King Saud University, College of Sciences Mathematical Department.

Mid-Term 2/S1/2018 Full Mark: 25. Time 1H30mn 15/11/2018

Question 1[4,4] a) Find the largest interval for which the following initial value problem has a unique solution

$$\begin{cases} (x-2)y'' + 3y = x\\ y(0) = 0, \ y'(0) = 1. \end{cases}$$

b) Solve the nonhomogeneous differential equation

$$y'' - y = 2e^x - 2x^2 + 5$$

Question 2 [4,3]. a) If $y_1 = e^x$ is a solution of the differential equation

$$y'' + 3y' - 4y = x_y$$

then use reduction of order method to obtain its general solution.

b) Determine a homogeneous linear differential equation with constant coefficients having the fundamental set of solutions:

$$y_1 = 7$$
, $y_2 = 8x$, $y_3 = e^{-x} \cos x$, $y_4 = e^{-x} \sin x$, $y_5 = 5x^2$.

Question 3 [5] Find the general solution of the differential equation

$$xy'' - 2y' + \frac{2}{x}y = 3x^3 + 2x; \quad x > 0.$$

Question 4 [5] Solve the following linear system of differential equations.

$$\begin{cases} 16x'' - y = 0\\ y'' - 16x = 32t \end{cases}$$