[Solution Key]

KING SAUD UNIVERSITY
COLLEGE OF SCIENCES
DEPARTMENT OF MATHEMATICS
Semester 461 / MATH-244 (Linear Algebra) / Mid-term Exam 1

Max. Marks: 25 **Max. Time:** $1\frac{1}{2}$ **hrs.**

Question 1: [Marks: 5]

Determine whether the following statements are true or false and justify your answer:

(i) If A and B are symmetric matrices compatible for the product AB, then AB is also symmetric.

False: for example, $A=\begin{bmatrix}1&1\\1&-1\end{bmatrix}, B=\begin{bmatrix}2&3\\3&2\end{bmatrix}$ are symmetric but $AB=\begin{bmatrix}5&5\\-1&1\end{bmatrix}$ is not symmetric.

(ii) If the matrix A^2 is invertible, then A itself is invertible.

True: A is not invertible $\Rightarrow |A|=0 \Rightarrow |A^2|=|A|^2=0 \Rightarrow A^2$ is not invertible.

(iii) If the matrix $\begin{bmatrix}-4&-3&-3\\1&0&1\\4&4&x\end{bmatrix}$ is its own adjoint, then $x=3$.

True: $adj\left(\begin{bmatrix}-4&-3&-3\\1&0&1\\4&4&x\end{bmatrix}\right)=\begin{bmatrix}-4&-3&-3\\1&0&1\\4&4&x\end{bmatrix} \Rightarrow x=c_{33}=(-1)^{3+3}M_{33}=\begin{vmatrix}-4&-3\\1&0\end{vmatrix}=3.$

(iv) If square matrices A and B are compatible for the product AB, then $|AB|=|BA|$.

True: $|AB|=|A||B|=|B||A|=|BA|$.

(v) If $A=\begin{bmatrix}1&2\\-5&1\end{bmatrix}$, then $(-\frac{1}{11},\frac{2}{11})$ is a solution of the equation $A^{-1}=xA+yI_2$.

True: $A^{-1}=\frac{1}{|A|}adj((A)=\frac{1}{11}\begin{bmatrix}1&-2\\5&1\end{bmatrix}=-\frac{1}{11}\begin{bmatrix}1&2\\-5&1\end{bmatrix}+\frac{2}{11}\begin{bmatrix}1&0\\0&1\end{bmatrix}=xA+yI_2.$

Question 2: [Marks: 4 + 3 + 3]

(a) Find the matrix A if $A^{-1}=\frac{1}{2}\begin{bmatrix}1&-1&1\\-1&3&1\\1&1&-2\end{bmatrix}$.

Solution: $A=(A^{-1})^{-1}=2\begin{bmatrix}1&-1&1\\-1&3&1\\1&1&-2\end{bmatrix}^{-1}=\frac{1}{5}\begin{bmatrix}7&1&4\\1&3&2\\4&2&-2\end{bmatrix}$.

(b) Show that $\begin{vmatrix}1&1&1\\b+c&c+a&a+b\\-(b+c-a)&-(c+a-b)&-(a+b-c)\end{vmatrix}=0$.

Solution: $\begin{vmatrix}1&1&1\\b+c&c+a&a+b\\-(b+c-a)&-(c+a-b)&-(a+b-c)\end{vmatrix}=\begin{vmatrix}1&1&1\\b+c&c+a&a+b\\a&b&c\end{vmatrix}$

$=\begin{vmatrix}1&1&1\\a+b+c&b+c+a&c+a+b\\a&b&c\end{vmatrix}=(a+b+c)\begin{vmatrix}1&1&1\\1&1&1\\a&b&c\end{vmatrix}=0.$

(c) Let A be a square matrix of size n with $|A|=3$ and $|adj(A)|=27$. Find n.

Solution: $adj(A)=|A|A^{-1}\Rightarrow |adj(A)|=|A|^{n-1}$. Hence, $3^3=27=3^{n-1}$ gives $n=3+1=4$.

