King Saud University, College of Science
Department of Mathematics
Midterm Exam 1447 H - (2025 − 2026 G)

First Midterm Exam of Math 5301.
Time allowed: 2 Hours

Given a simple and finite graph $G = (V; E)$ of order $n \geq 2$. Let $x \in V$.
We denoted, the complement of G by $\overline{G} = (V, \overline{E})$, and the degree of x in the graph G by $d_G(x)$.

Question 1. Show that:

1. If the graph G is self complementary, then its order n satisfies: $n = 4p$ or $n = 4p + 1$, where $p \in \mathbb{N} = \{1, 2, 3, ...\}$.
2. Find the values of n for the which a path P_n (resp. a cycle C_n) is self complementary.
3. (a) If $\exists\, x \in V$ such that $d_G(x) \in \{0, n-1\}$, then G is not a self complementary graph.
   (b) If G is a self complementary graph, then $\forall\, x \in V$, we have $d_G(x) \in \{1, ..., n-2\}$.
   (c) Find all self complementary graphs of order $n \leq 7$.

5 **Question 2.** Assume that: there are two vertices u and v in V, such that the length for any uv-path is greater than 3. Prove the following assertions:

1. $N_G(u) \cap N_G(v) = \emptyset$.
2. $N_G(u) \cup N_G(v) \subseteq V \setminus \{u, v\}$.
3. $d_G(u) + d_G(v) \leq n - 2$.
4. $|E| \leq \frac{n(n-2)}{2}$.
5. $|\overline{E}| \geq \frac{n}{2}$.

Question 3. Consider the two sequences:

- $D_1 = (1, 1, 2, 2, 3, 5, 6, 6)$
- $D_2 = (1, 1, 3, 3, 4, 4, 6, 6)$.

1. Show that D_1 is not graphic.
2. (a) Does there exist a simple bipartite graph G, such that $DEG(G) = D_2$?
   (b) Show that D_2 is graphic and find all graphs, up to isomorphism, G with $DEG(G) = D_2$.
   (c) Show that each graph G with $DEG(G) = D_2$ is self complementary graph.

Question 4. Prove the following statement:

(A graph $G = (V, E)$ is connected) if and only if
(for every non-empty subset X of V, with $X \neq V$, the edge boundary $\partial(X) \neq \emptyset$).