جامعة الملك سعود
King Saud University

King Saud University — MATH 132

Department of Mathematics — 2st midterm exam

1st semester 1446 H. — Duration: 75 Minutes

Question Number	I	II	III	IV	V	VI	Total
Mark							
	3	3	4	2	3	5	20

I. Let $R = \{(a,b) : b \geq 2a\}$, and $S = \{(x,y) : x - 2y = 0\}$. Where R and S are defined on $\mathbb{Z}$.

a. Find:

i. $R^\circ S.$ $= \{(x,y) : \exists z \ni : (x,z) \in S \wedge (z,y) \in R\}$

$= \{(x,y) : \exists z \ni : x - 2z = 0 \wedge y \geq 2z\}$

$= \{(x,y) : \exists z \ni : x = 2z \wedge y \geq 2z\}$

$= \{(x,y) : y \geq x\}$

ii. S^{-1}

$S^{-1} = \{(x,y) : (y,x) \in S\} = \{(x,y) : y - 2x = 0\}$

iii. $\bar{R}$

$\bar{R} = \{(x,y) : (x,y) \notin R\} = \{(x,y) : y < 2x\}$

iv. Symmetric closure of S.

$S \cup S^{-1} = \{(x,y) : x = 2y \text{ or } y = 2x\}$

b. Is S an equivalence relation? Justify your answer.

No, because it is not symetric.

$2 S 1$ but $1 \not S 2$.

II. Let T be a partial order relation defined on $A = \{1, 2, 3, 4, 5, 6\}$ defined by $T = \{(a, b): a - b = 2k,$ where k is a nonnegative integer$\}$.

a. List all ordered pairs.

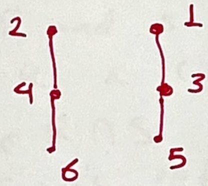
$\{(1,1), (2,2), (3,1), (3,3), (4,2), (4,4)$
$(5,1), (5,3), (5,5), (6,1), (6,2), (6,4)$
$(6,6)\}$.

b. Give an example of comparable elements and not comparable elements.

comparable 2 and 4

not comparable 2 and 3.

c. Drow the Hass diagram of T.



d. Is T a total order relation? Justify your answer.

No, because 2 and 1 are not comparable.

III. Let T be a relation on $\mathbb{Z}^+$ defined by $aTb \Leftrightarrow \frac{a}{b} \in \mathbb{Z}^+$. Is this relation

a. Reflexive?

Yes, because $\forall a \in \mathbb{Z}^+$, $\frac{a}{a} = 1 \in \mathbb{Z}^+ \Rightarrow aTa$.

b. Symmetric?

~~Yes~~ No, $2 T 1$ but $1 \not T 2$

$\frac{2}{1} \in \mathbb{Z}^+$ but $\frac{1}{2} \notin \mathbb{Z}^+$

c. Antisymmetric?

Yes, let $a, b \in \mathbb{Z}^+$ such that $aTb \wedge bTa$

$\Rightarrow \frac{a}{b} = k,\ \frac{b}{a} = t,\ k, t \in \mathbb{Z}^+ \Rightarrow 1 = \frac{a}{b} \cdot \frac{b}{a} = kt$

Since $k, t \in \mathbb{Z}^+$ Then we must have $k = 1, t = 1$

$\Rightarrow a = b$.

d. Transitive? yes,

let $aTb \wedge bTc \Rightarrow \frac{a}{b} \in Z^+ \wedge \frac{b}{c} \in Z^+$

$\Rightarrow \frac{a}{b} = k, \frac{b}{c} = t$ where $k, t \in Z^+$

$\Rightarrow \frac{a}{c} = \frac{a}{b} \cdot \frac{b}{c} = kt \in Z^+ \Rightarrow aTc$.

