Discrete Mathematics
Chapter 02

Logic


بسم الله الرحمن الرحيم

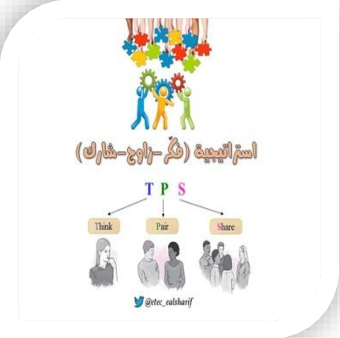
استراتيجيات التعليم

المناقشة

العرض التقدمي

العصف الذهني

التطبيق العملي



فكر – زاوج –شارك

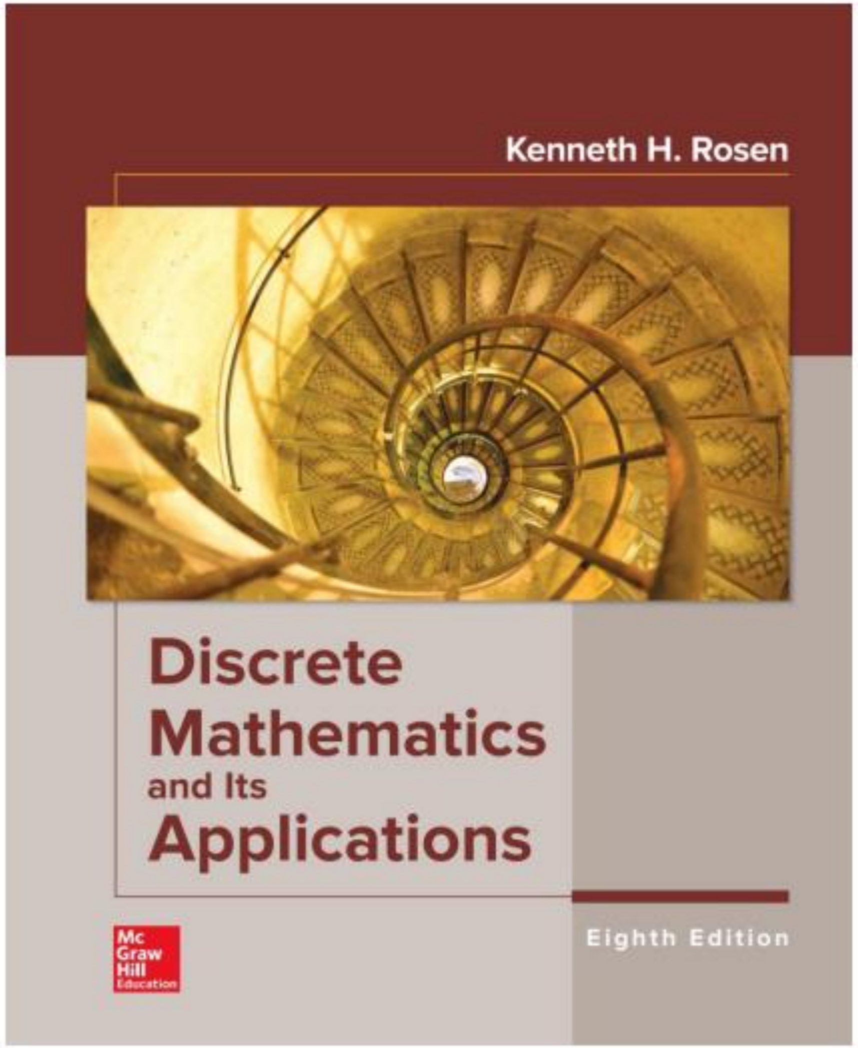
المحاضرة

الاكتشاف

البحث والاستقصاء

- Course code: 153 Math
- Course name: Discrete Mathematics
- Level: 1
- Third Semester 2st Year / B.Sc.
- Course Credit: 3 +2 credits

Lectures Reference



Textbook

2019

Course Outcomes

- Learn how to think mathematically.
- Grasp the basic logical and reasoning mechanisms of mathematical thought.
- Acquire logic and proof as the basics for abstract thinking.
- Improve problem-solving skills.
- Grasp the basic elements of induction, recursion, combination and discrete structures.
- Summarize the basic concepts of mathematical logic.

