King Saud University
College of Sciences
Mathematics Department

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Solution of the second midterm exam ACTU-472 (20%)

November 21, 2024, from 1 to 3 PM

Use ballpoint or ink–jet pens and keep five digits after dot

Problem 1 ***(4 points)***

1. *You are given: (i) The future lifetimes of (40) and (50) are independent. (ii) The survival function for (40) is based on a constant force of mortality, $\mu = 0.1$. (iii) The survival function for (50) follows De Moivre's law with $\omega = 100$.*
   Calculate the probability that (50) dies before (40) within 15 years.
   *(**Hint**: ${}_tq_{\overset{1}{x}:y} = \int_0^t \mathrm{P}\left(T_y > T_x \mid T_x = u\right) f_x(u)\, \mathrm{d}u$).*

2. *For a special fully continuous whole life insurance of 10000 on the last–survivor of (x) and (y), you are given: (i) T_x and T_y are independent. (ii) $\mu_{x+t} = 0.04$, $\mu_{y+t} = 0.06$ for $t > 0$ and $\delta = 0.05$. (iv) The level annual premium rates P are payable until the first death. Find P using equivalence principle (**Hint**: $\bar{A}_{\overline{xy}} = \bar{A}_x + \bar{A}_y - \bar{A}_{xy}$, $\bar{a}_{xy} = \frac{1-\bar{A}_{xy}}{\delta}$ and $\bar{a}_{\overline{xy}} = \frac{1-\bar{A}_{\overline{xy}}}{\delta}$).*

Solution:

1. We need to compute ${}_{10}q_{\overset{1}{50}:40}$ which is given by definition by

$$\begin{aligned} {}_{15}q_{\overset{1}{50}:40} &= \int_0^{15} \mathrm{P}\left(T_{40} > T_{50} \mid T_{50} = u\right) f_{50}(u)\, \mathrm{d}u = \int_0^{15} \mathrm{P}\left(T_{40} > u \mid T_{50} = u\right) f_{50}(u)\, \mathrm{d}u \\ &= \int_0^{15} \mathrm{P}\left(T_{40} > u\right) f_{50}(u)\, \mathrm{d}u = \int_0^{15} {}_up_{40}\ f_{50}(u)\, \mathrm{d}u, \end{aligned}$$

   where

$$\mathrm{f}_{50}(t) = \frac{1}{100-50} = \frac{1}{50} \text{ for } 0 < t < 50$$

   Therefore,

$${}_{10}q_{\overset{1}{50}:40} = \frac{1}{50}\int_0^{15} e^{-0.1t}\, \mathrm{d}t = \frac{1}{50}\frac{1-e^{-1.5}}{0.1} = \mathbf{0.15537}.$$

2. Since lifetimes are independent, T_{xy} has a force of failure of $\mu_{xy+t} = \mu_{x+t} + \mu_{y+t} = 0.04 + 0.06$, a constant, so that $\bar{A}_{xy} = \frac{0.1}{0.1+0.05} = \frac{10}{15} = \frac{2}{3} = 0.66667$, also, $\bar{A}_x = \frac{0.04}{0.04+0.05} = \frac{4}{9} = 0.44444$, $\bar{A}_y = \frac{0.06}{0.06+0.05} = \frac{6}{11} = 0.54545$ and hence by symmetric relation.

$$\bar{A}_{\overline{xy}} = \bar{A}_x + \bar{A}_y - \bar{A}_{xy} = \frac{4}{9} + \frac{6}{11} - \frac{2}{3} = \frac{32}{99} = 0.32323$$

   By the relation between insurance and annuity,

$$\bar{a}_{xy} = \frac{1-\bar{A}_{xy}}{\delta} = \frac{1-\frac{2}{3}}{\frac{5}{100}} = \frac{20}{3} = 6.66667$$

The level benefit premium is

$$P = 10000\frac{\bar{A}_{\overline{xy}}}{\bar{a}_{xy}} = 10000 \times \frac{\frac{32}{99}}{\frac{20}{3}} = \frac{16000}{33} = \mathbf{484.84848}.$$

