King Saud University | College of Sciences | Department of Mathematics

Second Examination | Math 132 | Semester 1 (1446) | Time:1H30

Question 1: (12 marks)

1. Let R be the relation on the set $A := \{0, 1, 2, 3, 4, 5, 6, 7\}$ defined as follows:

$$\text{for } x, y \in A, \quad (xRy) \Longleftrightarrow 2x - y = 4.$$

   (a) List all ordered pairs of R. **(2 marks)**

   (b) Find the domain and the image of R. **(1 marks)**

   (c) Represent the relation R by a matrix. **(1 marks)**

2. Let $S := \{(a, d), (b, a), (b, b), (b, d), (d, a), (d, d)\}$ be a relation on the set $B := \{a, b, c, d\}$.

   (a) Find $S^{-1} \circ S$. **(2 marks)**

   (b) Find $S \circ S^{-1}$. **(2 marks)**

3. Let T be the relation on $\mathbb{Z} - \{0\}$ defined as follows:

$$\text{for } m, n \in \mathbb{Z} - \{0\}, \quad (mTn) \Longleftrightarrow \frac{m}{n} > 0.$$

   Determine whether T is reflexive, symmetric, antisymmetric or transitive. (Justify your answers) **(4 marks)**

Question 2: (13 marks)

1. Let E be the relation on the set $\mathbb{Z}$ defined by:

$$\text{for } a, b \in \mathbb{Z}, \quad (aEb) \Longleftrightarrow 2|(a^2 + b^2).$$

   (a) Show that the relation E is an equivalence relation on ~~$\mathbb{X}$~~ $\mathbb{Z}$. **(3 marks)**

   (b) Show that $[x] = [-x]$, for all $x \in \mathbb{Z}$. **(1 marks)**

   (c) Determine whether $2 \in [-4]$. **(1 marks)**

   (d) Show that $[7] \cap [10] = \emptyset$. **(1 marks)**

2. Let $P := \{(1, 1), (1, 2), (1, 3), (1, 5), (2, 2), (3, 3), (4, 2), (4, 3), (4, 4), (4, 5), (5, 2), (5, 3), (5, 5)\}$ be a relation on the set $C := \{1, 2, 3, 4, 5\}$.

   (a) Show that P is a partial ordering relation on the set C. **(3 marks)**

   (b) Draw the digraph of P. **(1 marks)**

   (c) Determine whether P is a total order. **(1 marks)**

   (d) Draw the Hasse diagram of P on the set C. **(2 marks)** mid 2 (132)

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Q1)

1) a) $R = \{(3,2), (4,4), (5,6), (2,0)\}$.

2

b)

domain$(R) = \{3, 4, 5, 2\}$.

1

image$(R) = \{2, 4, 6, 0\}$.

c).

$$\begin{array}{c|c}
 & \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{matrix} \\
\begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{matrix} &
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\end{array}$$

1

2)

a) $S^{-1} = \{(d,a), (a,b), (b,b), (d,b), (a,d), (d,d)\}$.

2

$S^{-1} \circ S = \{(a,a), (a,b), (a,d), (b,d), (b,b), \cancel{(b,b)}, (b,a), (d,b), (d,d), (d,a)\}$.

$= \{(a,a), (a,b), (a,d), (b,a), (b,b), (b,d), (d,a), (d,b), (d,d)\}$.

b) $S \circ S^{-1} = \{\cancel{(d,d)}, (a,a), (a,d), (b,a), (b,b), (b,d), (d,a), (d,b), (d,d)\}$.

$= \{(a,a), (a,b), (a,d), (a,b), (b,a), (b,b), (b,d), (d,a), (d,b), (d,d)\}$.

2

3)

- $m \in \mathbb{Z} - \{0\}$; $\frac{m}{m} = 1 > 0 \Rightarrow m T m \Rightarrow T$ is reflexive.