Content

Week	Basic and support material to be covered
(4)	Logic: Proposition calculus and connectives,
(5)	Logic: Truth tables, Propositional Equivalence.

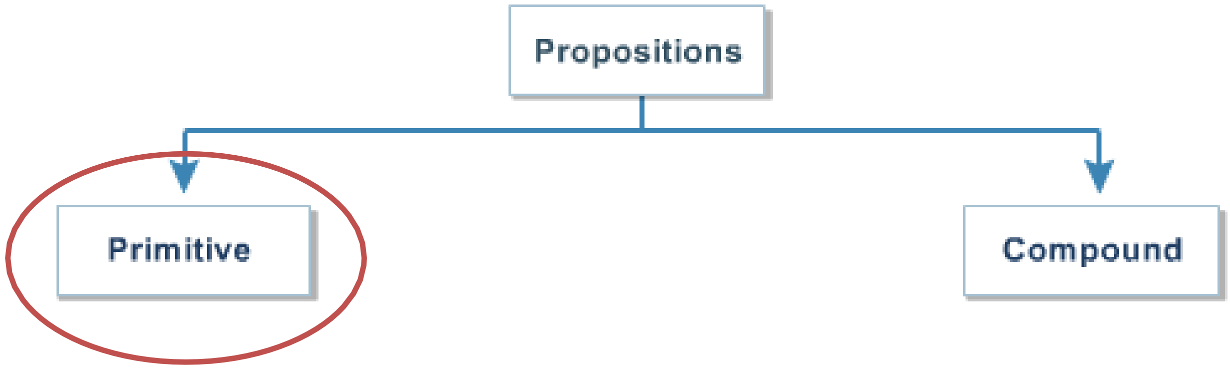
Introduction to Propositional Logic (1/4)

What is Logic?

- Logic is the discipline that deals with the methods of reasoning.
- On an elementary level, logic provides rules and techniques for determining whether a given argument is valid.
- Logical reasoning is used in mathematics to prove theorems.

Introduction to Propositional Logic (2/4)

- The basic building blocks of logic is **Proposition**
- A proposition (or statement) is a **declarative sentence** that is either **true** or **false**, but **not both**.
- The area of logic that deals with propositions is called **propositional logics**.



Introduction to Propositional Logic (3/4)

Examples:

Propositions	Truth value
$2 + 3 = 5$	**True**
$5 - 2 = 1$	**False**
Today is Friday	**False**
for $x = 4x + 3 = 7$,	**True**
Cairo is the capital of Egypt	**True**

Sentences	Is a Proposition
What time is it?	**Not** propositions
Read this carefully.	**Not** propositions
$x + 3 = 7$	**Not** propositions

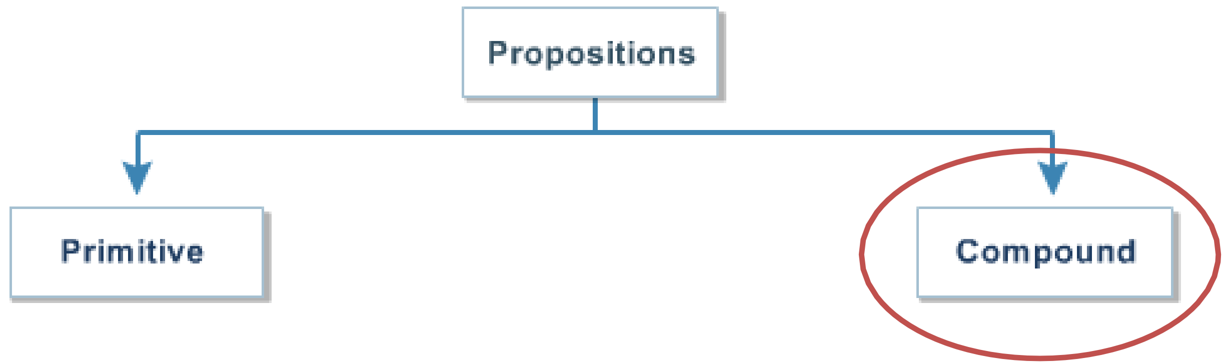
Introduction to Propositional Logic (4/4)

- We use letters to denote propositional variables $\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{r}, \boldsymbol{s}, \ldots$
- The truth value of a proposition is true, denoted by **T**, if it is a true proposition and false, denoted by **F**, if it is a false proposition.

