Multiple compounding

i% = 10 compounded annually ⟶ years

i% = 10 per year compounded monthly ⟶ months

$$I = {}^{r}/_{m} \times 100$$

- **m = number of compound periods per year**: For Example, m=12(monthly), m=4 (quarterly)…
- **r = nominal annual interest rate:** For Example, compounded monthly, compounded semiannually, compounded quarterly…

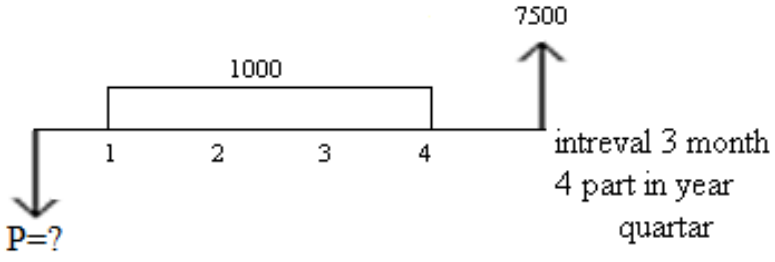
$$I_K = [(1 + {}^{r}/_{m})^{\frac{m}{k}} - 1] \times 100$$

$$I_{eff} = [(1 + {}^{r}/_{m})^{m} - 1] \times 100$$

Ex.1

Ali wishes to make a single deposit p at t=0 into a fund paying **15% per year compounded quarterly** such that $ 1000 payments are received at t=1,2,3 and 4 (**periods are 3 month intervals**), and a single payment of $7500 is received at t=12. What single deposit is required?

Solution

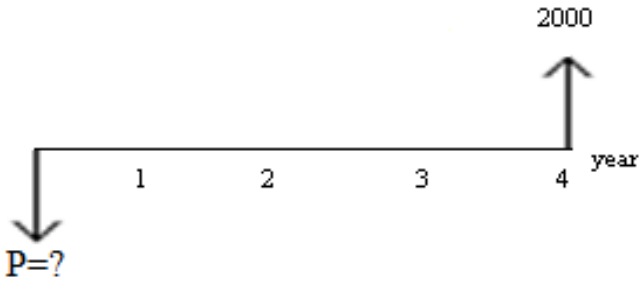


r=15% **compounded quarterly** and m=4 **quarterly**

$$i = \frac{r\%}{m} = \frac{0.15}{4} = 0.0375 = 3.75\%$$

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] + F(1+i)^{-n} = A\left[\frac{(1+0.0375)^4 - 1}{0.0375(1+0.0375)^4}\right] + (1+0.0375)^{-12} = \$\ 8473.12$$

Ex.2



Find the present worth if i=16% per year compounded monthly

$$I_{eff}=(1+\frac{r}{m})^{m}-1=(1+\frac{0.16}{12})^{12}-1=0.1723=17.23\ \%$$

$$P=F(1+i)^{-n}=2000(1+0.1723)^{-4}=SR\ 1059$$

Ex.3

A 20 **monthly** payment of SR 2000 each are made into an account that pays interest at a rate of 12.12 % **per year compounded quarterly**. Determine the present value of these payments if the first payment occur 3 months from today. Determine also annual effective interest.

Solution

$$I_K = [(1 + {}^{r}/_{m})^{\frac{m}{k}} - 1] \times 100$$

$$I_{monthly} = [(1 + {}^{.1212}/_{4})^{\frac{4}{12}} - 1] \times 100 = 1\%\ /\ \text{month}$$

$$p=A(p/A\ 1\%,20)(P/F\ 1\%,2)=2000(18.0456)(0.9803)=SR\ 8845$$

$$I_{eff} = [(1 + {}^{r}/_{m})^{m} - 1] \times 100$$

$$I_{eff} = [(1 + {}^{0.1212}/_{4})^{4} - 1] \times 100 = 12.682\%$$