King Saud University, Mid-Term Exam.1/S2/2024
College of Sciences Full Mark:25. Time 1H.30
Mathematical Department. 04/03/2024

Question 1. [4,4] **a)** Determine and sketch the largest local region of the $xy-$plane for which the initial value problem

$$\begin{cases} (y-4)\ln y.\frac{dy}{dx} = \sqrt{(x+1)(x-2)} \\ y(3) = 3, \end{cases}$$

has a unique solution.

b) Find the general solution of the differential equation

$$(y\cos x + 2xe^y)dx + (\sin x + x^2e^y - 1)dy = 0$$

Question 2. [4, 4]. **a)** Solve the differential equation

$$x\ln x\frac{dy}{dx} - y = -\frac{1+\ln x}{x}, \;\; x > 1.$$

and hence solve the differential equation.

b) obtain the general solution of the differential equation

$$2xy^{-1}\frac{dy}{dx} + (\sec^2 x + \tan x)y^2 = 1 + x, \quad xy \neq 0, \; x > 0.$$

Question 3. [4, 5]. **a)** Solve the initial value problem

$$\begin{cases} [x\sin(\frac{y}{x}) - y\cos(\frac{y}{x})]dx + x\cos(\frac{y}{x})dy = 0, \quad x \neq 0 \\ y(1) = \frac{\pi}{2}. \end{cases}$$

b) The number of bacteria in a culture grows by 5% in the first 5 hours, and becomes 2000 in 10 hours. If the rate of growth of bacteria is proportional to the number of bacteria at that instant, find the initial number of bacteria.