Compound Propositions (1/23)

Compound Proposition

- Compound Propositions are formed from existing propositions using **logical operators**.



Compound Propositions (2/23)

Negation

DEFINITION 1

Let p be a proposition. The *negation of* p, denoted by $\neg p$ (also denoted by $\overline{p}$), is the statement

"It is not the case that p."

The proposition $\neg p$ is read "not p." The truth value of the negation of p, $\neg p$, is the opposite of the truth value of p.

Other notations you might see are $\sim p$, $-p$, p', Np, and $!p$.

Compound Propositions (3/23)

Example

Find the negation of the proposition

p: “Cairo is the capital of Egypt”

P: Riyadh is the capital of Saudi Arabia

Compound Propositions (4/23)

Example: Solution

Find the negation of the proposition

p: “Cairo is the capital of

Egypt” <u>The negation is</u>

It is not the case that

$\neg p$: “It is not the case that Cairo is the capital of Egypt”

<u>This negation can be more simply expressed as</u>

$\neg p$: “Cairo is not the capital of Egypt”

Compound Propositions (5/23)

Truth Table

- Truth Table: is a table that gives the truth values of a compound statement.

The Truth Table for the Negation of a Proposition

	$\neg p$

Proposition

Truth Values

Compound Propositions (5/23)

Truth Table

- Truth Table: is a table that gives the truth values of a compound statement.

The Truth Table for the Negation of a Proposition

	$\neg p$
	F
	T

Proposition

Truth Values

Compound Propositions (6/23)

Negation

TABLE 1 The Truth Table for the Negation of a Proposition.

p	$\neg p$
T	F
F	T

Compound Propositions (7/23)

Logical Connectives

DEFINITION 2

Let p and q be propositions. The *conjunction* of p and q, denoted by $p \wedge q$, is the proposition "p and q." The conjunction $p \wedge q$ is true when both p and q are true and is false otherwise.

Example

p: Today is Friday.

q: It is raining today.

p A q: Today is Friday and it is raining today.

TABLE 2 The Truth Table for the Conjunction of Two Propositions.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Logical Connectives

DEFINITION 3

Let p and q be propositions. The *disjunction* of p and q, denoted by $p \vee q$, is the proposition "p or q." The disjunction $p \vee q$ is false when both p and q are false and is true otherwise.

Example

p: Today is Friday.

q: It is raining today.

$p \vee q$: Today is Friday or it is raining today.

TABLE 3 The Truth Table for the Disjunction of Two Propositions.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Compound Propositions (9/23)

Logical Connectives

DEFINITION 4

Let p and q be propositions. The *exclusive or* of p and q, denoted by $p \oplus q$ (or p XOR q), is the proposition that is true when exactly one of p and q is true and is false otherwise.

Example

They are parents.$p:$

They are children.$q:$

$p \oplus q:$ They are parents or children but not both.

TABLE 4 **The Truth Table for the Exclusive Or of Two Propositions.**

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Compound Propositions (10/23)

Logical Connectives

DEFINITION 5

Let p and q be propositions. The *conditional statement* $p \to q$ is the proposition "if p, then q." The conditional statement $p \to q$ is false when p is true and q is false, and true otherwise. In the conditional statement $p \to q$, p is called the *hypothesis* (or *antecedent* or *premise*) and q is called the *conclusion* (or *consequence*).

"if p, then q"
"if p, q"
"p is sufficient for q"
"q if p"
"q when p"
"a necessary condition for p is q"
"q unless $\neg p$"

TABLE 5 The Truth Table for the Conditional Statement $p \to q$.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

"p implies q"
"p only if q"
"a sufficient condition for q is p"
"q whenever p"
"q is necessary for p"
"q follows from p"

Logical Connectives

DEFINITION 5

Let p and q be propositions. The *conditional statement* $p \rightarrow q$ is the proposition "if p, then q." The conditional statement $p \rightarrow q$ is false when p is true and q is false, and true otherwise. In the conditional statement $p \rightarrow q$, p is called the *hypothesis* (or *antecedent* or *premise*) and q is called the *conclusion* (or *consequence*).

