

Mid-Term 2/S1/2018
Full Mark: 25. Time 1H30mn
15/11/2018

Question $1[4,4]$ a) Find the largest interval for which the following initial value problem has a unique solution

$$
\left\{\begin{array}{l}
(x-2) y^{\prime \prime}+3 y=x \\
y(0)=0, y^{\prime}(0)=1 .
\end{array}\right.
$$

b) Solve the nonhomogeneous differential equation

$$
y^{\prime \prime}-y=2 e^{x}-2 x^{2}+5
$$

Question $2[4,3]$. a) If $y_{1}=e^{x}$ is a solution of the differential equation

$$
y^{\prime \prime}+3 y^{\prime}-4 y=x,
$$

then use reduction of order method to obtain its general solution.
b) Determine a homogeneous linear differential equation with constant coefficients having the fundamental set of solutions:

$$
y_{1}=7, \quad y_{2}=8 x, \quad y_{3}=e^{-x} \cos x, \quad y_{4}=e^{-x} \sin x, y_{5}=5 x^{2} .
$$

Question 3 [5] Find the general solution of the differential equation

$$
x y^{\prime \prime}-2 y^{\prime}+\frac{2}{x} y=3 x^{3}+2 x ; \quad x>0 .
$$

Question 4 [5] Solve the following linear system of differential equations.

$$
\left\{\begin{array}{c}
16 x^{\prime \prime}-y=0 \\
y^{\prime \prime}-16 x=32 t
\end{array}\right.
$$

