KING SAUD UNIVERSITY
COLLEGE OF SCIENCES
DEPARTMENT OF MATHEMATICS

Semester 471 / MATH-244 (Linear Algebra) / Mid-term Exam 2

Max. Marks: 25 **Max. Time: $1\frac{1}{2}$ hrs.**

Note: Calculators are not allowed.

Question 1: [Marks: 3+2]

I. Select the correct choice:

(i) If W = {(1,1),(1,2)}, then W is:

(a) a vector space (b) a subspace of $\mathbb{R}^2$ (c) a basis of $\mathbb{R}^2$ (d) linearly dependent.

(ii) If B = {u, v, w} is an orthogonal set of non-zero vectors in an inner product space E and $\alpha u + \beta v + \gamma w = 0$ for some scalars α, β and γ, then:

(a) $\alpha = \beta = \gamma = 0$ (b) u = v = w (c) $\alpha = \gamma$, v = o, u = w (d) B is linearly dependent.

(iii) If A is an invertible matrix of order 3, then $rank(A)$ is equal to:

(a) 0 (b) 1 (c) 2 (d) 3.

II. Give an example for each of the following. You don't have to prove your answers.

(i) A linearly dependent subset of $\mathbb{R}^3$ consisting of three vectors in which every two vectors are linearly independent.

(ii) A matrix having *rank* 3 and *nullity* 4.

Question 2: [Marks: 4 + 3]

Let $A = \begin{bmatrix} 1 & 1 & 2 & -3 & 5 \\ 2 & 3 & 0 & -1 & -6 \\ 3 & 4 & 2 & -4 & -1 \end{bmatrix}$. Then:

(i) Find bases for $row(A)$ and $col(A)$.

(ii) Express the last three columns of the matrix A as linear combinations of the first two.

Question 3: [Marks: 3 + 4]

(i) Let $F = \{p = 2 - x,\ q = 5 + 3x - x^2 + 2x^3,\ r = 1 + 7x + x^2 + 4x^3\}$. Find the real numbers α, β, γ so that $-3 + 5x^2 + 2x^3 = \alpha p + \beta q + \gamma r$. Also, show that the set F is linearly independent in the vector space P_3.

(ii) Let $G = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ be a basis of the vector space $\mathbb{R}^6$. Let A be a 4×6 matrix such that $A(v_1)$ and $A(v_2)$ are linearly independent vectors in $\mathbb{R}^4$ and $A(v_i) = 0$, for all $i = 3,4,5,6$. Show that the null space $N(A) = span(\{v_3, v_4, v_5, v_6\})$. Also, find $nullity(A^T)$.

Question 4: [Marks: 3 + 3]

(i) Consider a vector space E of dimension 3. Let $B = \{u_1, u_2, u_3\}$ and $C = \{v_1, v_2, v_3\}$ be two ordered bases for E such that the transition matrix ${}_CP_B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ from B to C. Compute the coordinate vector $[v_1 - 2v_2 + 3v_3]_B$.

(ii) Consider the matrices $A = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ in the inner product space $M_2(\mathbb{R})$, consisting of all real matrices of size 2, with inner product $< A, B > = trace\ (AB^T)$. Find the angle between the matrices A and B. Also, verify the Pythagorean theorem for the same matrices.

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