"if p, then q"
"if p, q"
"p is sufficient for q"
"q if p"
"q when p"
"a necessary condition for p is q"
."q unless $\neg p$"

TABLE 5 The Truth Table for the Conditional Statement $p \rightarrow q$.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

"p implies q"
"p only if q"
"a sufficient condition for q is p"
"q whenever p"
"q is necessary for p"
"q follows from p"

Compound Propositions (11/23)

Logical Connectives

EXAMPLE 1

"If you get 100% on the final, then you will get an A."

If you manage to get a 100% on the final, then you would expect to receive an A. If you do not get 100% you may or may not receive an A depending on other factors. However, if you do get 100%, but the professor does not give you an A, you will feel cheated.

Logical Connectives

EXAMPLE 2

Let p be the statement "Maria learns discrete mathematics" and q the statement "Maria will find a good job." Express the statement $p \rightarrow q$ as a statement in English.

Compound Propositions (12/23)

Logical Connectives

EXAMPLE 2

Let p be the statement "Maria learns discrete mathematics" and q the statement "Maria will find a good job." Express the statement $p \to q$ as a statement in English.

"If Maria learns discrete mathematics, then she will find a good job."

"Maria will find a good job when she learns discrete mathematics."

Compound Propositions (13/23)

Logical Connectives

EXAMPLE3

“If today is Friday, then $2 + 3 = 6$.”

Compound Propositions (13/23)

Logical Connectives

EXAMPLE

"If today is Friday, then $2 + 3 = 6$."

is true every day except Friday, even though $2 + 3 = 6$ is false.

Compound Propositions (14/23)

Logical Connectives

DEFINITION 6

Let p and q be propositions. The *biconditional statement* $p \leftrightarrow q$ is the proposition "p if and only if q." The biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise. Biconditional statements are also called *bi-implications*.

"p is necessary and sufficient for q"
"if p then q, and conversely"
"p iff q." "p exactly when q."

TABLE 6 **The Truth Table for the Biconditional $p \leftrightarrow q$.**

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

"You can take the flight if and only if you buy a ticket."

Truth Tables of Compound Propositions

EXAMPLE 1

Construct the truth table of the compound proposition $(p \vee \neg q) \rightarrow (p \wedge q)$.

Truth Tables of Compound Propositions

1

EXAMPLE

Construct the truth table of the compound proposition $(p \vee \neg q) \rightarrow (p \wedge q)$.

TABLE 7 The Truth Table of $(p \vee \neg q) \rightarrow (p \wedge q)$.

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T				
T	F				
F	T				
F	F				

Truth Tables of Compound Propositions

EXAMPLE 1

Construct the truth table of the compound proposition $(p \lor \neg q) \to (p \land q)$.

TABLE 7 The Truth Table of $(p \lor \neg q) \to (p \land q)$.

p	q	$\neg q$	$p \lor \neg q$	$p \land q$	$(p \lor \neg q) \to (p \land q)$
T	T	F			
T	F	T			
F	T	F			
F	F	T			

Truth Tables of Compound Propositions

EXAMPLE 1

Construct the truth table of the compound proposition $(p \vee \neg q) \rightarrow (p \wedge q)$.

TABLE 7 The Truth Table of $(p \vee \neg q) \rightarrow (p \wedge q)$.

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F	T		
T	F	T	T		
F	T	F	F		
F	F	T	T		

Truth Tables of Compound Propositions

EXAMPLE 1

Construct the truth table of the compound proposition $(p \vee \neg q) \rightarrow (p \wedge q)$.

TABLE 7 The Truth Table of $(p \vee \neg q) \rightarrow (p \wedge q)$.

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F	T	T	
T	F	T	T	F	
F	T	F	F	F	
F	F	T	T	F	

Compound Propositions (16/23)

Truth Tables of Compound Propositions

EXAMPLE 1

Construct the truth table of the compound proposition $(p \vee \neg q) \rightarrow (p \wedge q)$.

TABLE 7 The Truth Table of $(p \vee \neg q) \rightarrow (p \wedge q)$.

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

Compound Propositions (17/23)

Precedence of Logical Operators

TABLE 8
Precedence of Logical Operators.