Problem 2 *(4 points)*

1. *For a temporary life annuity–immediate on independent lives* (30) *and* (40)*: (i)* $i = 0.05$ *(ii)* $\ell_{30} = 95015$, $\ell_{50} = 89510$ *(iii)* $\ddot{a}_{30:40} = 14.2070$, $\ddot{a}_{40:50} = 12.4785$.
   Calculate $a_{30:40:\overline{10}|}$ *(**Hint**:* $a_{x:y:\overline{n}|} = a_{x:y} - {}_{n}E_{x.y}\ a_{x+n:y+n}$ *and* $a_{x:y} = \ddot{a}_{x:y} - 1$*).*

2. *For a multiple state model of an insurance on* (x) *and* (y)*: (i) The death benefit of* 50000 *is payable at the moment of the second death. (ii) You use the states: State* 0 = *both alive. State* 1 = *only* (x) *is alive. State* 2 = *only* (y) *is alive State* 3 = *neither alive (iii)* $\mu^{03}_{x+t:y+t} = 0$ *(no common shock component) for* $t \geq 0$ *(iv)* $\mu^{13}_{x+t} = \mu^{23}_{y+t} = 0.1$*, for* $t \geq 0$ *(v)* $\delta = 0.04$ *and (vi)* ${}_{t}p^{01}_{xy} = {}_{t}p^{02}_{xy} = 3\left(e^{-0.1t} - e^{-0.12t}\right)$*. Calculate the expected present value of this insurance on* (x) *and* (y)*.*
   *(**Hint**:* $\bar{A}_{\overline{xy}} = \int_0^\infty v^t\left({}_{t}p^{01}_{xy}\ \mu^{13}_{x+t} + {}_{t}p^{02}_{xy}\ \mu^{23}_{y+t}\right)\mathrm{d}t$ *for no common shock component).*

Solution:

1. We use the relation between annuity due and annuity immediate to calculate $a_{30:40:\overline{10}|}$:

$$\begin{aligned}
a_{30:40:\overline{10}|} &= a_{30:40} - {}_{10}E_{30.40}\ a_{40:50} = \ddot{a}_{30:40} - 1 - v^{10}\ {}_{10}p_{30:40}\ \left(\ddot{a}_{40:50} - 1\right) \\
&= \ddot{a}_{30:40} - 1 - v^{10}\ {}_{10}p_{30}\ {}_{10}p_{40}\left(\ddot{a}_{40:50} - 1\right) \\
&= \ddot{a}_{30:40} - 1 - v^{10}\frac{\ell_{50}}{\ell_{30}}\left(\ddot{a}_{40:50} - 1\right) \\
&= 13.2070 - (1.05)^{-10}\frac{89510}{95015} \times 11.4785 = \mathbf{6.56847}.
\end{aligned}$$

2. This is a model with no common shock component. We use

$$\bar{A}_{\overline{xy}} = \int_0^\infty v^t\left({}_{t}p^{01}_{xy}\ \mu^{13}_{x+t} + {}_{t}p^{02}_{xy}\ \mu^{23}_{y+t}\right)\mathrm{d}t,$$

where

$$\begin{aligned}
{}_{t}p^{01}_{xy} &= \int_0^t {}_{s}p^{00}_{xy}\ \mu^{01}_{x+s:y+s}\ {}_{t-s}p^{11}_{x+s}\ \mathrm{d}s = \int_0^t e^{-0.12s} 0.06 e^{-0.1(t-s)}\mathrm{d}s \\
&= 0.06e^{-0.1t}\int_0^t e^{-0.02s}\ \mathrm{d}s = 3e^{-0.1t}\left(1 - e^{-0.02t}\right) = 3e^{-0.1t} - 3e^{-0.12t}.
\end{aligned}$$

Observe that ${}_{t}p^{02}_{xy}$ has the same expression, thus

$$\begin{aligned}
\bar{A}_{\overline{xy}} &= \int_0^\infty e^{-\delta t}\left({}_{t}p^{01}_{xy}\ \mu^{13}_{x+t} + {}_{t}p^{02}_{xy}\ \mu^{23}_{y+t}\right)\mathrm{d}t = \int_0^\infty e^{-0.04t} \times 0.2 \times \left(3e^{-0.1t} - 3e^{-0.12t}\right)\mathrm{d}t \\
&= 0.6\int_0^\infty \left(e^{-0.14t} - e^{-0.16t}\right)\mathrm{d}t = 0.6\left(\frac{1}{0.14} - \frac{1}{0.16}\right) = 0.5357143
\end{aligned}$$

hence the APV of the insurance is $50000\bar{A}_{\overline{xy}} = 50000 \times 0.5357143 = \mathbf{26785.715}$.

