King Saud University Department of Mathematics

Math 481 Midterm 1

Time: 60 minutes (Total: 25 points)

Instructions. Show all essential steps and justify your claims. You may use standard results from the course (clearly cite them). No calculators, notes, or books. Simplify final answers.

Name: _____ **Student #:** _____

1. (10 points) Determine whether each function is Riemann integrable on $[0,1]$. If it is, compute the integral; if not, explain precisely why (e.g., via an ε–criterion or explicit partitions).

   (a) (4 points)
   $$f(x) = \begin{cases} 1, & x \in [0, \frac{1}{2}), \\ 0, & x \in [\frac{1}{2}, 1]. \end{cases}$$

   (b) (3 points)
   $$f(x) = \begin{cases} 1, & x \in \mathbb{Q}, \\ 0, & x \notin \mathbb{Q}, \end{cases} \qquad x \in [0,1].$$

   (c) (3 points)
   $$f(x) = \begin{cases} x, & x \in \mathbb{Q}, \\ 0, & x \notin \mathbb{Q}, \end{cases} \qquad x \in [0,1].$$

2. (6 points) Evaluate the limits below. State clearly any theorems you use.

   (a) (3 points) $\displaystyle\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n+k}$

   (b) (3 points) $\displaystyle\lim_{n\to\infty} \frac{\sum_{k=1}^{3^n} k^3}{3^{4n}}$

3. (9 points) Evaluate the following improper integrals. For each part, (i) justify convergence, and (ii) compute the value.

   (a) (5 points) $\displaystyle\int_1^2 \frac{1}{x \log x}\, dx$

   (b) (4 points) $\displaystyle\int_0^\infty \frac{dx}{\sqrt{x}\,(x+1)}$

Question:	1	2	3	Total
Points:	20	12	18	50