MID TERM II EXAM. SEMESTER II, 1445

Dept. Math., College of Science
KING SAUD UNIVERSITY

Math: 107 Full Mark: 25 Time: 90 Minutes

Q1. [Marks: 4+2+3=9]

(a) Let a constant force $\mathbf{F} = \langle 2, 2, 0\rangle$ is applied on a particle displacing it from point $(0, 0, 0)$ to $(0, 3, 0)$. Then, find: (i) work done by the force $\mathbf{F}$, and (ii) angle θ between the force $\mathbf{F}$ and the displacement $\mathbf{d}$.

(b) Let $\mathbf{u} = \langle 3, -1, -4\rangle$, $\mathbf{v} = \langle 2, 5, -2\rangle$, and $\mathbf{w} = \langle -1, 0, 6\rangle$.
Compute $comp_{\mathbf{u}}(\mathbf{v} \times \mathbf{w})$.

(c) Find the area of the triangle $\triangle ABC$, where $A(2, -1, 1)$, $B(-3, 2, 0)$, and $C(4, -5, 3)$.

Q2. [Marks: 2+3+3=8]

(a) Find an equation of the plane through $P(2, 5, -6)$ and parallel to the plane $3x - y + 2z = 10$.

(b) Let l_1 be the line passing through $A(1, 3, 0)$ and $B(0, 4, 5)$, and l_2 be the line passing through $C(-2, -1, 2)$ and $D(5, 1, 0)$. Determine whether l_1 and l_2 are skew lines, that is, neither parallel nor intersecting.

(c) Identify the surface $y = 6x^2 + z^2$. Give its traces, and sketch it.

Q3. [Marks: 2+3+3=8]

(a) Find the domain of the vector-valued function $\mathbf{r}(t) = \ln(1 - t)\mathbf{i} + \sin t\mathbf{j} + t^3\mathbf{k}$.

(b) If $\mathbf{r}(t) = (1 + t)\mathbf{i} + 2t\mathbf{j} + (2 + 3t)\mathbf{k}$ is the position vector of a moving point P, find its velocity, acceleration, and speed at $t = 2$.

(c) Find $\mathbf{r}(t)$ subject to the given conditions: $\mathbf{r}'(t) = 2\mathbf{i} - 4t^3\mathbf{j} + 6\sqrt{t}\mathbf{k}$, $\mathbf{r}(0) = \mathbf{i} + 5\mathbf{j} + 3\mathbf{k}$.