

## MT 2 solution

①

Q1 (a) Work done =  $\underline{F} \cdot \underline{d}$   
(i)  $= \langle 2, 2, 0 \rangle \cdot \langle 0, 3, 0 \rangle$   
 $= 6 \text{ units} \quad (\text{Mark 2})$

(ii)  $\theta = \cos^{-1} \left( \frac{6}{\sqrt{8} \sqrt{9}} \right) = \cos^{-1} \left( \frac{6}{(2\sqrt{2})(3)} \right) = \cos^{-1} \left( \frac{1}{\sqrt{2}} \right)$

$\Rightarrow \theta = \cos^{-1} \left( \frac{1}{\sqrt{2}} \right) = 45^\circ$   
(Mark 2)

(b)  $\text{comp}_{\underline{u}}(\underline{v} \times \underline{w})$   
 $= \underline{v} \times \underline{w} \cdot \frac{\underline{u}}{\|\underline{u}\|}$   
 $= \langle 30, -10, 5 \rangle \cdot \frac{\langle -3, -1, -4 \rangle}{\sqrt{26}}$   
 $= \frac{80}{\sqrt{26}} \quad (\text{Mark 1+1})$

(c)  $\Delta ABC = \frac{1}{2} \|\vec{AB} \times \vec{AC}\|$   
 $= \frac{1}{2} (2\sqrt{66}) = \sqrt{66}$   
(Mark 3)

(2)

Q<sub>2</sub> (a) Equation of the plane is

$$3x - y + 2z = -11 \quad (\text{Mark 2})$$

(b) Here  $\underline{a} = \langle -1, 1, 5 \rangle$ ,  $\underline{b} = \langle 7, 2, -2 \rangle$ 

$$l_1: x = 1 - t, y = 3 + t, z = 5t, \quad t \in \mathbb{R}$$

$$l_2: x = -2 + 7u, y = -1 + 2u, z = 2 - 2u, \quad u \in \mathbb{R}$$

Not Parallel because

$$-\frac{1}{7} \neq \frac{1}{2} \neq \frac{-5}{2} \quad (\text{Mark 1})$$

Not intersecting because as  $t = \frac{-22}{9}$   
and  $u = \frac{7}{9}$  we get

$$z = -\frac{110}{9} \text{ and } z = \frac{4}{9}$$

which means  $l_1$  and  $l_2$  non-intersecting

(Mark 2)

(c) Paraboloid having y-axis as its axis. Traces: xy-trace  $y = 6x^2$  a parabola; yz-trace:  $y = z^2$  a parabola and xz-trace  $6x^2 + z^2 = 0$ , origin.

(3)

Q<sub>3</sub> (a) Domain of  $\underline{r}$  is

$$D = \{t; t < 1\}$$

since the domain of  $\ln(1-t)$  is

$$\{t; t < 1\}.$$

(Mark 2)

(b) For all  $t$ ,

$$\underline{v}(t) = \underline{i} + 2\underline{j} + 3\underline{k}$$

$$\underline{a}(t) = 0, \text{ and}$$

$$\|\underline{v}(t)\| = \sqrt{14} \approx 3.74$$

(Mark 1+1+1)

$$(c) \quad \underline{r}'(t) = 2\underline{i} - 4t^3\underline{j} + 6\sqrt{t}\underline{k} \Rightarrow$$

$$\underline{r}(t) = 2t\underline{i} - t^4\underline{j} + 4t^{3/2}\underline{k} + \underline{c}$$

$$\text{Thus, } \underline{r}(t) = (2t+1)\underline{i} + (5-t^4)\underline{j} +$$

$$(4t^{3/2}+3)\underline{k}$$

(Mark 2+1)