Problem 3 *(4 points)*
Consider a 4–year pure endowment policy sold to (x). The policy pays a survival benefit of 20000 at the end of the fourth year if the policyholder is alive at that time. Premiums are paid at the beginning of each year as long as the policyholder is alive. You are given: (i) $v(t)$ is the price of a zero–coupon bond that pays 1 with certainty t years from now. (ii) The following interest rate scenario model:

Scenario ω_i	*Probability*	$v(1)$	$v(2)$	$v(3)$	$v(4)$
1	0.4	0.96	0.92	0.88	0.8
2	0.1	0.94	0.88	0.85	0.75
3	0.3	0.99	0.94	0.89	0.85
4	0.2	0.9	0.8	0.7	0.6

(ii) $\mu_x = 0.02$ *for all* $x \geq 0$.

1. *Find the premium $P(\omega_i)$ for scenario ω_i, $i = 1, 2, 3, 4$. Then calculate the **expected value** of the net annual premium. (**Hint**: ${}_nE_x = v(n)\ {}_np_x$ and $\ddot{a}_{x:\overline{n}|} = \sum_{k=0}^{n-1} v(k)\ {}_kp_x$).*

2. *Let $f(t, t+k)$ be the forward interest rate, contracted at time 0, effective from time t to $t+k$. (**Hint:** $v(t) = \frac{1}{(1+y_t)^t}$ and $(1 + f(t, t+k))^k = \frac{(1+y_{t+k})^{t+k}}{(1+y_t)^t} = \frac{v(t)}{v(t+k)}$ where y_t is the spot rate). Calculate the average of $f(1, 4)$.*

Solution:

1. The net annual premium can be expressed as

$$1000 \frac{{}_4E_x}{\ddot{a}_{x:\overline{4}|}} = \frac{1000v(4)\ {}_4p_x}{1 + v(1)\ p_x + v(2)\ {}_2p_x + v(3)\ {}_3p_x}$$

Since $\mu_x = 0.02$ for all x, we have ${}_tp_x = e^{-0.02t}$. The net annual premium $P(\omega_i)$ for the scenario ω_i are given by:

$$\begin{aligned}
P(\omega_1) &= \frac{20000 \times 0.80 \times e^{-0.08}}{1 + 0.96e^{-0.02} + 0.92e^{-0.04} + 0.88e^{-0.06}} = \frac{14769.86154}{3.65367} = \mathbf{4042.47301} \\
P(\omega_2) &= \frac{20000 \times 0.75 \times e^{-0.08}}{1 + 0.94e^{-0.02} + 0.88e^{-0.04} + 0.85e^{-0.06}} = \frac{13846.74520}{3.56738} = \mathbf{3881.48871} \\
P(\omega_3) &= \frac{20000 \times 0.85 \times e^{-0.08}}{1 + 0.99e^{-0.02} + 0.94e^{-0.04} + 0.89e^{-0.06}} = \frac{15692.97789}{3.71171} = \mathbf{4227.96536} \\
P(\omega_2) &= \frac{20000 \times 0.60 \times e^{-0.08}}{1 + 0.90e^{-0.02} + 0.80e^{-0.04} + 0.70e^{-0.06}} = \frac{11077.39616}{3.31005} = \mathbf{3346.59933}
\end{aligned}$$

The expected value of the net annual premium is given by

$$\begin{aligned}
\mathrm{E}[P] &= 0.4P(\omega_1) + 0.1P(\omega_2) + 0.3P(\omega_3) + 0.2P(\omega_4) \\
&= 0.4 \times 4042.47301 + 0.1 \times 3881.48871 + 0.3 \times 4227.96536 + 0.2 \times 3346.59933 \\
&= \mathbf{3942.84755}.
\end{aligned}$$