IV. **Let A be a set, R and S be relations on A. Prove that :**

a. $R \cup R^{-1}$ is symmetric.

let $(a,b) \in R \cup R^{-1}$ for some a,b

$\Rightarrow (a,b) \in R$ or $(a,b) \in R^{-1}$

$\Rightarrow (b,a) \in R^{-1}$ or $(b,a) \in R$

$\Rightarrow (b,a) \in R^{-1} \cup R = R \cup R^{-1}$

$\Rightarrow R \cup R^{-1}$ is symetric.

b. If both R and S reflexive relations on A, then $R \cap S$ is reflexive.

Suppose R and S are reflexive $\forall a$

$\Rightarrow (a,a) \in R \wedge (a,a) \in S$

$\Rightarrow (a,a) \in R \cap S \quad \forall a$

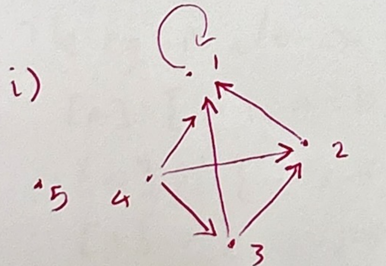
$\Rightarrow R \cap S$ is reflexive.

V. **Let $R = \{(1,1),(2,1),(3,1),(3,2),(4,1),(4,2),(4,3)\}$ be a relation on $A = \{1,2,3,4,5\}$.**

i) **Represent R using a directed graph.**

ii) **Is R reflexive, symmetric, transitive? Justify your answer in each case.**

i)



ii) R is not reflexive because $(2,2) \notin R$

R is not symetric because $(2,1) \in R$ but $(1,2) \notin R$

R is transitive because

$(2,1) \wedge (1,1) \in R \rightarrow (2,1) \in R$

$(3,1) \wedge (1,1) \in R \rightarrow (3,1) \in R$

$(3,2) \wedge (2,1) \in R \rightarrow (3,1) \in R$

$(4,1) \wedge (1,1) \in R \rightarrow (4,1) \in R$

$(4,2) \wedge (2,1) \in R \rightarrow (4,1) \in R$

$(4,3) \wedge (3,2) \in R \rightarrow (4,2) \in R$.

1. **Prove that the relation $\equiv \pmod 5$ is an equivalence relation and find all equivalent classes.**

$\equiv \pmod 5$ is reflexive, symetric and transitive

$a \equiv b \pmod m \Leftrightarrow m \mid a-b \quad \forall a,b \in \mathbb{Z}$

1) reflexive.

$\forall a \in \mathbb{Z} \quad a-a=0 \Rightarrow 5 \mid a-a \Rightarrow a \equiv a \pmod 5$

2) symetric

let $a,b \in \mathbb{Z} \ni: a \equiv b \pmod 5$

$\Rightarrow a-b=5K$ for some $K \in \mathbb{Z}$

$\Rightarrow b-a=5(-K) \Rightarrow b \equiv a \pmod 5$

3.5

3) transitive.

let $a,b,c \in \mathbb{Z} \ni: a \equiv b \pmod 5$ and $b \equiv c \pmod 5$

$\Rightarrow a-b=5K$ and $b-c=5t$ for $K,t \in \mathbb{Z}$

$\Rightarrow a-c=a-b+b-c=5(K+t) \Rightarrow a \equiv c \pmod 5$

From 1, 2 and 3 $\equiv \pmod 5$ is an equivalence relation.

The equivalence classes

$[a]=\{b \mid a \equiv b \pmod 5\}=\{\ldots, a-25, a-5, a, a+5, a+25, \ldots\}$

$[0]=\{\ldots,-10,-5,0,5,10,\ldots\}$

$[1]=\{\ldots,-9,-4,1,6,11,\ldots\}$

$\frac{1.5}{1.5}$

$[2]=\{\ldots,-8,-3,2,7,12,\ldots\}$

$[3]=\{\ldots,-7,-2,3,8,13,\ldots\}$

Good Luck

$[4]=\{\ldots,-6,-1,4,9,14,\ldots\}$