Question 3: [Marks: 3 + 3 + 4]

(a) Solve the matrix equation: $XA=B$, where $A=\begin{bmatrix}1&2&-1\\2&-1&-1\\-4&0&3\end{bmatrix}$ and $B=\begin{bmatrix}-2&1&1\\8&1&-5\\4&3&-3\end{bmatrix}$.

Solution: Since $|A|=-3$, A^{-1} exists. $XA=B\Rightarrow X=BA^{-1}=\begin{bmatrix}-2&1&1\\8&1&-5\\4&3&-3\end{bmatrix}\left(\frac{1}{3}\begin{bmatrix}3&6&3\\2&1&1\\4&8&5\end{bmatrix}\right)=\begin{bmatrix}0&-1&0\\2&3&0\\2&1&0\end{bmatrix}$.

(b) Solve the following system of linear equations by using Cramer's Rule:

$$\begin{aligned}x+3y+3z&=1\\x+3y+5z&=-1\\x+2y-z&=2.\end{aligned}$$

Solution: $|A|=\begin{vmatrix}1&3&3\\1&3&5\\1&2&-1\end{vmatrix}=2$, $|A_x|=\begin{vmatrix}1&3&3\\-1&3&5\\2&2&-1\end{vmatrix}=-10$, $|A_y|=\begin{vmatrix}1&1&3\\1&-1&5\\1&2&-1\end{vmatrix}=6$ and $|A_z|=\begin{vmatrix}1&3&1\\1&3&-1\\1&2&2\end{vmatrix}=-2$.

Hence. $x=\frac{|A_x|}{|A|}=-\frac{10}{2}=-5$, $y=\frac{|A_y|}{|A|}=\frac{6}{2}=3$ and $z=\frac{|A_z|}{|A|}=-\frac{2}{2}=-1$.

(c) Find the value of m for which the following system of linear equations admits a unique solution and then find this uniquely existing solution.

$$\begin{aligned}x+y+z&=1\\x+y+2z&=0\\2x-y-z&=-1\\x-2y+z&=m.\end{aligned}$$

Solution: $[A:B]=\begin{bmatrix}1&1&1&:&1\\1&1&2&:&0\\2&-1&-1&:&-1\\1&-2&1&:&m\end{bmatrix}\sim\begin{bmatrix}1&1&1&:&1\\0&1&1&:&1\\0&0&1&:&-1\\0&0&0&:&m+5\end{bmatrix}$. So, $m=-5$ gives a unique solution $x=0, y=2, z=-1$.

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[Solution Key]

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Semester 461 / MATH-244 (Linear Algebra) / Mid-term Exam 1

Max. Marks: 25 **Max. Time: $1\frac{1}{2}$ hrs.**

Question 1: [Marks: 5]

Determine whether the following statements are true or false:

(i) If A is a square matrix and $A^2 = 0$, then $(I + A)^{-1} = I - A$. [True] [Mark 1]

(ii) If A and B are row equivalent square matrices, then $|A| = |B|$. [False] [Mark 1]

(iii) If $Aadj(A) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then $|A| = 9$. [False] [Mark 1]

(iv) There is a homogeneous linear equation for which (1, 0, 2) is a solution but not (2, 0, −4). [True] [Mark 1]

(v) If $RREF(A)$ has a zero row, then $AX = B$ must have infinitely many solutions. [False] [Mark 1]

Question 2: [Marks: 4 + 3 + 3]

(a) Find inverse of the matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ and find the matrix B satisfying the equation $BA = A^2 + 5A$.

Solution: $[A|I] = \begin{bmatrix} 2 & 0 & 0 & : & 1 & 0 & 0 \\ 0 & 1 & 0 & : & 0 & 1 & 0 \\ 0 & 0 & 3 & : & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & : & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & : & 0 & 1 & 0 \\ 0 & 0 & 1 & : & 0 & 0 & \frac{1}{3} \end{bmatrix}$, and so $A^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$. [Marks 2]

Next, $B = (BA)A^{-1} = (A^2 + 5A)A^{-1} = A + 5I = \begin{bmatrix} 7 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 8 \end{bmatrix}$. [Marks 2]

(b) Let A be an invertible matrix. Show that $(adj(A))^{-1} = adj(A^{-1})$.