2. The missing value is that of $f(1,4)$ for the Scenario 4 and can be found by

$$\begin{aligned}
f(1,4)(\omega_1) &= \left(\frac{v(1)(\omega_1)}{v(4)(\omega_1)}\right)^{\frac{1}{3}} - 1 = \left(\frac{0.96}{0.80}\right)^{\frac{1}{3}} - 1 = 0.06266 \\
f(1,4)(\omega_2) &= \left(\frac{v(1)(\omega_2)}{v(4)(\omega_2)}\right)^{\frac{1}{3}} - 1 = \left(\frac{0.94}{0.75}\right)^{\frac{1}{3}} - 1 = 0.07817 \\
f(1,4)(\omega_3) &= \left(\frac{v(1)(\omega_3)}{v(4)(\omega_3)}\right)^{\frac{1}{3}} - 1 = \left(\frac{0.99}{0.85}\right)^{\frac{1}{3}} - 1 = 0.05214 \\
f(1,4)(\omega_4) &= \left(\frac{v(1)(\omega_4)}{v(4)(\omega_4)}\right)^{\frac{1}{3}} - 1 = \left(\frac{0.90}{0.60}\right)^{\frac{1}{3}} - 1 = 0.14471
\end{aligned}$$

The average of $f(1,4)$ is given by

$$\begin{aligned}
\mathrm{E}[f(1,4)] &= 0.4f(1,4)(\omega_1) + 0.1f(1,4)(\omega_2) + 0.3f(1,4)(\omega_3) + 0.2f(1,4)(\omega_4) \\
&= 0.4 \times 0.06266 + 0.1 \times 0.07817 + 0.3 \times 0.05214 + 0.2 \times 0.14471 = \mathbf{0.077465}.
\end{aligned}$$

Problem 4 ***(6 points)*** *An insurer issues a 5–year fully discrete endowment insurance of* 10000 *to* (60). *The insurer assumes that initial expenses will be* 50, *and renewal expenses, which are incurred at the beginning of the second and subsequent years in which a premium is payable, will be* 5% *of the premium. There is no settlement expense. The insurer holds net premium reserves, using an interest rate of* 4%. *The survival model used to calculate the premium and reserves is*

$$q_{60} = 0.0138,\ q_{61} = 0.0150,\ q_{62} = 0.0164,\ q_{63} = 0.0179,\ \textit{and } q_{64} = 0.0195.$$

The funds invested for the policy are expected to earn interest rate of 5%. *You are given:*
Under $i = 5\%$, $\ddot{a}_{60:\overline{5|}} = 4.25$, *and under* $i = 10\%$, $\ddot{a}_{60:\overline{5|}} = 4.058$.

1. *Suppose the insurer charges the gross premium. Calculate the gross premium for the policy using an interest rate of* 5%.

2. *Calculate all net premium reserves, using an interest rate of* 4%, *net premium of* 1876.7507, *and* ${}_2V = 3771.15787$ *and* ${}_4V = 7952.39709$.

3. *Suppose that the insurer sets a level premium to be found so that the profit margin on the policy is* 15%, *using a risk-adjusted discount rate of* 10%. *Calculate the premium under the following profit test basis:*
   Survival model*: same as that in the reserve basis*
   Expenses*: initial expenses of* 50 *incorporated in the profit at time* 0, *no expenses for the profit during the first policy year, a renewal expense of* 5% *of the premium payable at the beginning of the year for the remaining policy years.*
   Interest on assets*:* 5%. *In the calculation of profits, use net premium reserves obtained in 2 and* ${}_1V(1.05) - {}_2V\ p_{61} = -1783.42871$ *and* ${}_3V(1.05) - {}_4V\ p_{63} = -1714.77045$.

Hints:

$$\mathbf{NPV}(r) = \mathrm{Pr}_0 + \sum_{k=0}^{5-1} \frac{{}_kp_x}{(1+r)^{k+1}}\ \mathrm{Pr}_{k+1} \quad \text{and profit margin} = \frac{\mathbf{NPV}(r)}{\text{Premium } \sum_{k=0}^{4} v^k\ {}_kp_x}$$

$$
\begin{aligned}
\mathrm{Pr}_0 &= -e_0 - c_0 G_0 \\
\mathrm{Pr}_1 &= G(1+i_1) - (b_1 + E_1)\, q_x + {}_0V(1+i_1) - {}_1V\ p_x \\
\mathrm{Pr}_{h+1} &= \left(G_h\,(1-c_h) - e_h\right)(1+i) - (b_{h+1} + E_{h+1})\, q_{x+h} + {}_hV(1+i) - {}_{h+1}V\ p_{x+h} \text{ for } h \geq 1.
\end{aligned}
$$

