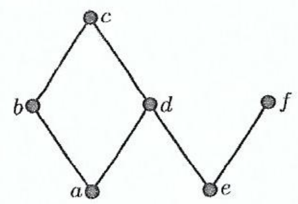
King Saud University	College of Sciences	Department of Mathematics
Final Examination	Math 132	Semester I (1446) Time:3 Hours

Question 1: (8 marks)

1. Without using truth tables, prove that the following conditional statement is a Tautology:
$$[(p \to q) \wedge p] \vee (q \to \neg p). \quad \text{(3 marks)}$$
2. Let $n \in \mathbb{N}$. Show that: if n^2 is odd, then $1-n$ is even. (2 marks)
3. Use mathematical induction to prove the following statement:
$$9+13+17+\cdots+(4n+5) = n(2n+7), \quad \text{for each integer } n, \text{ with } n \geq 1. \quad \text{(3 marks)}$$

Question 2: (15 marks)

1. Let R be the relation on the set $\mathbb{Z}$ defined by
$$a, b \in \mathbb{Z};\ aRb \iff b = -a$$
Decide whether the relation R is reflexive, symmetric, antisymmetric or transitive. Justify your answers. (4 marks)
2. Let E be the relation on the set $\mathbb{Z} - \{0\}$ defined by
$$m, n \in \mathbb{Z} - \{0\};\ mEn \iff 3mn > 0$$
   (a) Show that E is an equivalence relation. (3 marks)
   (b) Find $[1]$ and $[-1]$. (2 marks)
3. Let T be the equivalence relation on $A := \{1, 2, 3, 4, 5, 6\}$ with equivalence classes $\{1, 5, 6\}$, $\{2, 4\}$ and $\{3\}$.
   (a) Draw the digraph of T. (1 marks)
   (b) List all ordered pairs of T. (2 marks)
4. Let P be the partial ordering relation on the set $B := \{a, b, c, d, e, f\}$ represented by the following Hasse diagram.



   (a) List all ordered pairs of P. (2 marks)
   (b) Is P a total order. Justify your answer. (1 mark)

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Q1) 8

1) $[(P\to q)\wedge P]\vee(q\to \neg P)\equiv[(\neg P\vee q)\wedge P]\vee[\neg q\vee\neg P]$.

$\equiv[(\neg P\wedge P)\vee(q\wedge P)]\vee(\neg q\vee \neg P)$ 3

$\equiv[F\vee(P\wedge q)]\vee\neg(P\wedge q)$

$\equiv(P\wedge q)\vee\neg(P\wedge q)\equiv T$.

2) By contraposition, we show that, if ~~1-n~~ is ~~odd~~ then n^2 is even.

we assume that $1-n$ is odd $\Rightarrow 1-n=2k+1;\ k\in\mathbb{Z}$.

$\Rightarrow -n=2k\Rightarrow n=-2k$. 2

$\Rightarrow n^2=(-2k)^2=4k^2=2(2k^2)$ is even.

3) $P(n)$: "$9+13+17+\dots+(4n+5)=n(2n+7)$". ; $n\geq 1$.

B.S: $P(1)$; $9=1(2+7)=9$ true.

I.S. Let $k\geq 1$, we assume that $P(k)$ is true

$\Rightarrow 9+13+17+\dots+(4k+5)=k(2k+7)$.

we prove that $P(k+1)$ is true.

$P(k+1)$: $9+13+17+\dots+(4k+9)=(k+1)(2k+9)$?? $=2k^2+9k+2k+9$ $=2k^2+11k+9$.

$9+13+17+\dots+(4k+5)+(4k+9)=k(2k+7)+(4k+9)$

$=2k^2+7k+4k+9$

$=2k^2+11k+9$. 3

$\Rightarrow P(k+1)$ is true

$\Rightarrow \forall n\geq 1;\ P(n)$ is true.

Q2) 15

1) • $1 \in \mathbb{Z}$; $1 \neq -1 \Rightarrow 1 \not R (1) \Rightarrow R$ is not reflexive.

• $a, b \in \mathbb{Z}$; $aRb \Rightarrow b = -a \Rightarrow a = -b \Rightarrow bRa \Rightarrow R$ is Symmetric.

4 • $1, -1 \in \mathbb{Z}$; $1 = -(-1)$ and $-1 = -(1) \Rightarrow 1 R (-1)$ and $(-1) R 1$

but $1 \neq (-1) \Rightarrow R$ is not antisymmetric.

• $2, (-2) \in \mathbb{Z}$; $2R(-2)$ and $-2 R 2$ but $2 \not R 2$.

$\Rightarrow R$ is not transitive.

2) a) • Let $m \in \mathbb{Z} - \{0\}$; $3.m.m = 3m^2 > 0$

$\Rightarrow mEm \Rightarrow E$ is reflexive

3 • Let $m, n \in \mathbb{Z} - \{0\}$; $mEn \Rightarrow 3m.n > 0 \Rightarrow 3n.m > 0 \Rightarrow nEm$

$\Rightarrow E$ is symmetric.

• Let $m, n, p \in \mathbb{Z} - \{0\}$; mEn and nEp.