Operator	*Precedence*
$\neg$	1
$\land$	2
$\lor$	3
$\rightarrow$	4
$\leftrightarrow$	5

Truth Tables of Compound Propositions

EXAMPLE 2

Construct the truth table of the compound proposition $(p \wedge \neg q) \rightarrow r$

Truth Tables of Compound Propositions

EXAMPLE 2

Construct the truth table of the compound proposition $(p \wedge \neg q) \to r$

p	q	r	$\neg q$	$p \wedge \neg q$	$(\to rp \wedge) \neg q$

Compound Propositions (19/23)

Truth Tables of Compound Propositions

EXAMPLE 2

Construct the truth table of the compound proposition $(p \wedge \neg q) \rightarrow r$

p	q	r	$q\neg$	$q\neg \wedge p$	$(\rightarrow rp \wedge) \neg q$
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

Truth Tables of Compound Propositions

EXAMPLE 2

Construct the truth table of the compound proposition $(p \wedge \neg q) \rightarrow r$

p	q	r	$\neg q$	$p \wedge \neg q$	$(p \wedge \neg q) \rightarrow r$
T	T	T	F		
T	T	F	F		
T	F	T	T		
T	F	F	T		
F	T	T	F		
F	T	F	F		
F	F	T	T		
F	F	F	T		

Truth Tables of Compound Propositions

EXAMPLE 2

Construct the truth table of the compound proposition $(p \wedge \neg q) \rightarrow r$

p	q	r	$\neg q$	$p \wedge \neg q$	$(\rightarrow rp \wedge) \neg q$
T	T	T	F	F	
T	T	F	F	F	
T	F	T	T	T	
T	F	F	T	T	
F	T	T	F	F	
F	T	F	F	F	
F	F	T	T	F	
F	F	F	T	F	

Compound Propositions (19/23)

Truth Tables of Compound Propositions

EXAMPLE 2

Construct the truth table of the compound proposition $(p \wedge \neg q) \rightarrow r$

p	q	r	$\neg q$	$p \wedge \neg q$	$(p \wedge \neg q) \rightarrow r$
T	T	T	F	F	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	T	F	F	F	T
F	F	T	T	F	T
F	F	F	T	F	T

Compound Propositions (20/23)

Logic and Bit Operations

- Computers represent information using **bits**. A bit is a symbol with two possible values, namely, 0 (zero) and 1 (one).

Truth Value	*Bit*
T	1
F	0

Compound Propositions (21/23)

Computer Bit Operations

- We will also use the notation OR, AND, and XOR for the operators ∨, ∧, and ⊕, as is done in various programming languages.

TABLE 9 Table for the Bit Operators *OR*, *AND*, and *XOR*.

x	y	$x \vee y$	$x \wedge y$	$x \oplus y$
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	0

Compound Propositions (22/23)

Bit Strings

- Information is often represented using bit strings, which are lists of zeros and ones. When this is done, operations on the bit strings can be used to manipulate this information.

A *bit string* is a sequence of zero or more bits. The *length* of this string is the number of bits in the string.

101010011 is a bit string of length nine.

Example

- Find the bitwise OR, bitwise AND, and bitwise XOR of the bit strings 01 1011 0110 and 11 0001 1101

01 1011 0110	
11 0001 1101	
11 1011 1111	bitwise *OR*
01 0001 0100	bitwise *AND*
10 1010 1011	bitwise *XOR*

Revision

Propositional Logic

Introduction

- A **proposition** is a declarative sentence (a sentence that declares a fact) that is either true or false, but not both.
- Are the following sentences propositions?
  - Riyadh is the capital of Saudi Arabia. (Yes)
  - Read this carefully. (No)
  - 1+2=3 (Yes)
  - x+1=2 (No)
  - What time is it? (No)

DEFINITION

Let p be a proposition. The negation of p, denoted by ¬p, is the statement
"It is not the case that p."

The proposition ¬p is read "not p." The truth value of the negation of p, ¬p is the opposite of the truth value of p.

* Example
  * Find the negation of the proposition "Today is Friday." and express this in simple English.

    Solution: The negation is "It is not the case that today is Friday."
    In simple English, "Today is not Friday." or "It is not Friday today."