The table below maybe useful

k	$(1.1)^{-k}$	${}_kp_{60}$	$(1.1)^{-(k+1)}\ {}_kp_{60}$	$(1.1)^{-k}\ {}_kp_{60}$
0	1	1	0.909091	1
1	0.909091	0.986200	0.815041	0.896545
2	0.826446	0.971407	0.729832	0.802816
3	0.751315	0.955476	0.652603	0.717863
4	0.683013	0.938373	0.582656	0.640921
5	0.620921			

<u>Solution</u>:

1. Let G be the gross premium. By the equivalence principle,

$$10000 A_{60:\overline{5}|} + 50 + 0.05G\left(\ddot{a}_{60:\overline{5}|} - 1\right) = G\ddot{a}_{60:\overline{5}|}$$

where $A_{60:\overline{5}|} = 1 - \frac{0.05}{1.05} \times 4.25 = 0.79762$. As a result

$$G = \frac{10000 A_{60:\overline{5}|} + 50}{0.95\ddot{a}_{60:\overline{5}|} + 0.05} = \frac{10000 \times 0.79762 + 50}{0.95 \times 4.25 + 0.05} = \mathbf{1963.59633}$$

2. Starting with ${}_0V = 0$, recursion formula gives

$$
\begin{aligned}
{}_1V &= \frac{({}_0V + P)(1+i) - 10000q_{60}}{1 - q_{60}} = \frac{1876.7507 \times 1.04 - 138}{1 - 0.0138} = 1839.20171 \\
{}_3V &= \frac{({}_2V + P)(1+i) - 10000q_{62}}{1 - q_{62}} = \frac{(3771.15787 + 1876.7507) \times 1.04 - 164}{1 - 0.0164} = 5805.02736
\end{aligned}
$$

Of course ${}_5V = 10000$.

3. Let the annual premium be H. The APV of the premium, discounted using 10% interest, is $H\ddot{a}_{60:\overline{5}|} = 4.058H$. Then we compute the profit vector is

$$
\begin{aligned}
\mathrm{Pr}_0 &= -50 \\
\mathrm{Pr}_1 &= H\,(1.05) - 138 + 0 - 0.9862 \times 1839.20171 \\
&= 1.05H - 1951.8207 \\
\mathrm{Pr}_2 &= (0.95H)\,(1.05) - 150 + 1839.20171\,(1.05) - 0.985 \times 3771.15787 \\
&= 0.9975H - 1933.42871 \\
\mathrm{Pr}_3 &= (0.95H)\,(1.05) - 164 + 3771.15787\,(1.05) - 0.9836 \times 5805.02736 \\
&= 0.9975H - 1914.10915 \\
\mathrm{Pr}_4 &= (0.95H)\,(1.05) - 179 + 5805.02736\,(1.05) - 0.9821 \times 7952.39709 \\
&= 0.9975H - 1893.77045 \\
\mathrm{Pr}_5 &= (0.95H)\,(1.05) + 7952.39709\,(1.05) - 10000 \\
&= 0.9975H - 1649.98306.
\end{aligned}
$$

Now, The NPV of the project is given

$$\begin{aligned}
\mathbf{NPV}(0.1) &= \Pr_0 + \sum_{k=0}^{5-1} v^{k+1}\,{}_kp_x\,\Pr_{k+1} \\
&= -50 + 0.909091\,\Pr_1 + 0.815041\,\Pr_2 \\
&\quad +0.729832\,\Pr_3 + 0.625203\,\Pr_4 + 0.582656\,\Pr_5 \\
&= -50 + 0.909091\,(1.05H - 1951.8207) \\
&\quad +0.815041\,(0.9975H - 1933.42871) \\
&\quad +0.729832\,(0.9975H - 1914.10915) \\
&\quad +0.625203\,(0.9975H - 1893.77045) \\
&\quad +0.582656\,(0.9975H - 1649.98306) \\
&= 3.70039H - 6942.54791
\end{aligned}$$

The profit margin is

$$\text{profit margin} = \frac{\mathbf{NPV}(0.1)}{H\ddot{a}_{60:\overline{5}|}} = \frac{3.70039H - 6942.54791}{4.058H} = 0.15.$$

On solving, we get $H = \mathbf{2245.55111}$.

