

**Question 1[5].** Find and sketch the largest local region of the  $xy$ -plane for which the initial value problem

$$\begin{cases} \sqrt{y^2 - 4}dy - (x - y)^2 \ln x dx = 0 \\ y(2) = -3, \end{cases}$$

has a unique solution.

**Question 2[4+4].** a) Solve the initial value problem

$$\begin{cases} \frac{dy}{dx} = \frac{y}{x}(\ln x - \ln y), & x > 0, y > 0. \\ y(1) = 1. \end{cases}$$

b) By using an appropriate substitution, solve the differential equation

$$2xe^{2y} \frac{dy}{dx} = 3x^4 + e^{2y}, \quad x > 0$$

**Question 3[4+4].** a) Solve the differential equation

$$2y + 2x^2y^2 + (x + x^3y) \frac{dy}{dx} = 0, \quad x + x^3y \neq 0.$$

b) Find the general solution of the differential equation

$$(2x^{-1}y^{3/2} - x^{-2}e^x)dx + \sqrt{y}dy = 0, \quad x > 0, \quad y > 0.$$

**Question 4[4].** The population of a town is doubled in 5 years and became 20000 in 10 years. What is the initial population if the rate of growth of population is proportional to the population at that instant.