aligned}$$

$$\begin{aligned} E(x) &= \sum_{\text{all } x} x f(x) \\ &= 0 + \frac{1}{2} + 0 + \frac{1}{2} = 1 \end{aligned}$$

$$\begin{aligned} E(x^2) &= \sum x^2 f(x) \\ &= 0 + \frac{1}{2} + 0 + \frac{3}{2} = \frac{4}{2} = 2 \end{aligned}$$

Substitute ① and ② into * we get:

$$\begin{aligned} E(y) &= E(x^2) - 2E(x) + 1 \\ &= 2 - 2(1) + 1 \\ &= 1 \end{aligned}$$

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$$\begin{aligned} V(X) &= E((X - \mu)^2) \\ &= E(X^2 - 2X\mu + \mu^2) \\ &= E(X^2) - 2E(X)\mu + \mu^2 \quad \because \mu = E(X) \\ &= E(X^2) - 2(E(X))^2 + E(X)^2 \end{aligned}$$

Note: $E(X)^2 = [E(X)]^2$

$$\text{Var}(X) = V(X) = \sum_x^2 = E(X^2) - E(X)^2$$

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Hiber Geometric Distribution

$$\text{Max}(0, n-(N-k)) \leq x \leq \min(n, k)$$

الحد الأعلى: لا يمكن الحصول على نجاحات أكثر من الموجود k أو أكثر من عدد السحبات n .

الحد الأدنى: لا يمكن الحصول على نجاحات أقل من الصفر. وإذا لم يكن لدي مرآت فشل كافية، فلا بد أن أحصل على عدد معين من النجاحات.

Why $x \leq \min(n, k)$?

we cannot have number of successes more than available k , or more than objects drawn n .

Why $x \leq \text{Max}(0, n-(N-k))$?

We cannot have the number of successes less than zero.

Also, if there are not enough failures to fill the sample, we must have at least a certain number of successes.

Example: $N=10, k=7, n=6$

Number of failures is: $N-K=3$

$$\text{Min}(7,6) = 6$$

$$\text{Max}(0, 6-3) = \text{Max}(0,3) = 3$$

Therefore: $3 \leq x \leq 6$; So $x = 3,4,5,6$

Why can't $x=0$?

Because we are drawing 6 objects, but there are only 3 failures available.

Therefore at least 3 successes must be included.

To Clarify: If number of successes is zero ($x=0$) in our sample of size 6, we would need 6 failures. But there are only 3 failures in the population, so $x=0$ impossible.

$$\begin{aligned} x &\neq 1 & N-K &= 3 \\ n-1 &= 6-1 = 5 & &\neq 3 \\ x &\neq 2 \\ n-2 &= 6-2 = 4 & &\neq 3 \\ x &= 3 \\ n-3 &= 6-3 = 3 & &= 3 \end{aligned}$$

