Question 1. **[5,4]** **a)** Obtain the partial differential equation of lower derivatives by eliminating the arbitrary functions

$$u(x, y) = xf(y + cx) + yg(y + cx).$$

b) Solve the Cauchy problem

$$\begin{cases} \cos y.\frac{\partial u}{\partial x} + \cos x.\frac{\partial u}{\partial y} = \sin^2 x \cos y \\ \qquad\qquad u(y, 0) = 1. \end{cases}$$

Question 2. **[4, 4].** **a)** Discuss the existence of the solution of the partial differential equations

$$y\frac{\partial u}{\partial x} - x\frac{\partial u}{\partial y} = u(x^2 + y^2),$$

near the point $(1, 0, 0)$ situated on the curve $\Gamma = \{x = \cos s, \ y = \sin s, \ z = s\}$.

b) Discuss the existence of the solution of the partial differential equation

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0,$$

under the condition

$$u(0, y, z) = y + z$$

Question 3. **[4, 4]** **a)** Find the solution of the partial differential equation subject to the given condition

$$\begin{cases} x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = x^2 - y \\ \qquad\quad u(1, y) = y. \end{cases}$$

b) Consider the differential equation

$$\frac{\partial^2 u}{\partial x^2} - 9\frac{\partial^2 u}{\partial y^2} = x^2 + \sin y + 2.$$

Classify the differential equation, reduce it to its canonical form and deduce its solution.