KING SAUD UNIVERSITY
COLLEGE OF SCIENCE
DEPARTMENT OF MATHEMATICS
M - 203 (DIFFERENTIAL AND INTEGRAL CALCULUS)
FINAL EXAMINATION (II SEMESTER 1447/1448, 2025/2026

Max. Marks: 40 Time: 3 Hours

Marking Scheme: [Q.1. (4+4+4); Q.2. (4+4+4); Q.3. (4+4+4+4)]

Note: No calculator is allowed.

Q#1) (a) Test the convergence or divergence of the series

$$\sum_{n=1}^{\infty}\left(\frac{1}{2n+1}-\frac{1}{2n+2}\right)$$

(b) Find the interval of convergence and the radius of convergence for the power series

$$\sum_{n=1}^{\infty}\frac{(-1)^n x^n}{n\sqrt{n}\,4^{n+2}}$$

(c) Find the Maclaurin series of e^x and use its first three non-zero terms to approximate the series

$$\int_0^1 \frac{e^{x^2}-1-x^2}{x^4}\,dx$$

Q#2) (a) Evaluate the integral by reversing the order of integration

$$\int_0^1\int_{\sin^{-1}y}^{\frac{\pi}{2}} \cos^2(x)\,dxdy$$

(b) Use polar coordinates to evaluate the integral

$$\int_1^2\int_0^x \frac{1}{\sqrt{x^2+y^2}}\,dydx$$

(c) Evaluate the iterated integral

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{x^2+y^2}^{2}(x^2+y^2)\,dzdydx$$

by changing to cylindrical coordinates.

Q#3) (a) Show that the line integral

$$\int_{(1,2,3)}^{(3,2,1)}(2xy-z^2)dx+(x^2+2z)dy+(2z+2y-2xz)dz$$

is independent of path and find its value.

(b) Use Green's theorem to find the area enclosed by the ellipse $9x^2+4y^2=36$.

(c) Let S be the surface that is boundary of the region bounded by the graphs of the equations $z=2-x^2-y^2$, $z=x^2+y^2$ and let $\mathbf{F}$ be the force give by $\mathbf{F}=y\mathbf{i}+x\mathbf{j}+z\mathbf{k}$ acting on the points of S. Find the flux of $\mathbf{F}$ through S.

(d) Use Stokes' theorem to evaluate the surface integral

$$\int\int_S ((curl\ \mathbf{F})\cdot\mathbf{n}\,dS$$

where $\mathbf{F}(x,y,z)=y^2z\mathbf{i}+xz\mathbf{j}+x^2y^2\mathbf{k}$ and S is the part of the paraboloid $z=x^2+y^2$ that lies inside the cylinder $x^2+y^2=1$, oriented upward.

King Saud University, College of Science
Department of Mathematics
M-203 (Diff. & Integral Calculus)
Max. Marks 40 Final Exam (II Sem 1447/1448 2025/2026). Time: 3 Hours
Solutions to the Questions

Q #1(a) Test the Convergence or divergence of the Series:

$$\sum_{n=1}^{\infty}\left(\frac{1}{2n+1}-\frac{1}{2n+2}\right) \qquad \text{[Marks: 4]}$$

Soln. Let $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty}\left(\frac{1}{2n+1}-\frac{1}{2n+2}\right) = \sum_{n=1}^{\infty}\frac{1}{4n^2+6n+2}$ (1)

Let $\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty}\frac{1}{n^2}$ which is Conv. by p-series test

$$\lim_{n\to\infty}\frac{b_n}{a_n} = \lim_{n\to\infty}\frac{1}{4n^2+6n+2}\times n^2 \quad (2)$$

$$= \lim_{n\to\infty}\frac{n^{2}}{n^{2}\left(4+\frac{6}{n}+\frac{2}{n^2}\right)} = \frac{1}{4} > 0$$

Hence by LCT, $\sum_{n=1}^{\infty}\frac{1}{4n^2+6n+2} = \sum_{n=1}^{\infty}\left(\frac{1}{2n+1}-\frac{1}{2n+2}\right)$ is Conv. (1)

#1(b) Find the Interval of Conv. and radius of Conv. for the power Series: $\sum_{n=1}^{\infty}\frac{(-1)^n x^n}{n\sqrt{n}\,4^{n+2}}$. [Marks: 4]

Soln. We apply Abs. Ratio test:

$$\lim_{n\to\infty}\left|\frac{(-1)^{n+1}x^{n+1}}{(n+1)\sqrt{n+1}\,4^{n+3}}\times\frac{n\sqrt{n}\,4^{n+2}}{(-1)^n x^n}\right|$$

$= \frac{1}{4}|x|$. For Abs. Conv. $\frac{1}{4}|x|<1$

$\Leftrightarrow |x|<4 \Leftrightarrow -4<x<4$ (1)