Solution: $adj(A) = |A|\, A^{-1} \Longrightarrow (adj(A^{-1}))^{-1} = (|A|^{-1} A)^{-1} = |A|\, A^{-1} = adj(A) \Longrightarrow (adj(A))^{-1} = adj(A^{-1})$.

[Marks 1+1+1]

(c) Show that the matrix $\begin{bmatrix} 1 & 0 & 1+x \\ 1 & 1 & 0 \\ -x & 0 & 1 \end{bmatrix}$ is invertible for all $x\varepsilon\mathbb{R}$, where $\mathbb{R}$ denotes the set of real numbers.

Solution: Since $\begin{vmatrix} 1 & 0 & 1+x \\ 1 & 1 & 0 \\ -x & 0 & 1 \end{vmatrix} = \mathbf{1 + x(x+1) \neq 0}$ **for all** $x\varepsilon\mathbb{R}$, $\begin{bmatrix} 1 & 0 & 1+x \\ 1 & 1 & 0 \\ -x & 0 & 1 \end{bmatrix}$ is invertible for all $x\varepsilon\mathbb{R}$.

[Marks 2+1]

Question 3: [Marks: 2 + 4 + 4]

(a) If E is an elementary matrix, then show that the linear system $EX = O$ has only the trivial solution.

Solution: Since E being an elementary matrix is invertible, $X = E^{-1}(EX) = E^{-1}O = O$; only trivial solution.

[Marks 1+1]

(b) Solve the following system of linear equations:

$$x + y = 1$$
$$x + 2y + z = -1$$
$$x + 3y - z = 2.$$

Solution: Observe that $|A| = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 1 & 3 & -1 \end{vmatrix} = \mathbf{-3} \neq, |A_x| = \begin{vmatrix} 1 & 1 & 0 \\ -1 & 2 & 1 \\ 2 & 3 & -1 \end{vmatrix} = \mathbf{-4}, |A_y| = \begin{vmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \\ 1 & 2 & -1 \end{vmatrix} = \mathbf{1},$

and $|A_z| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & -1 \\ 1 & 3 & 2 \end{vmatrix} = \mathbf{5}$. **So, by Cramer's Rule,** $x = \frac{|A_x|}{|A|} = \frac{4}{3}, y = \frac{|A_y|}{|A|} = \frac{-1}{3}, z = \frac{|A_z|}{|A|} = \frac{-5}{3}$.

[Marks 1+1.5+1.5]

(c) What conditions must a, b, c, and d satisfy for the following system to be consistent?

$$x_1 + x_2 - x_4 = a$$
$$x_2 - x_3 - 2x_4 = b$$
$$2x_1 + 2x_3 + 2x_4 = c$$
$$2x_1 + x_2 + x_3 = d.$$

Solution: Since $[A|I] = \begin{bmatrix} 1 & 1 & 0 & -1 & : & a \\ 0 & 1 & -1 & -2 & : & b \\ 2 & 0 & 2 & 2 & : & c \\ 2 & 1 & 1 & 0 & : & d \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & -1 & : & a \\ 0 & 1 & -1 & -2 & : & b \\ 0 & 0 & 0 & 0 & : & c - 2a + 2b \\ 0 & 0 & 0 & 0 & : & d - 2a + b \end{bmatrix}$, the given system would be consistent for real numbers a, b, c, d satisfying the conditions $c = 2(a - b); d = c + b$. [Marks 2+2]

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[Solution Key]

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Semester 452 / MATH-244 (Linear Algebra) / Mid-term Exam 1

Max. Marks: 25 **Max. Time: $1\frac{1}{2}$ hr**

Question 1: [Marks: 3 + 4 + 3]

(a). Let $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & -2 & 1 \\ 2 & 1 & -2 \end{bmatrix}$. Compute A^2 and then use A^2 to find A^{-1}.

