KING SAUD UNIVERSITY
COLLEGE OF SCIENCES
DEPARTMENT OF MATHEMATICS

Semester 471 / MATH-244 (Linear Algebra) / Make-up of Mid-term Exams

Max. Marks: 25 **Max. Time: ~~3 Hrs.~~ 90 min**

Note: Calculators are not allowed.

Question 1: [Marks: 1+1+1+1+1]

Which of the given choices is correct?

(i) If A is a square matrix satisfying $A^2 = I$, then $A - 2A^{-5}$ is equal to:

a) $A - 2I$ b) $A^2 - A$ c) $2A^{-1}$ d) $-A$.

(ii) If $(adj(A))^{-1} = A$, then the determinant $|A|$ is equal to:

a) 4 b) 3 c) 1 d) 2.

(iii) Let $B = \{u_1, u_2, u_3, u_4, u_5\}$ be a linearly dependent subset of $\mathbb{R}^5$. Then the $span(B)$ must be:

a) equal to $\mathbb{R}^5$ b) of dimension 4 c) of dimension ≤ 3 d) of dimension ≤ 4.

(iv) Let ${}_BP_C = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ be the transition matrix from ordered basis $C = \{v_1, v_2, v_3\}$ to ordered basis B of a vector space V. Then, the coordinate vector $[2v_1 + v_2 - 3v_3]_B$ is equal to:

a) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ b) $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ c) $\begin{bmatrix} -5 \\ 2 \\ 4 \end{bmatrix}$ d) $\begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}$.

(v) If $\{(r, s, 0, 2r - s) | r, s \in \mathbb{R}\}$ is the solution space of homogeneous system $AX = 0$, then $rank(A)$ is equal to:

a) 1 b) 3 c) 2 d) 4

Question 2: [Marks: 4 + 2 + 4]

a) Find inverse of the matrix $A = \begin{bmatrix} 3 & 2 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 2 & 1 & 3 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ by using cofactors.

b) Let B be a 3×3 matrix with $|B| = 2$. Evaluate the determinant $|adj(adj(B^{-1}))|$.

c) Solve the following system of linear equations:

$$\begin{aligned} 2w - x - 4y + 3z &= 0 \\ 3w - 3x - 2y &= 1 \\ 3w - 2x - 5y + 4z &= 0 \end{aligned}$$

Question 3: [Marks: 3 + 3 + 4]

a) Show that the vectors $u_1 = (1, 2, 1, 0)$, $u_2 = (1, 2, 0, 0)$, $u_3 = (1, 0, 0, 1)$ are linearly independent, and then find a basis B of the vector space $\mathbb{R}^4$ such that $u_1, u_2, u_3 \in B$.

b) Find nullity and rank of the matrix $\begin{bmatrix} 0 & -1 & 0 & -1 \\ -1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 2 \end{bmatrix}$.

c) Let $B = \{(0,1,2), (1,0,0), (0,0,1)\}$ and $C = \{(1,1,0), (1,0,2), (0,1,1)\}$ be ordered bases for the vector space $\mathbb{R}^3$ and $x \in \mathbb{R}^3$ with the coordinate vector $[x]_B = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$. Find the vector x, the transition matrix ${}_CP_B$ from basis B to C, and the coordinate vector $[x]_C$.

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