At $x=-4$, we have $\sum_{n=1}^{\infty}\frac{(-1)^n(-4)^n}{n\sqrt{n}\,4^{n+2}} = \sum\frac{1}{n^{3/2}\,4^2}$ which is Conv. by p-series test. (1)

At $x = 4$, we have $\sum_{n=1}^{\infty} \frac{(-1)^n 4^n}{n\sqrt{n}\, 4^{n+2}} = \sum_{n=1}^{\infty} (-1)^n \frac{1}{4^2 \cdot n^{3/2}}$

which conv. by AST. (1)

Hence Interval of conv. $[-4, 4]$ and

Radius of conv. $r = 4$ (1)

1(c) Find the Maclaurin Series of e^x and use its first three non-zero terms to approximate the series: $\int_0^1 \frac{e^{x^2} - 1 - x^2}{x^4} dx$. [Marks: 4]

Soln: We have the Maclaurin Series as

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \cdots + \frac{x^n}{n!} f^{(n)}(0) + \cdots \quad \text{---(1)}$$

We have $f(x) = e^x \Rightarrow f(0) = 1$

$f'(x) = e^x \Rightarrow f'(0) = 1$

$f''(x) = e^x \Rightarrow f''(0) = 1$

$\vdots$

$f^{(n)}(x) = e^x \Rightarrow f^{(n)}(0) = 1$

$\vdots$

Substituting these values in (1), we get

$$f(x) = e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots \quad (2)$$

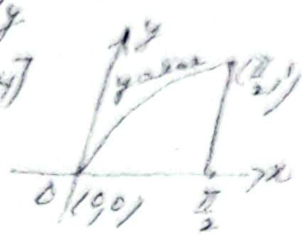
$$\therefore e^{x^2} = 1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \frac{x^8}{4!} + \cdots$$

$$\therefore \frac{e^{x^2} - 1 - x^2}{x^4} = \frac{\left(\cancel{1} + \cancel{x^2} + \frac{x^4}{2!} + \frac{x^6}{3!} + \frac{x^8}{4!} + \cdots\right) - \cancel{1} - \cancel{x^2}}{x^4}$$

$$= \frac{1}{2} + \frac{x^2}{3!} + \frac{x^4}{4!} + \cdots \quad (1)$$

Now, $\int_0^1 \frac{e^{x^2} - 1 - x^2}{x^4} dx = \left[\frac{x}{2} + \frac{x^3}{3 \times 6} + \frac{x^5}{5 \times 24}\right]_0^1 \approx \frac{1}{2} + \frac{1}{18} + \frac{1}{120}$ (1)

Q # 2(a) Evaluate the Integral by reversing the order of integration: $\int_0^1 \int_{\sin^{-1} y}^{\frac{\pi}{2}} \cos^2(x)\,dx\,dy$ [Marks: 4]



Soln. Given: $\sin^{-1} y \le x \le \frac{\pi}{2}$ and $0 \le y \le 1$

Changing, we get $0 \le y \le \sin x$; $0 \le x \le \frac{\pi}{2}$

We have $I = \int_0^{\pi/2} \int_0^{\sin x} \cos^2(x)\,dy\,dx$ (2)

$= \int_0^{\frac{\pi}{2}} \sin(x)\cos^2(x)\,dx$

Put $u = \cos x \Rightarrow du = -\sin x\,dx$

$\therefore \int -u^2\,du = -\frac{u^3}{3}$

$= -\int_1^0 u^2\,du = \left[\frac{u^3}{3}\right]_0^1$

If $x = 0$, $u = 1$ and if $x = \frac{\pi}{2}$, $u = 0$

$\therefore I = \frac{1}{3}$ (2)

Q # 2(b) Use polar Coordinates to evaluate the Integral:

$I = \int_1^2 \int_0^x \frac{1}{\sqrt{x^2+y^2}}\,dy\,dx.$ [Marks: 4]

Soln. $I = \int_0^{\frac{\pi}{4}} \int_{\sec\theta}^{2\sec\theta} \frac{1}{r} r\,dr\,d\theta = \int_0^{\frac{\pi}{4}} [r]_{\sec\theta}^{2\sec\theta}\,d\theta$ (2)

$= \int_0^{\pi/4} \sec\theta\,d\theta = [\ln|\sec\theta + \tan\theta|]_0^{\pi/4}$ (1)

$= \ln(1+\sqrt{2})$ (1)

Q # 2(c) Evaluate the Integral: $\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{x^2+y^2}^{2}(x^2+y^2)\,dz\,dy\,dx$

by changing to cylindrical Coordinates.

