

**King Saud University, College of Sciences**  
**Mathematical Department.**  
**Mid-Term 2/S2/2024 Full Mark:25. Time 1H30mn**

**Question 1[5,4].** a) Obtain the largest interval about  $x_0 = 2$  for which the following initial value problem has a unique solution

$$\begin{cases} (x^2 + 1)y'' + \frac{1}{\sqrt{3-x}}y' + \frac{1}{\sqrt{2x}}y = 0 \\ y(2) = 0, \quad y'(2) = 2. \end{cases}$$

b) Find a linear homogeneous differential equation that has the fundamental set of solutions

$$\{7, \cos 2x, 3x, 8 \sin 2x, 5x^2, 2e^{-x}\}$$

**Question 2[4,4].** a) Solve the initial value problem

$$\begin{cases} y'' - 3y' + 2y = 5x - 3 + 4xe^x \\ y(0) = 0, \quad y'(0) = 0 \end{cases}$$

b) Use the reduction of order method to find the general solution of the differential equation

$$xy'' - y' + 4x^3y = 0, \quad x > 0,$$

provided that  $y = \sin(x^2)$  satisfies the given differential equation

**Question 3[4,4].** a) Solve the differential equation

$$4y'' + 36y = \csc 3x.$$

b) Write down the form of the particular solution  $y_p$  for the differential equation

$$y^{(4)} - 16y = 1 - 3x + 4xe^{-2x} + 7 \cos 3x + 5 \sin 2x.$$