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- $m, n \in \mathbb{Z} - \{0\}$; $\frac{m}{n} > 0 \Rightarrow m > 0$ and $n > 0$
  or $m < 0$ and $n < 0$
  $\Rightarrow \frac{n}{m} > 0 \Rightarrow n T m \Rightarrow T$ is symmetric
- T is not antisymmetric.
- T transitive; $m, n, p \in \mathbb{Z} - \{0\}$. $\frac{m}{n} > 0$ $\frac{n}{p} > 0$
  $\Rightarrow \frac{m}{p} > 0 \Rightarrow m T p$
  $\Rightarrow T$ is transitive.

Q2).

1)

a) $\cdot\ a\in\mathbb{Z};\ a^2+a^2=2a^2 \Rightarrow 2\mid(a^2+a^2) \Rightarrow a\,E\,a$

$\Rightarrow E$ is reflexive

$\cdot\ a,b\in\mathbb{Z};\ aEb \Rightarrow 2\mid(a^2+b^2) \Rightarrow 2\mid(b^2+a^2)$

$\Rightarrow bEa \Rightarrow E$ is symmetric

$\cdot\ a,b,c\in\mathbb{Z};\ aEb$ and $bEc \Rightarrow 2\mid(a^2+b^2)$ and $2\mid(b^2+c^2)$

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$\Rightarrow a^2+b^2=2q_1$ and $b^2+c^2=2q_2$

$\Rightarrow a^2+c^2=2\underbrace{(q_1+q_2-b^2)}_{\in\mathbb{Z}}$

$\Rightarrow 2\mid(a^2+c^2)$

$\Rightarrow aEc \Rightarrow E$ is transitive

$\Rightarrow E$ is an equivalence relation.

b) $x\in\mathbb{Z};\ x^2+(-x)^2=2x^2 \Rightarrow 2\mid(x^2+(-x)^2)$

1 $\Rightarrow x\,E\,(-x) \Rightarrow [x]=[-x]$.

c) $\cdot\ 2^2+(-4)^2=20,\ 2\mid 20 \Rightarrow 2\,E\,(-4) \Rightarrow 2\in[-4]$.

1

d) $\cdot\ 7^2+10^2=149 \quad 2\nmid 149 \Rightarrow 7\not{E}\,10$

1 $\Rightarrow [7]\cap 10=\emptyset$.

2) a) $\cdot\ (1,1),(2,2),(3,3),(4,4),(5,5)\in P \Rightarrow P$ is reflexive

$\cdot\ (1,2)\in P,(2,1)\notin P,(1,3)\in P,(3,1)\notin P,(1,5)\in P,(5,1)\notin P,(4,2)\in P,(2,4)\notin P$

$(4,3)\in P,(3,4)\notin P,(4,5)\in P,(5,4)\notin P, \Rightarrow P$ is antisymmetric

3

$\cdot\ (1,5),(5,3)\to(1,3)\in P$

$(1,5)(5,2)\to(1,2)\in P$ $\Big) \Rightarrow P$ is transitive

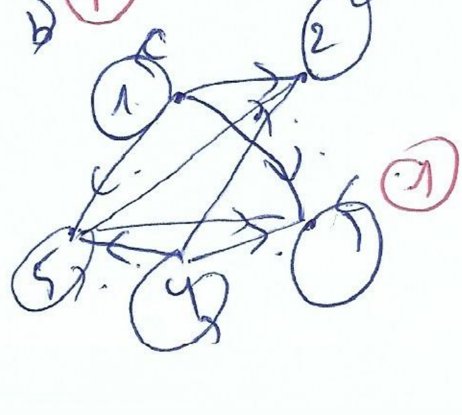
$(4,5)(5,3)\to(4,3)\in P \quad \Rightarrow P$ is a partial ordering relation

$(4,5)(5,2)\to(4,2)$

1

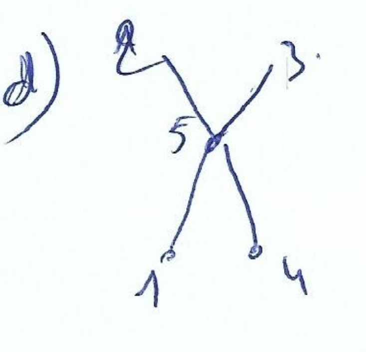
c) No, 1, 4 are incomparable

b)



1

d)



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2)