[Marks: 4]

Soln. We get $\int_0^{2\pi}\int_0^{1}\int_{r^2}^{2} r^2\, r\,dz\,dr\,d\theta$ (3)

$$= \int_0^{2\pi}\int_0^{1} r^3(2-r^2)\,dr\,d\theta$$

$$= \int_0^{2\pi}\left[\cancel{2}\frac{r^4}{\cancel{4}_2}-\frac{r^6}{6}\right]_0^1 d\theta = 2\pi\left(\frac{1}{2}-\frac{1}{6}\right)$$

$$= \frac{2\pi}{3}$$ (1)

Q # 3(a) Show that the line integral

$$\int_{(1,2,3)}^{(3,2,1)} (2xy-z^2)\,dx + (x^2+2z)\,dy + (2z+2y-2xz)\,dz$$

is independent of path and find its value. [Marks: 4]

Soln. We need to find f such that $\vec{F} = \nabla f$

$f_x = 2xy - z^2 \Rightarrow f(x,y,z) = x^2y - xz^2 + g(y,z)$ —(1)

$f_y = x^2 + 2z \Rightarrow f(x,y,z) = x^2y + 2yz + h(x,z)$ —(2)

$f_z = 2z + 2y - 2xz \Rightarrow f(x,y,z) = z^2 + 2yz - xz^2 + J(x,y)$ —(3)

$\Rightarrow f(x,y,z) = x^2y - xz^2 + 2yz + z^2 + C$ (2)

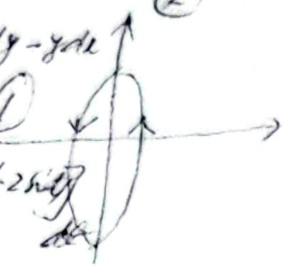
Hence, by Fundamental theorem of Line Integral, we have $\left[f(x,y,z)\right]_{(1,2,3)}^{(3,2,1)} = \left[x^2y - xz^2 + 2yz + z^2\right]_{(1,2,3)}^{(3,2,1)}$

$= (18-3+4+1) - (2-9+12+9) = 6$ (1)

Q # 3(b). Use Green's theorem to find the area enclosed by the ellipse $9x^2+4y^2=36$ [Marks: 4]

Soln. $C: x = 2\cos t,\ y = 3\sin t,\ 0 \le t \le 2\pi$ 2

By Green's theorem Area $A = \frac{1}{2}\oint_C x\,dy - y\,dx$ 1



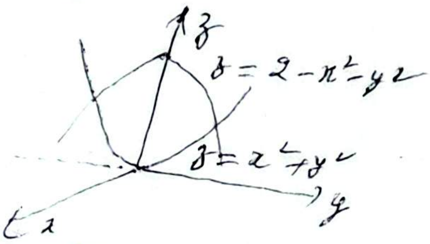
$\therefore A = \frac{1}{2}\int_0^{2\pi}[2\cos t(3\cos t) - 3\sin t(-2\sin t)]\,dt$

$= 3\int_0^{2\pi}(\cos^2 t + \sin^2 t)\,dt = 3\times 2\pi = 6\pi$ 1

Q # 3(c) Let S be the surface that is boundary of the region bounded by the graphs of the equations $z = 2 - x^2 - y^2$, $z = x^2+y^2$ and let $\vec{F}$ be the force given by $\vec{F} = y\vec{i} + x\vec{j} + z\vec{k}$ acting on the points of S. Find the Flux of $\vec{F}$ through S. [Marks: 4]

Soln. The surface S is closed

$\therefore$ Flux $= \iint_S \vec{F}\cdot\vec{n}\,ds$



By Divergence theorem

$= \iiint_Q (\nabla\cdot\vec{F})\,dV = \int_0^{2\pi}\int_0^1\int_{r^2}^{2-r^2} r\,dz\,dr\,d\theta$ 3

$= 2\pi\int_0^1 r(2-r^2-r^2)\,dr$

$= 2\pi\left[2\frac{r^2}{2} - 2\frac{r^4}{4}\right]_0^1$

$= 2\pi\cdot\left[\frac{1}{2}\right] = \pi$ 1

Q-A 3(a) Use Stokes' theorem to evaluate the surface integral $\iint_S (\text{curl}\,\vec{F})\cdot\vec{n}\,ds$, where $\vec{F} = y^2 z\,\hat{i} + xz\,\hat{j} + x^2y^2\,\hat{k}$ and S is the part of the paraboloid $z = x^2+y^2$ that lies inside the cylinder $x^2+y^2=1$, oriented upward.

[Marks: 4]

Soln. By Stokes' theorem, we have

$$\iint_S \text{curl}\,\vec{F}\cdot\vec{n}\,ds = \oint_C \vec{F}\cdot d\vec{r} \quad (1)$$

where C has the parametrization as

$$C: \quad x=\cos t,\ y=\sin t,\ z=1;\ 0\le t\le 2\pi \quad (1)$$

$$\therefore \oint_C \vec{F}\cdot d\vec{r} = \oint_C y^2 z\,dx + xz\,dy + x^2y^2\,dz$$

$$= \int_0^{2\pi} \sin^2 t(-\sin t)\,dt + \cos t\cdot\cos t\,dt + 0$$

$$= \int_0^{2\pi}\left[-(1-\cos^2 t)\sin t + \left(\frac{1+\cos 2t}{2}\right)\right]dt$$

$$= \pi \quad (2)$$