College of Sciences
Department of Statistics & OR

كلية العلوم
قسم الإحصاء وبحوث العمليات

King Saud University
جامعة الملك سعود

Mid Term Exam 1, Academic Year 1442 H, Second semester

Exam Information		معلومات الامتحان	
Course name	Actuarial probability	احتمال اكتواري	اسم المقرر
Course Code	STAT 216	216 احص	رمز المقرر
Exam Date	2021-03-15	1442-08-02	تاريخ الامتحان
Exam Time	07:00PM	07:00 م	وقت الامتحان
Exam Duration	2 hours	02 ساعات	مدة الامتحان
Classroom No.			رقم قاعة الاختبار
Instructor Name			اسم استاذ المقرر

Student Information		معلومات الطالب	
Student's Name			اسم الطالب
ID number			الرقم الجامعي
Section No.			رقم الشعبة
Serial Number			الرقم التسلسلي

General Instructions:
- Your Exam consists of 2 pages (except this paper)
- Keep your mobile and smart watch out of the classroom .

تعليمات عامة:
- عدد صفحات الامتحان 2 صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف والساعات الذكية خارج قاعة الامتحان.

هذا الجزء خاص بأستاذ المادة ***This section is ONLY for instructor***

#	Course Learning Outcomes (CLOs)	Related Question(s)	Points	Final Score
1	General Probability	Q3--Q4	4	
2	Univariate random variables	Q1-Q2-Q3-Q5	3	30

Student's Answers

1	2	3	4	5	6

Exercise 1: An insurance policy pays 1000 per day for up to 2 days of hospitalization and then 500 per day for each day of hospitalization thereafter. The maximum of reimbursement is 4 days of hospitalization. The number of days of hospitalization, X, is a discrete random variable with probability distribution

$$P(X=k)=\begin{cases}\dfrac{5-k}{15}, & if \quad k=0,1,..,4,\\ 0, & if \quad not.\end{cases}$$

Determine the expected payment for hospitalization under this policy.

(A) 1200 (B) 2125 (C) 1700 (D) 1750 (E) None

Exercise 2: An insurance policy reimburses a loss up to a benefit limit of 10 (in thousands riyals). The policyholder's loss, X, follows a distribution with density function:

$$f_X(x)=\begin{cases}\dfrac{3}{x^4}, & if \ x>1\\ 0, & if \ not.\end{cases}$$

The benefit paid under the insurance policy, Y, is given by

$$Y=\begin{cases}X, & if \quad X\le 10\\ 10, & if \quad X>10.\end{cases}$$

What is the expected value of the benefit ?

(A) 2.064 (B) 1.723 (C) None (D) 1.219 (E) 3.257

Exercise 3: In a factory, machines A, B, and C produce the same Item. Of their production, machines A, B, and C produce 4%, 3%, and 1% defective items, respectively. Of the total production in the factory, machine A produces 30%, machine B produces 20%, and machine III produces 50%.

An item is selected at random from the production, compute the probability $P(D)$ that it is defective.

(A) 18/100 (B) None (C) 154/1000 (D) 23/1000 (E) 1.5/100

Exercise 4: An electronic system contains three cooling components that operate independently. The probability of each component's failure is 0.05. The system will overheat if and only if at least two components fail. Calculate the probability that the system will overheat.

(A) 0.9456 (B) None (C) 0.4268 (D) 0.0219 (E) 0.0073

Exercise 5: The time to failure of a component in an electronic device has an exponential distribution with a mean of 4 hours. Calculate the probability that the component will work without failing for at least 5 hours.

(A) None (B) 2/100 (C) $e^{-\frac{5}{4}}$ (D) $1 - e^{-\frac{5}{4}}$ (E) $e^{-\frac{3}{4}} - e^{-\frac{5}{4}}$

Exercise 6: An actuary studied the likelihood that different types of drivers would be involved in at least one collision during any one-year period. The results of the study are presented below.

Type of driver	Percentage of all drivers	Probability of at least one collision
Teen	8%	0.15
Young adult	16%	0.08
Midlife	45%	0.04
Senior	31%	0.05

Given that a driver has been involved in at least one collision in the past year, what is the probability that the driver is a young adult driver?

(A) 0.0629 (B) None (C) 0.1987 (D) 0.2196 (E) 0.2545

Correction

Exercise 1: Define Y to be hospitalization payments made by the insurance policy. Then

$$Y = \begin{cases} 0, & if \;\; X = 0 \\ 1000, & if \;\; X = 1 \\ 2000, & if \;\; X = 2 \\ 2500, & if \;\; X = 3 \\ 3000, & if \;\; X = 4 \end{cases}$$

We deduce $E[Y] = \frac{1000}{15}(0 + 4 + 6 + 5 + 3) = 1200$.

Exercise 2: Since $Y = g(X)$ with $g(x) = \min(x, 10)$, then

$$E[Y] = \int_{-\infty}^{\infty} \min(x, 10)\, f_X(x)\, dx$$

and finally

$$E[Y] = 3\left(\int_1^{10} x\, \frac{1}{x^4}\, dx + \int_{10}^{\infty} \frac{10}{x^4}\, dx\right) = 3\left(\frac{99}{200} + \frac{1}{300}\right) = \frac{299}{200} = 1.495\,.$$

Exercise 3:

P(D|A) = 0.04, P(A) = 0.3; P(D|B) = 0.03, P(B) = 0.2; P (D|C) = 0.01, P (C) = 0.5.
If one item is selected at random from production, the probability that it is defective equals, according to the total probability formula to:

P(D) = P(D|A) P(A) + P(D|B) P(B) + P(D|C) P(C) = 23/1000.

If the selected item is defective, the conditional probability that it was produced by machine A, B, or C can be calculated by

P(A|D) = P(D|A) P(A) / P(D) = 12/23;
P(B|D) = P(D| B) P(B) / P(D) = 6/23;
P(C|D) = P(D|C) P(C) / P(D) = 5/23.

Exercise 4: Let $X =$ of failing components amongst the 3 has $Binomial\ (3, 0.05)$ distribution.

P(system will overheat)$= P(X \geq 2) = \binom{3}{2} 0.05^2\, 0.95^1 + \binom{3}{3} 0.05^3\, 0.95^0 = 0.00725$.

Exercise 5: Let $X =$ time to failure has $Exp(1/4)$ distribution. $P(X > 5) = e^{-\frac{5}{4}}$.

Exercise 6: We have

P(C) = P(C|T) P(T) + P(C|Y) P(Y) + P(C|M) P(M)+P(C|S)P(S) = 0.0583.

Then, P(Y|C) = P(C|Y) P(Y)/ P(C)=0.2196