* Note: Always assume fixed times, fixed places, and particular people unless otherwise noted.
* Truth table:

The Truth Table for the Negation of a Proposition.	
p	$\neg p$
T	F
F	T

* Logical operators are used to form new propositions from two or more existing propositions. The logical operators are also called connectives.

DEFINITION

Let p and q be propositions. The conjunction of p and q, denoted by $p \wedge q$, is the proposition "p and q". The conjunction $p \wedge q$ is true when both p and q are true and is false otherwise.

* Example
  * Find the conjunction of the propositions p and q where p is the proposition "Today is Friday." and q is the proposition "It is raining today.", and the truth value of the conjunction.

    Solution: The conjunction is the proposition "Today is Friday and it is raining today." The proposition is true on rainy Fridays.

DEFINITION

Let p and q be propositions. The disjunction of p and q, denoted by p ν q, is the proposition "p or q". The conjunction p ν q is false when both p and q are false and is true otherwise.

* Note:

  inclusive or : The disjunction is true when at least one of the two propositions is true.

  * E.g. "Students who have taken calculus or computer science can take this class." – those who take one or both classes.

  exclusive or : The disjunction is true only when one of the proposition is true.

  * E.g. "Students who have taken calculus or computer science, but not both, can take this class." – only those who take one of them.

DEFINITION

Let p and q be propositions. The exclusive or of p and q, denoted by p $\oplus$ q, is the proposition that is true when exactly one of p and q is true and is false otherwise.

The Truth Table for the Conjunction of Two Propositions.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

The Truth Table for the Disjunction of Two Propositions.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Conditional Statements

DEFINITION

Let p and q be propositions. The conditional statement $p \rightarrow q$, is the proposition "if p, then q." The conditional statement is false when p is true and q is false, and true otherwise. In the conditional statement $p \rightarrow q$, p is called the hypothesis (or antecedent or premise) and q is called the conclusion (or consequence).

- A conditional statement is also called an implication.
- Example: "If I am elected, then I will lower taxes." $p \rightarrow q$

implication:

	p	q	$p \rightarrow q$
elected, lower taxes.	T	T	T
not elected, lower taxes.	F	T	T
not elected, not lower taxes.	F	F	T
elected, not lower taxes.	T	F	F

* Example:
    * Let p be the statement “Omar learns discrete mathematics.” and q the statement “Omar will find a good job.” Express the statement $p \rightarrow q$ as a statement in English.

Solution: Any of the following -

“If Omar learns discrete mathematics, then he will find a good job.

“Omar will find a good job when he learns discrete mathematics.”

“For Omar to get a good job, it is sufficient for him to learn discrete mathematics.”

“Omar will find a good job unless he does not learn discrete mathematics.”

* Other conditional statements:
  * Converse of $p \rightarrow q: q \rightarrow p$
  * Contrapositive of $p \rightarrow q: \neg q \rightarrow \neg p$
  * Inverse of $p \rightarrow q: \neg p \rightarrow \neg q$

DEFINITION

Let p and q be propositions. The biconditional statement $p \leftrightarrow q$ is the proposition "p if and only if q." The biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise. Biconditional statements are also called bi-implications.

* $p \leftrightarrow q$ has the same truth value as $(p \rightarrow q) \wedge (q \rightarrow p)$
* "if and only if" can be expressed by "iff"
* Example:
  * Let p be the statement "You can take the flight" and let q be the statement "You buy a ticket." Then $p \leftrightarrow q$ is the statement

    "You can take the flight if and only if you buy a ticket."

    Implication:

    If you buy a ticket you can take the flight.

    If you don't buy a ticket you cannot take the flight.

The Truth Table for the Biconditional $p \leftrightarrow q$.		
p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Truth Tables of Compound Propositions

- We can use connectives to build up complicated compound propositions involving any number of propositional variables, then use truth tables to determine the truth value of these compound propositions.
- Example: Construct the truth table of the compound proposition

$$(p \vee \neg q) \longrightarrow (p \wedge q).$$

The Truth Table of $(p \vee \neg q) \rightarrow (p \wedge q)$.

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

Translating English Sentences

* English (and every other human language) is often ambiguous. Translating sentences into compound statements removes the ambiguity.
* Example: How can this English sentence be translated into a logical expression?

“You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old.”

