King Saud University, College of Sciences
Mathematical Department.
Mid-Term 2(M316)/S2/2025 Full Mark:25. Time 1H30min

Question 1[5,4]. a) Find the Fourier series for the 2π-periodic function $f(x) = x \sin x$,and deduce that

$$\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} + ... = \frac{\pi - 2}{4}$$

b) Let $h(x) = \cos x$ be defined on $(0, \pi)$. Find the Fourier cosine series for h and draw the extension of h over $(-4\pi, 4\pi)$.

Question 2[4,5]. a) Show that the change of variable $x = \cos t$ transforms the legendre quation

$$(1 - x^2)y'' - 2xy' + n(n+1)y = 0$$

to the equation

$$(\sin t)y'' + (\cos t)y' + n(n+1)(\sin t)y = 0.$$

b) Consider the legendre polynomials $P_n(x)$, obtain the first four terms of the legendre expansion of the function $f(x) = |x|$, for $x \in [-1, 1]$.

Question 3[7]. If $e^{2xt-t^2} = \sum_{n=0}^{\infty} \frac{1}{n!} H_n(x) t^n$, where $H_n(x)$ are the Hermit polynomials, then show that

$$H_n'(x) = 2nH_{n-1}(x)$$

and

$$H_{2n}(0) = \frac{(-1)^n (2n)!}{n!}, \quad H_{2n+1}(0) = 0 \text{ for all } n \in \mathbb{N}_0.$$