Solution: $A^2 = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} = 9\mathbf{I}$ and so $A^{-1} = \frac{1}{9}\text{A} = \frac{1}{9}\begin{bmatrix} 1 & 2 & 2 \\ 2 & -2 & 1 \\ 2 & 1 & -2 \end{bmatrix}$. (Marks 1 + 2)

(b). Let $A = \begin{bmatrix} 1 & 1 & -1 & -2 \\ 1 & 1 & 0 & -2 \\ -1 & 0 & 1 & 1 \\ -2 & 0 & 0 & 1 \end{bmatrix}$. Find A^{-1} and then use A^{-1} to find $adj(A)$.

Solution: $[A|\text{I}] = \left[\begin{array}{cccc|cccc} 1 & 1 & -1 & -2 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & -2 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ -2 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \sim \dots \sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 1 & 0 & 0 & 3 & -2 & 3 & -1 \\ 0 & 0 & 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & -2 & 2 & -1 \end{array}\right]$ and so $A^{-1} = \begin{bmatrix} 1 & -1 & 1 & -1 \\ 3 & -2 & 3 & -1 \\ -1 & 1 & 0 & 0 \\ 2 & -2 & 2 & -1 \end{bmatrix}$. ((Marks 1.5)

Next, $det(A) = 1$. Hence, $adj(A) = det(A)A^{-1} = \begin{bmatrix} 1 & -1 & 1 & -1 \\ 3 & -2 & 3 & -1 \\ -1 & 1 & 0 & 0 \\ 2 & -2 & 2 & -1 \end{bmatrix}$. (Marks 1 + 1.5)

(c). Let A be a 3×3 matrix with $det(A) = 2$. Evaluate $det(adj(A))$.

Solution: $det(adj(A)) = det(det(A)A^{-1}) = (det(A))^3(\det(A))^{-1} = 4$. (Marks 1 + 1.5 + 0.5)

Question 2: [Marks: 4 + 4]

(a). Let $A = \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}$. Find all matrices $M = \begin{bmatrix} x & y \\ z & t \end{bmatrix}$ such that

Solution: $AM = MA$ implies $\begin{cases} y - z = 0 \\ x - 3z - t = 0 \\ x - 3y - t = 0. \end{cases}$ This system of linear equations has solution set (Mark 1.5)

$\{(3z + t, z, z, t) |\ t, z \in \mathbb{R}\}$. Hence, $M = \begin{bmatrix} 3z + t & z \\ z & t \end{bmatrix}$ for all $t, z \in \mathbb{R}$. (Marks 2 + 0.5)

(b). Find the values of a, b and c so that (1,−2,3) is the solution of following system of linear equations:

$$ax + 2by + cz = 6$$
$$ax + 6by + cz = -2$$
$$3ax + 4by + cz = -8.$$

Solution: Since (1,−2,3) is the solution of the above given system, we get the following system of linear equations:

$$a - 4b + 3c = 6$$
$$a - 12b + 3c = -2$$
$$3a - 8b + 3c = -8.$$

(Marks 2)

Which has the unique solution $a = -5$, $b = 1$ and $c = 5$. (Marks 2)

Question 3: [Marks: 3 + 4]

(a). Use the Gauss-Jordan elimination method to solve the linear system $AX = B$, where:

$$A = \begin{bmatrix} 2 & -1 & -4 & 3 \\ 3 & -2 & -5 & 4 \\ 3 & -3 & -2 & 0 \end{bmatrix},\ X = \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

Solution: $[A|\text{I}] = \left[\begin{array}{cccc|c} 2 & -1 & -4 & 3 & 1 \\ 3 & -2 & -5 & 4 & 1 \\ 3 & -3 & -2 & 0 & 1 \end{array}\right] \sim \dots \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & 4 \\ 0 & 1 & 0 & -5 & 3 \\ 0 & 0 & 1 & -3 & 1 \end{array}\right]$ (RREF). (Marks 1.5)

