King Saud University | College of Sciences | Department of Mathematics

Second Examination | Math 132 | Semester 1 (1446) | Time:1H30

Question 1: (12 marks)

1. Let R be the relation on the set $A := \{0,1,2,3,4,5,6,7\}$ defined as follows:

$$\text{for } x, y \in A, \quad (xRy) \iff 2x - y = 4.$$

   (a) List all ordered pairs of R. **(2 marks)**

   (b) Find the domain and the image of R. **(1 marks)**

   (c) Represent the relation R by a matrix. **(1 marks)**

2. Let $S := \{(a,d),(b,a),(b,b),(b,d),(d,a),(d,d)\}$ be a relation on the set $B := \{a,b,c,d\}$.

   (a) Find $S^{-1} \circ S$. **(2 marks)**

   (b) Find $S \circ S^{-1}$. **(2 marks)**

3. Let T be the relation on $\mathbb{Z} - \{0\}$ defined as follows:

$$\text{for } m, n \in \mathbb{Z} - \{0\}, \quad (mTn) \iff \frac{m}{n} > 0.$$

   Determine whether T is reflexive, symmetric, antisymmetric or transitive. (Justify your answers) **(4 marks)**

Question 2: (13 marks)

1. Let E be the relation on the set $\mathbb{Z}$ defined by:

$$\text{for } a, b \in \mathbb{Z}, \quad (aEb) \iff 2|(a^2 + b^2).$$

   (a) Show that the relation E is an equivalence relation on $\mathbb{Z}$. **(3 marks)**

   (b) Show that $[x] = [-x]$, for all $x \in \mathbb{Z}$. **(1 marks)**

   (c) Determine whether $2 \in [-4]$. **(1 marks)**

   (d) Show that $[7] \cap [10] = \emptyset$. **(1 marks)**

2. Let $P := \{(1,1),(1,2),(1,3),(1,5),(2,2),(3,3),(4,2),(4,3),(4,4),(4,5),(5,2),(5,3),(5,5)\}$ be a relation on the set $C := \{1,2,3,4,5\}$.

   (a) Show that P is a partial ordering relation on the set C. **(3 marks)**

   (b) Draw the digraph of P. **(1 marks)**

   (c) Determine whether P is a total order. **(1 marks)**

   (d) Draw the Hasse diagram of P on the set C. **(2 marks)**