PART A: CHOOSE THE CORRECT ANSWER, EACH QUESTION CREDITS 2 MARKS:

1. If $i^{(4)} = 8\%$, what is the equivalent $i^{(12)}$:

$i^{(12)} = 12\left[\left(1 + \frac{i^{(4)}}{4}\right)^{1/3} - 1\right]$

(a) 7.9 %

(b) 1.9 %

(c) 5.9 %

2. Suppose the accumulation function for your account is $a(t) = 3t + 1$. If you invest \$ 2000 at time 1 in this fund how much your fund worth at time 8?

1 8

$2000 \cdot \frac{a(8)}{a(1)}$

(a) 320

(b) 12,500

(c) 50,000

3. Based on 4% annual interest rate, for a 10-year annuity with level payments of 20 each, and with the first payment occurring 6 years from now, then the present value equals:

20 6-year 10-year

(a) $20\ v^6\ a_{\overline{10}|i}$

(b) $20\ v^5\ a_{\overline{10}|i}$

(c) $20\ \frac{a_{\overline{15}|i}}{a_{\overline{5}|i}}$

4. The present value of a series of forever payments of 10 at the beginning of every 8-years, with $i = 3\%$, is:

$10\,\ddot{a}_{\overline{\infty}|}\,(1+i)^8-1$

$= 10\,\dfrac{(1+i)^8-1}{[(1+i)^8-1]+1}$

(a) $\frac{10*1.03}{0.03}$

(b) $\frac{10*(1.03)^8}{(1.03)^8-1}$

(c) $\frac{10*(1-(1.03)^{-10})*(1.03)^8}{0.03}$

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5. A loan for 20,000 must be repaid by 5 year-end payments with interest at an annual effective rate of interest 10%. Then the annual payment is:

20,000 — 5-years

$20{,}000 = P\,a_{\overline{5}|\,0.10}$

(a)5,500

(b)5,456.12

(c)5,275.95

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PART B: SOLVE 5 QUESTIONS OF THE FOLLOWING: EACH QUESTION CREDITS 4 MARKS:

1. John invested $12000 on January 1, 1995. Assuming compound interest at 5 % per year,

1 Jan. 1995 — 1 Jan 2002

1200 — 7 years

(A) Find the following:

i- The accumulated value on January 1, 2002.

ii- The amount of money needed to invest 10 years in the past to accumulating $10,000.

(B) If interest is convertible semi-annually, find the rate of discount convertible semi-annually.

(A) (i) F.V = $12000\,(1+0.05)^7 = 16{,}885.21$ (1)

(ii)

? — 10 years, 10,000

$$P.V = \frac{10{,}000}{(1+0.05)^{10}} = 6{,}139.11$$ (1)

(B) if $i^{(2)} = 5\%$, $d^{(2)} = ?$

$$\left.\begin{aligned}\left(1+\frac{i^{(m)}}{m}\right)^m &= \left(1-\frac{d^{(m)}}{m}\right)^{-m}\\ \Rightarrow \left(1+\frac{0.05}{2}\right)^2 &= \left(1-\frac{d^{(2)}}{2}\right)^{-2}\end{aligned}\right\}$$ (1)

$$\Rightarrow 1+\frac{0.05}{2} = \frac{1}{1-\frac{d^{(2)}}{2}}$$

$$\Rightarrow 1-\frac{d^{(2)}}{2} = \frac{1}{1+\frac{0.05}{2}} \Rightarrow d^{(2)} = 2\left[1-\frac{1}{1+\frac{0.05}{2}}\right]$$

$$= 4.88\%$$ (1)

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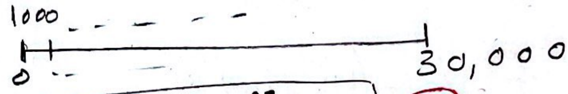
2. You want to accumulate at least 30,000 in an account earning a 6% annual effective rate. You will make a level deposit of 1,000 at the beginning of each year for n years.

(a) Find n. (b) What is the account balance after n years

2 2



(a) $\boxed{30{,}000 = 1000\, \ddot{s}_{\overline{n}|i}}$ 1

$$= 1000 \frac{(1.06)^n - 1}{\frac{0.06}{1.06}}$$

$$\Rightarrow n = \frac{\ln\left[30 * \left(\frac{0.06}{1.06}\right) + 1\right]}{\ln(1.06)}$$

$$= \frac{0.99255}{0.05827} = 17.034$$

we need more than 17 for payments periods.

So number of payments periods:

$\boxed{n = 18}$ 1

(b) The account balance after $n = 18$ years:

$$1000\, \ddot{s}_{\overline{18}|i} = 1000 \frac{(1.06)^{18} - 1}{\frac{0.06}{1.06}} \quad 1$$

$$= 32{,}759.99 \quad 1$$

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3. (a) Derive a formula of the present value for arithmetic decreasing annuities. 2

(b) A 10-year annuity immediate makes quarterly payments of 100 in year 1, 90 in year 2, decreasing to 10 per quarter in year 10. Find the present value of this annuity at an interest rate of 6% convertible monthly.