Hence, solution set of the given linear system is {(4+7t, 3+5t, 1+3t, t) | t ∈ ℝ}. (Marks 1.5)

(b). Find all the non-trivial solutions of the following homogeneous system:

$$2x + 2y + 4z = 0$$
$$\text{w} - y - 3z = 0$$
$$2\text{w} + 3x + \text{y} + z = 0$$
$$-2\text{w} + x + 3\text{y} - 2z = 0.$$

Solution: $[A|\text{I}] = \left[\begin{array}{cccc|c} 0 & 2 & 2 & 4 & 0 \\ 1 & 0 & -1 & -3 & 0 \\ 2 & 3 & 1 & 1 & 0 \\ -2 & 1 & 3 & -2 & 0 \end{array}\right] \sim \dots \sim \left[\begin{array}{cccc|c} 1 & 0 & -1 & -3 & 0 \\ 0 & 1 & 1 & -8 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$. (Marks 1.5)

Hence, the set of non-trivial solutions of the given linear system is {(t, - t, t, 0) | 0 ≠ t ∈ ℝ. (Marks 2 + 0.5)

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Solution Key: Mid-term Exam I of MATH-244 (Linear Algebra) / Semester 451

Question 1: [Marks: 4+2+3]**:**

a) Find the reduced row echelon form of the matrix $A=\begin{bmatrix}1&0&3&0\\-1&2&-3&-1\\0&-2&0&0\\0&0&0&-2\end{bmatrix}$ and use it to find non-trivial solutions of the linear system AX = O, where O = $[0\ \ 0\ \ 0\ \ 0]^{\mathrm{T}}$.

Solution: $A=\begin{bmatrix}1&0&3&0\\-1&2&-3&-1\\0&-2&0&0\\0&0&0&-2\end{bmatrix}\sim\begin{bmatrix}1&0&3&0\\0&2&0&-1\\0&-2&0&0\\0&0&0&-2\end{bmatrix}\sim\begin{bmatrix}1&0&3&0\\0&2&0&-1\\0&1&0&0\\0&0&0&1\end{bmatrix}\sim\begin{bmatrix}1&0&3&0\\0&1&0&0\\0&0&0&1\\0&0&0&0\end{bmatrix}$ (REF). [2 marks]

Hence, $(-3t,0,t,0), \forall\, 0\neq t\in\mathbb{R}$, is a non-trivial solution of the system AX = O. [2 marks]

b) Let B be a 3×3 matrix with $det(B)=2$. Compute $det(B^{-1}+adj(B))$.

Solution: $det(B^{-1}+adj(B)) = det(B^{-1}+det(B)\,B^{-1}) = det(B^{-1}+2B^{-1}) = det(3B^{-1}) = \frac{3^3}{det(B)} = \frac{27}{2}$. [2 marks]

c) Let $P=\begin{bmatrix}1&1&0\\1&2&1\\1&3&-1\end{bmatrix}$. Compute $adj(P)$ and use it to find P^{-1}.

Solution: $adj(P)=c^{\mathrm{T}}=\begin{bmatrix}-5&2&1\\1&-1&-2\\1&-1&1\end{bmatrix}^{\mathrm{T}}=\begin{bmatrix}-5&1&1\\2&-1&-1\\1&-2&1\end{bmatrix}$. $det(P)=\begin{vmatrix}1&1&0\\1&2&1\\1&3&-1\end{vmatrix}=-3$. [1.5+.5 marks]

Hence, $P^{-1}=\frac{1}{\det(P)}adj(P)=\frac{1}{-3}\begin{bmatrix}-5&1&1\\2&-1&-1\\1&-2&1\end{bmatrix}$. [1 mark]

Question 2: [Marks: 2+3+3]**:**

a) Give example of an invertible matrix A with $\mathrm{tr}(A)=0$.

