Second Mid-term, Semester II, 1447

Department of Mathematics, College of Science, KSU

Course: Math 209 Maximum Marks: 25 Duration: 1.5 Hours

Question 1 [4+4+4 points]

(1) Find the interval and radius of convergence for the power series:

$$\sum_{n=1}^{\infty} \frac{2^n}{n!} x^n, \qquad \sum_{n=1}^{\infty} \frac{1}{4^n n^2}(x+1)^n$$

(2) Find the power series expansion centered at 0 for the functions:

$$e^{2x}, \qquad \frac{x}{1+x^4}$$

(3) Use the result from part (2) to find the power series for:

$$\log\left(\sqrt{1+x^4}\right), \qquad \int_0^x \frac{e^{2t}-1}{t}\, dt$$

Question 2 [4+3 points]

Consider the periodic function defined by:

$$f(x) = x^2, \qquad x \in [-2\pi, 2\pi],$$

and extended periodically with $f(x+4\pi) = f(x)$.

(a) Find the Fourier series representation of $f(x)$.

(b) Deduce that:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$$

Question 3 [3+3 points]

Find the Fourier integral representation of the function:

$$f(x) = \begin{cases} 3, & |x| \le 2, \\ 0, & |x| > 2 \end{cases}$$

Then deduce the result:

$$\int_0^{\infty} \frac{\sin(2x)}{x}\, dx = \frac{\pi}{2}$$