2

(a)

$$(Da)_{\overline{n}|} + (Ia)_{\overline{n}|} = (n+1)a_{\overline{n}|} \quad 1$$

$$\Rightarrow (Da)_{\overline{n}|} = (n+1)a_{\overline{n}|} - (Ia)_{\overline{n}|}$$

$$= (n+1)\frac{1-v^n}{i} - \frac{\ddot{a}_{\overline{n}|} - nv^n}{i}$$

$$1 \quad = \frac{(n+1) - \ddot{a}_{\overline{n}|} - v^n}{i} = \frac{n - (\ddot{a}_{\overline{n}|} - 1 + v^n)}{i}$$

$$= \frac{n - a_{\overline{n}|}}{i}$$

(b) 

$$i^{(12)} = 6\%$$

quarterly annual

$$P.V. = \boxed{(4 * 10)\, D a^{(4)}_{\overline{10}|6\%} = (4 * 10)\frac{i}{i^{(4)}}\,(Da)_{\overline{10}|6\%}} \quad 1$$

$$1+i = \left(1+\frac{i^{(12)}}{12}\right)^{12} = \left(1+\frac{i^{(4)}}{4}\right)^4$$

$$\Rightarrow 1+i = \left[1+\frac{0.06}{12}\right]^{12} = [1.005]^{12}$$

$\frac{1}{2}$

$$\frac{i^{(4)}}{4} = (1+i)^{1/4} - 1$$

$$= [1.005]^{12/4} - 1$$

$$= [1.005]^3 - 1$$

So,

$$P.V. = (4 * 10) \frac{\cancel{i}}{i^{(4)}} \left[\frac{10 - a_{\overline{10}|i}}{\cancel{i}} \right]$$

$$= (10 * \cancel{4}) \frac{1}{\cancel{4}[(1.005)^3 - 1]} \left[10 - \frac{1 - \left(\frac{1}{1.005}\right)^{120}}{(1.005)^{12} - 1} \right]$$

$$= 1,789.75$$

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4. An insurance company has an obligation to pay medical costs for a client. The annual claim costs 5,000 and it starts one year from today. The cost increases in such a way that each payment is 7% greater than the previous cost. If claim payments made at annual interest rate 5% and the client is expected to live an additional 20 years, find the present value of this obligation.

$g = 5\%$

$i = 7\%$

5,000   5,000(1+0.05)

0   1   2   - - -   20-years

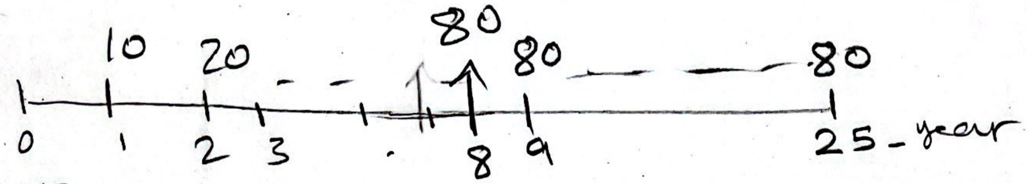
$$P.V. = 5{,}000 \; \frac{1 - \left(\frac{1+0.05}{1+0.07}\right)^{20}}{0.07 - 0.05}$$

1   formula   2

$$= 78{,}584.36$$

1

5. Find the present value at time 0 of an annuity such that the payments are made at the end of each year. Payments start at \$10, each payment thereafter increases by \$10 until reaching \$80, and they remain at that level until 25 payments in total are made. The annual effective rate of interest is 4%.



P.V?

$i = 4\%$ $\quad 10(1, 2, \ldots, 8)$

1.25 $\qquad$ 1.25

$$P.V = 10 * (Ia)_{\overline{8}|4\%} + v^8 * 80 * a_{\overline{17}|i}$$

2.5

$$= 10 * \frac{\ddot{a}_{\overline{8}|0.04} - 8v^8}{0.04} + v^8 * 80 * \frac{1 - v^{17}}{0.04}$$

1.5

$$= 10 * \frac{\frac{1-v^8}{d \, \frac{0.04}{1.04}} - 8v^8}{0.04} + v^8 * 80 * \frac{1-v^{17}}{0.04}$$

$$= 1{,}000.28$$

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6. Payments are made to an account at a continuous rate of (20k+tk), where $0 \leq t \leq 5$. Interest is credited at a force of interest $\delta_t = \frac{1}{20+t}$. The present value equals 5000.

Calculate k.

$$5{,}000 = P.V.$$

$$1 \quad = \int_0^5 K(20+t)\, e^{-\int_0^t \frac{1}{20+s} ds}\, dt$$

$$= K \int_0^5 (20+t) \cdot e^{-\int_0^t \frac{1}{20+s} ds}\, dt$$

$$\tfrac{1}{2} \quad -\int_0^t \frac{1}{20+s} ds = -\ln(20+s)\Big|_0^t = -\ln(1+0.05t) \quad \tfrac{1}{2}$$

$$\text{So, } 5{,}000 = K \int_0^5 (20+t) \cdot \boxed{\frac{1}{1+0.05t}}\, dt$$

$$20 * 5 = 100 \quad \frac{100\,[20+t]}{100+5t} = \frac{20\,[\cancel{100}+5t]}{[\cancel{100}+5t]}$$

1

$$= K \int_0^5 20\, dt$$

$$\Rightarrow 5{,}000 = K\,(20 * 5) \Rightarrow K = \frac{5{,}000}{100}$$

$$\Rightarrow K = 50 \quad 1$$