Solution: Let q, r, and s represent “You can ride the roller coaster,” “You are under 4 feet tall,” and “You are older than 16 years old.” The sentence can be translated into:

$$(r \land \neg s) \rightarrow \neg q.$$

* Example: How can this English sentence be translated into a logical expression?

"You can access the Internet from campus only if you are a computer science major or you are not a freshman."

Solution: Let a, c, and f represent "You can access the Internet from campus," "You are a computer science major," and "You are a freshman." The sentence can be translated into:

$$a \rightarrow (c \vee \neg f).$$

2.2 Logic and Bit Operations

- Computers represent information using bits.
- A **bit** is a symbol with two possible values, 0 and 1.
- By convention, 1 represents T (true) and 0 represents F (false).
- A variable is called a Boolean variable if its value is either true or false.
- Bit operation – replace true by 1 and false by 0 in logical operations.

Table for the Bit Operators *OR*, *AND*, and *XOR*.

x	y	$x \vee y$	$x \wedge y$	$x \oplus y$
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	0

DEFINITION

A bit string is a sequence of zero or more bits. The length of this string is the number of bits in the string.

* Example: Find the bitwise OR, bitwise AND, and bitwise XOR of the bit string 01 1011 0110 and 11 0001 1101.

Solution:

01 1011 0110

11 0001 1101

----

11 1011 1111 bitwise OR
01 0001 0100 bitwise AND
10 1010 1011 bitwise XOR

2.3 Logical Equivalences

DEFINITION

The compound propositions p and q are called logically equivalent if p ↔ q is a tautology. The notation p ≡ q denotes that p and q are logically equivalent.

* Compound propositions that have the same truth values in all possible cases are called logically equivalent.
* Example: Show that ¬p ν q and p → q are logically equivalent.

Truth Tables for $\neg p \vee q$ and $p \rightarrow q$.

p	q	$\neg p$	$\neg p \vee q$	$p \rightarrow q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

* Example: Show that $\neg(p \rightarrow q)$ and $p \land \neg q$ are logically equivalent.

Example: Show that $(p \land q) \rightarrow (p \lor q)$ is a tautology.

DEFINITION 2: The compound propositions p and q are called *logically equivalent* if $p \leftrightarrow q$ is a tautology. The notation $p \equiv q$ denotes that p and q are logically equivalent.

TABLE 1 **Examples of a Tautology and a Contradiction.**

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

TABLE 2 **De Morgan's Laws.**

$\neg(p \wedge q) \equiv \neg p \vee \neg q$
$\neg(p \vee q) \equiv \neg p \wedge \neg q$

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

$$p \to q \equiv \neg p \vee q$$
$$p \to q \equiv \neg q \to \neg p$$
$$p \vee q \equiv \neg p \to q$$
$$p \wedge q \equiv \neg(p \to \neg q)$$
$$\neg(p \to q) \equiv p \wedge \neg q$$
$$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$$
$$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$$
$$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$$
$$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$$

$$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$$
$$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$$
$$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$$
$$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$$

Decide whether the following propositions are tautology or a contradiction or a contingency :

1) $(p \wedge q) \rightarrow (\neg p \rightarrow q)$

(By rules " without using the truth tables")

$$(p \wedge q) \rightarrow (\neg p \rightarrow q) \equiv \neg(p \wedge q) \vee (p \vee q)$$ *(Conditional Rule)*

$$\equiv \neg p \vee \neg q \vee p \vee q$$ *(DeMorgan's Rule)*

$$\equiv (\neg p \vee p) \vee (\neg q \vee q)$$ *(Commutative and Associative Rules)*

$$\equiv T \vee T \equiv T$$ *(Negation Rule)*

2) $[\neg p \wedge (p \vee q)] \rightarrow q$

(By rules " without using the truth tables)

$$
\begin{aligned}
[\neg p \wedge (p \vee q)] \rightarrow q &\equiv \neg[\neg p \wedge (p \vee q)] \vee q && \textit{(Conditional Rule)} \\
&\equiv p \vee \neg(p \vee q) \vee q && \textit{(DeMorgan's Rule)} \\
&\equiv (p \vee q) \vee \neg(p \vee q) && \textit{(Commutative and Associative Rules)} \\
&\equiv T && \textit{(Negation Rule)}
\end{aligned}
$$

شكرًا لحسن استماعكم

Thank you