Problem 5 *(**6 points**) For a universal life insurance with a death benefit of* 50000 *plus account value, you are given:*
(i)

Policy year	*Monthly Premium*	*Percent of Premium Charge*	*Cost of Insurance Rate Per Month*	*Monthly Expense Charge*	*Surrender Charge*
1	500	30%	0.001	5	1500
2	500	10%	0.002	5	1000

(ii) The credited interest rate is $i^{(12)} = 0.048$.
(iii) The actual cash surrender value at the end of month 11 *is* 5000.
(iv) The policy remains in force for months 12 *and* 13, *with the monthly premiums of* 500 *being paid at the start of each month.*

1. *(**2 points**) Find the Cost of Insurance the month* 12 *and* 13.
2. *(**3 points**) Calculate the account values at the end of the months* 11, 12 *and* 13.
3. *(**1 point**) Calculate the cash surrender value at the end of month* 13.
   *(**Hints**: (*$\mathrm{CoI}_t = q_t^* \times v_q \times \mathrm{ADB}_t$ *and*

Additional Death Benefit	
Specified Amount (Type A)	$\mathrm{ADB}_t = \max(\mathrm{FA} - \mathrm{AV}_t, (\gamma_t - 1)\,\mathrm{AV}_t)$
Specified Amount plus the Account Value (Type B)	$\mathrm{ADB}_t = \max(\mathrm{FA}, (\gamma_t - 1)\,\mathrm{AV}_t)$

$\mathrm{CV}_t = \max(\mathrm{AV}_t - \mathrm{SC}_t, 0)$ *and* $\mathrm{AV}_t = (\mathrm{AV}_{t-1} + P_t - \mathrm{EC}_t - \mathrm{CoI}_t)(1 + i_t^c)$*).*

<u>**Solution**</u>:

1. Since the question does not specify another interest rate for calculating the cost of insurance, we should also use a per month rate of 0.004 to calculate the cost of insurance.

$$\mathrm{CoI}_{12} = q^*_{12} \times v_q \times \mathrm{ADB}_{12} = 0.001 \times (1.004)^{-1} \times 50000 = \mathbf{49.80079}.$$

and

$$\mathrm{CoI}_{13} = q^*_{13} \times v_q \times \mathrm{ADB}_{13} = 0.002 \times (1.004)^{-1} \times 50000 = \mathbf{99.60159}.$$

2. This is a specified amount plus account value (Type B) contract. Since the question does not mention anything about corridor factor requirement, we may assume that there is no corridor factor requirement.

   (a) We shall begin with the recursion at the end of month 11. According to statement (iii), the actual cash surrender value at the end of month 11 is 1000. It follows that

   $$\mathrm{CV}_{11} = \max\left(\mathrm{AV}_{11} - \mathrm{SC}_{11}, 0\right) = \max\left(\mathrm{AV}_{11} - 1500, 0\right) = 5000 = \mathrm{AV}_{11} - 1500,$$

   which implies $\mathrm{AV}_{11} = 5000 + 1500 = 6500$.

   (b) We can then project the values of AV_{12} and AV_{13} using the following equation:

   $$\mathrm{AV}_t = \left(\mathrm{AV}_{t-1} + P_t - \mathrm{EC}_t - \mathrm{CoI}_t\right)\left(1 + i^c_t\right).$$

   The interest rate to be credited per month is $i^c_{12} = \frac{0.048}{12} = 0.004$. For month 12, we have $P_{12} = 500$, $\mathrm{EC}_{12} = 500 \times 0.3 + 5 = 155$,

   $$\begin{aligned} \mathrm{AV}_{12} &= \left(\mathrm{AV}_{11} + P_{12} - \mathrm{EC}_{12} - \mathrm{CoI}_{12}\right)\left(1 + i^c_{12}\right) \\ &= (6500 + 500 - 155 - 49.80079)(1.004) = \mathbf{6822.38001}. \end{aligned}$$

   (c) For month 13, we have $P_{13} = 500$, $\mathrm{EC}_{13} = 500 \times 0.1 + 5 = 55$,

   $$\begin{aligned} \mathrm{AV}_{13} &= \left(\mathrm{AV}_{12} + P_{13} - \mathrm{EC}_{13} - \mathrm{CoI}_{13}\right)\left(1 + i^c_{13}\right) \\ &= (6822.38001 + 500 - 55 - 99.60159)(1.004) = \mathbf{7196.44953}. \end{aligned}$$

3. The cash surrender value is given by

$$\mathrm{CV}_{13} = \max\left(\mathrm{AV}_{13} - \mathrm{SC}_{13}, 0\right) = \max(7196.44953 - 1000, 0) = \mathbf{6196.44953}.$$