King Saud University

College of Sciences

Department of Mathematics

Final Exam, S1 1445

M 380 – Stochastic Processes

Time: 3 hours – **Marks: 40**

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Answer all questions.

Q1: 8[2+2+2+2]

In an insurance company, the loss amount takes on a uniform distribution with the following density:

$f_x(x) = \frac{1}{4000}$,$0 \le x \le 4000$, assume a deductible of 400\$ applies for every loss.

(a) Calculate the expected loss, X

(b) Calculate the variance and standard deviation of the loss amount

(c) Find the mean amount of the payment made to the insured by the firm

(d) What is the expected amount of the portion of loss not covered by insurance

Q2: 4[2.5 +1.5]

(a) Consider a sequence of items from a production process, with each item being graded as good or defective. Suppose that a good item is followed by another good item with probability 0.4 and is followed by a defective item with probability 0.6. Similarly, a defective item is followed by another defective item with probability 0.3, and is followed by a good item with probability 0.7. If the first item is good, what is the probability that the first defective item to appear is the sixth item?

(b) A Markov chain $\{X_n\}$ on the states 1,2 has the transition probability matrix:
$$P = \left\| \begin{matrix} 0.1 & 0.9 \\ 0.3 & 0.7 \end{matrix} \right\|$$
What is $\Pr(X_5 = 2 | X_3 = 1)$?

Q3: 6[1+5]

Suppose that the weather on any day depends on the weather conditions for the previous 2 days. Suppose also that if it was sunny today but cloudy yesterday, then it will be sunny tomorrow with probability 0.5, if it was cloudy today but sunny yesterday, then it will be sunny tomorrow with probability 0.4, if it was sunny today and yesterday, then it will be sunny tomorrow with probability 0.7, if it was cloudy for the last 2 days, then it will be sunny tomorrow with probability 0.2.

(a) Transform this model into a Markov chain, and then find the transition probability matrix.

(b) Find the long run fraction of days in which it is cloudy, and also the long run fraction of days in which it is sunny.

Q4: 5[2.5+2.5]

(a) Messages arrive at a telegraph office as a Poisson process with mean rate of 2 messages per hour. What is the probability that 1 message arrives during the morning hours 8:00 a.m. to noon?

(b) Suppose that customers arrive at a clinic according to a non-homogenous Poisson process having the rate function:

$$\lambda(t) = \begin{cases} 2t + 1, & 0 \le t \le 1, \\ 3, & 1 \le t < 2, \\ 4t^2, & 2 \le t \le 4, \end{cases}$$

where t is measured in hours from the opening time of the clinic. What is the probability that 5 customers arrive in the first 2 hours?

Q5: 8[4+4]

(a) Let X_n denote the condition of a machine at the end of period n for n=1,2,.. . Let X_0 be the condition of the machine at the start. Consider the condition of the machine at any time can be observed and classified as being in one of the following three states: **State 0:** Good operating order, **State 1:** Damaged operating order and **State 2:** In repair. Assume that $\{X_n\}$ is a Markov chain with transition probability matrix

$$\mathbf{P}= \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{ccc} 0 & 1 & 2 \\ \begin{Vmatrix} 0.6 & 0.3 & 0.1 \\ 0.3 & 0.3 & 0.4 \\ 0.4 & 0.1 & 0.5 \end{Vmatrix} \end{array}$$

and starts in state $X_0 = 1$.
  (i)   Find $\Pr\{X_2 = 1\}$.
  (ii)   Calculate the limiting distribution.
  (iii)   What is the long run rate of repairs per unit time?

(b) A pure death process starting from $X(0) = 3$ has death parameters $\mu_0 = 0$, $\mu_1 = 2$, $\mu_2 = 3$ and $\mu_3 = 5$. Determine $P_n(t)$ for n=0,1,2,3.

Q6: 9[2+3.5+3.5]

For each of the following, let $B(t)$ be a Brownian Motion.

(a) Find the number c for which $\Pr\{B(4) > c|B(0) = 1\} = 0.1587$
(b) Show that the process:

$$X(t) = cB(t/C^2), c > 0,$$

is a Brownian Motion.

(c) For any u, $e^{uB(t)-\frac{u^2}{2}t}$, is a martingale.

NORMAL DISTRIBUTION TABLE

Entries represent the area under the standardized normal distribution from -∞ to z, Pr(Z<z)
The value of z to the first decimal is given in the left column. The second decimal place is given in the top row.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Values of z for selected values of Pr(Z<z)							
z	0.842	1.036	1.282	1.645	1.960	2.326	2.576
Pr(Z<z)	0.800	0.850	0.900	0.950	0.975	0.990	0.995