# MID TERM II EXAM. SEMESTER II, 1445

Dept. Math., College of Science
KING SAUD UNIVERSITY

Math: 107 Full Mark: 25 Time: 90 Minutes

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**Q1**. [Marks: 4+2+3=9]

(a) Let a constant force $\mathbf{F} = \langle 2, 2, 0 \rangle$ is applied on a particle displacing it from point $(0, 0, 0)$ to $(0, 3, 0)$. Then, find: (i) work done by the force $\mathbf{F}$, and (ii) angle $\theta$ between the force $\mathbf{F}$ and the displacement $\mathbf{d}$.

(b) Let $\mathbf{u} = \langle 3, -1, -4 \rangle$, $\mathbf{v} = \langle 2, 5, -2 \rangle$, and $\mathbf{w} = \langle -1, 0, 6 \rangle$.
Compute $comp_{\mathbf{u}}(\mathbf{v} \times \mathbf{w})$.

(c) Find the area of the triangle $\triangle ABC$, where $A(2, -1, 1)$, $B(-3, 2, 0)$, and $C(4, -5, 3)$.

**Q2**. [Marks: 2+3+3=8]

(a) Find an equation of the plane through $P(2, 5, -6)$ and parallel to the plane $3x - y + 2z = 10$.

(b) Let $l_1$ be the line passing through $A(1, 3, 0)$ and $B(0, 4, 5)$, and $l_2$ be the line passing through $C(-2, -1, 2)$ and $D(5, 1, 0)$. Determine whether $l_1$ and $l_2$ are skew lines, that is, neither parallel nor intersecting.

(c) Identify the surface $y = 6x^2 + z^2$. Give its traces, and sketch it.

**Q3**. [Marks: 2+3+3=8]

(a) Find the domain of the vector-valued function $\mathbf{r}(t) = \ln(1 - t)\mathbf{i} + \sin t\mathbf{j} + t^3\mathbf{k}$.

(b) If $\mathbf{r}(t) = (1 + t)\mathbf{i} + 2t\mathbf{j} + (2 + 3t)\mathbf{k}$ is the position vector of a moving point $P$, find its velocity, acceleration, and speed at $t = 2$.

(c) Find $\mathbf{r}(t)$ subject to the given conditions: $\mathbf{r}'(t) = 2\mathbf{i} - 4t^3\mathbf{j} + 6\sqrt{t}\mathbf{k}$, $\mathbf{r}(0) = \mathbf{i} + 5\mathbf{j} + 3\mathbf{k}$.

MT 2 solution



Q1 (a) (i) Work done $= \underline{F}\cdot\underline{d}$

$$= \langle 2,2,0\rangle\cdot\langle 0,3,0\rangle$$

$$= 6 \text{ units} \quad \text{(Mark 2)}$$

(ii) $\theta = \cos^{-1}\left(\dfrac{6}{\sqrt{8}\,\sqrt{9}}\right) = \cos^{-1}\left[\dfrac{6}{(2\sqrt{2})(3)} = \dfrac{1}{\sqrt{2}}\right]$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ$$

(Mark 2)

(b) $\text{comp}_{\underline{u}}(\underline{v}\times\underline{w})$

$$= \underline{v}\times\underline{w}\cdot\frac{\underline{u}}{\|\underline{u}\|}$$

$$= \langle 30,-10,5\rangle\cdot\frac{\langle -3,-1,-4\rangle}{\sqrt{26}}$$

$$= \frac{80}{\sqrt{26}} \quad \text{(Mark 1+1)}$$

(c) $\Delta ABC = \dfrac{1}{2}\|\overrightarrow{AB}\times\overrightarrow{AC}\|$

$$= \frac{1}{2}\left(2\sqrt{66}\right) = \sqrt{66}$$

(Mark 3)

$Q_2$ (a) Equation of the plane is

$$3x - y + 2z = -11 \quad \text{(Mark 2)}$$

(b) Here $\underline{a} = \langle -1, 1, 5 \rangle$, $\underline{b} = \langle 7, 2, -2 \rangle$

$l_1$: $x = 1 - t$, $y = 3 + t$, $z = 5t$, $t \in \mathbb{R}$

$l_2$: $x = -2 + 7u$, $y = -1 + 2u$, $z = 2 - 2u$, $u \in \mathbb{R}$

Not parallel because

$$-\frac{1}{7} \neq \frac{1}{2} \neq \frac{-5}{2} \quad \text{(Mark 1)}$$

Not intersecting because as $t = \frac{-22}{9}$ and $u = 7/9$ we get

$$z = -\frac{110}{9} \text{ and } z = \frac{4}{9}$$

which means $l_1$ and $l_2$ non-intersecting

(Mark 2)

(c) Paraboloid having $y$-axis as its axis. Traces: xy-trace $y = 6x^2$ a parabola; yz-traces: $y = z^2$ a parabola and xy-trace $6x^2 + z^2 = 0$, origin.

$Q_3$ (a) Domain of $\underline{r}$ is

$$D = \{t : t < 1\}$$

since the domain of $\ln(1-t)$ is

$$\{t ; t < 1\}.$$

(Mark 2)

(b) For all $t$,

$$\underline{v}(t) = \underline{i} + 2\underline{j} + 3\underline{k}$$

$$\underline{a}(t) = 0, \text{ and}$$

$$\|\underline{v}(t)\| = \sqrt{14} \simeq 3.74$$

(Mark 1+1+1)

(c) $\underline{r}'(t) = 2\underline{i} - 4t^3\underline{j} + 6\sqrt{t}\,\underline{k} \Rightarrow$

$$\underline{r}(t) = 2t\underline{i} - t^4\underline{j} + 4t^{3/2}\underline{k} + \underline{c}$$

Thus, $\underline{r}(t) = (2t+1)\underline{i} + (5-t^4)\underline{j} +$

$$(4t^{3/2} + 3)\underline{k}$$

(Mark 2+1)