Solution: For any non-zero real number x, the matrix $\boldsymbol{A}=\begin{bmatrix}-x&x\\x&x\end{bmatrix}$ is invertible because [1 mark]

$|A|=-2x^2\neq 0$. However, $\mathrm{tr}(A)=0$. [.5+.5 mark]

b) Find the values of λ for which the matrix $C=\begin{bmatrix}1&1&1\\1&2&\lambda\\1&-1&3-2\lambda\end{bmatrix}$ is not invertible.

Solution: Since $det(C)=0$ for any real value of λ, the matrix C is non-invertible for all $\lambda\in\mathbb{R}$. [1+2 marks]

c) Solve the matrix equation $AZ=X+Y$ for Z, where A is an invertible matrix of size 3,

$X=\begin{bmatrix}-1\\2\\3\end{bmatrix}$, $Y=\begin{bmatrix}5\\0\\-4\end{bmatrix}$, $AX=\frac{1}{3}X$ and $AY=\frac{1}{2}Y$.

Solution: $AX=\frac{1}{3}X$ gives $A^{-1}X=3X=\begin{bmatrix}-3\\6\\9\end{bmatrix}$; similarly, $AY=\frac{1}{2}Y$ gives $A^{-1}Y=\begin{bmatrix}10\\0\\-8\end{bmatrix}$. [2 marks]

Hence, $AZ=X+Y$ implies $Z=A^{-1}X+A^{-1}Y=\begin{bmatrix}-3\\6\\9\end{bmatrix}+\begin{bmatrix}10\\0\\-8\end{bmatrix}=\begin{bmatrix}7\\6\\1\end{bmatrix}$. [1 mark]

Question 3: [Marks: 4+4]

a) Find the values of δ for which the following linear system of equations

$$\begin{aligned}x+y+z+t&=4\\x+\delta y+z+t&=4\\x+y+\delta z+(3-\delta)\,t&=6\\2x+2y+2z+(\delta-5)\,t&=6\end{aligned}$$

has: (i) no solution (ii) infinitely many solutions.

Solution: Augmented matrix of the given system $\begin{bmatrix} 1 & 1 & 1 & 1 & :4 \\ 1 & \delta & 1 & 1 & :4 \\ 1 & 1 & \delta & 3-\delta & :6 \\ 2 & 2 & 2 & \delta-5 & :6 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 1 & : & 4 \\ 0 & \delta-1 & 0 & 0 & : & 0 \\ 0 & 0 & \delta-1 & -5 & : & 0 \\ 0 & 0 & 0 & \delta-7 & : & -2 \end{bmatrix}$. [2 marks]

Hence, the system has no solution if $\delta \in \{1,7\}$; and infinitely many solutions if $\delta \in \{\ \}$. [2 marks]

b) Use Cramer's rule to solve the following linear system of equations:

$$\begin{aligned} x + \ \ y \ \ \ \ \ \ &= \ \ 1 \\ x + 2y + z &= -1 \\ x + 3y - z &= \ \ 2. \end{aligned}$$

Solution: Let A denote the matrix of coefficients. Then, $|A| = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 1 & 3 & -1 \end{vmatrix} = -3$. So, the Cramer's is applicable on the given system. Therefore, [1 mark]

$x = \dfrac{\begin{vmatrix} 1 & 1 & 0 \\ -1 & 2 & 1 \\ 2 & 3 & -1 \end{vmatrix}}{|A|} = \dfrac{-4}{-3} = \dfrac{4}{3}$; $y = \dfrac{\begin{vmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \\ 1 & 2 & -1 \end{vmatrix}}{|A|} = \dfrac{1}{-3} = \dfrac{-1}{3}$ and $z = \dfrac{\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & -1 \\ 1 & 3 & 2 \end{vmatrix}}{|A|} = \dfrac{5}{-3} = \dfrac{-5}{3}$. [1+1+1 marks]