$\Rightarrow 3mn > 0$ and $3np > 0$

$\Rightarrow 9mn^2p > 0$

$\Rightarrow 3mp > 0 \Rightarrow mEp \Rightarrow E$ is transitive

$\Rightarrow E$ is an equivalence relation.

b)

$[1] = \{m \in \mathbb{Z} - \{0\}; 3m.1 > 0\}$

$= \{m \in \mathbb{Z} - \{0\}; m.1 > 0\}$. 1

$= \{m \in \mathbb{Z} - \{0\}; m > 0\}$.

$= \{1, 2, 3, 4, \ldots\}$.

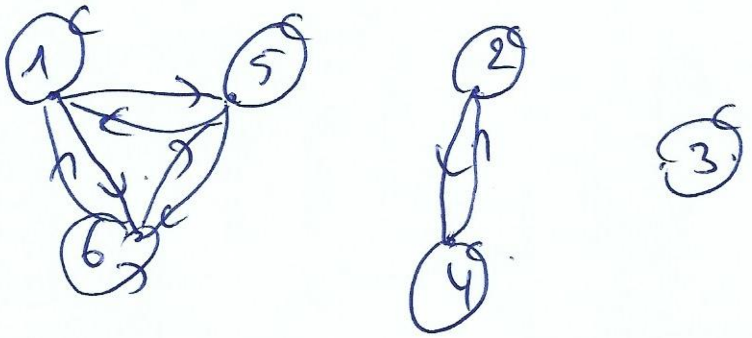
$[-1] = \{m \in \mathbb{Z} - \{0\}; 3m(-1) > 0\}$. 1

$= \{m \in \mathbb{Z} - \{0\}; 3m < 0\}$.

$= \{m \in \mathbb{Z} - \{0\}; m < 0\} = \{\ldots, -3, -2, -1\}$.

2)

3).

a) 

(1)

b) $T = \{(1,1),(1,5),(1,6),(5,1),(5,5),(5,6),(6,1),(6,5),(6,6),(2,2),(2,4),(4,2),(4,4),(3,3)\}$ (2)

4)

a). $P = \{(a,a),(a,b),(a,d),(a,c),(e,e),(e,d),(e,f),(e,c),(b,b),(b,c),(d,d),(d,c),(f,f),(c,c)\}$. (2)

b) No, P is not a total order, a,e are incomparables. (1)

Q3) 14

1) a) $X\times Y=\{(a,0),(a,1),(a,2),(b,0),(b,1),(b,2),(c,0),(c,1),(c,2)\}$.

$(X\times Y)-Z=\{(a,0),(b,0),(c,1),(c,2)\}$. 1

b) $X\cap Y=\phi$.

$(X\cap Y)\times X=\phi\times X=\phi$. 1

c) $\{\phi\}\times Z=\{(\phi,(a,1)),(\phi,(a,2)),(\phi,(b,1)),(\phi,(b,2));(\phi,(c,0))\}$. 1

2) a) $f(\{a,b\})=\{0,4\}$; $f(\{a,c,d\})=\{0,1\}$. 1

b) $f^{-1}(\{0,4\})=\{a,b\}$; $f^{-1}(\{1\})=\{c,d\}$. 1

c) f is not one to one because $f(c)=f(d)$. 1

f is not onto because the is not $x\in C$ s.t. $f(x)=2$. 1

3) a) $goh(x)=g(h(x))=g(3-3x)=2(3-3x)-1$
$=6-6x-1=5-6x$. 1

$hog(x)=h(g(x))=h(2x-1)=3-3(2x-1)=3-6x+3=6-6x$ 1

b). Let $x,y\in\mathbb{R}$; such that $g(x)=g(y)\Rightarrow 2x-1=2y-1$
$\Rightarrow x=y$
$\Rightarrow g$ is one to one.

Let $y\in\mathbb{R}$; $\exists ?\, x\in\mathbb{R}$; s.t. $g(x)=y$.

$g(x)=y\Rightarrow 2x-1=y\Rightarrow x=\frac{y+1}{2}$.

for $y\in\mathbb{R}$, $\exists x=\frac{y+1}{2}\in\mathbb{R}$, s.t. $g(x)=y$
$\Rightarrow g$ is onto. 2

$\Rightarrow g$ is one to one correspondence.

c) $g(x)=y\Rightarrow 2x-1=y\Rightarrow 2x=y+1\Rightarrow x=\frac{y+1}{2}$;
$\forall y\in\mathbb{R}$; $g^{-1}(y)=\frac{y+1}{2}$. 1

d) $hog(x)=6-6x$ is onto and one to one
if $hog(x)=hog(y)\Rightarrow x=y$, $\forall y\in\mathbb{R}$; $\exists x=\frac{6-y}{6}\in\mathbb{R}$; $hog(x)=y$. 2

4)

Q4) 3

1) a) $|A_1| = \aleph_0$ ($A_1 \subseteq \mathbb{Z}$). 1

b) $|A_2| = \aleph_0$ ($A_2 \subseteq \mathbb{Q}^+$). 1

2) $O \subseteq \mathbb{Z}^+$ and $\mathbb{Z}^+$ is countable

$\Rightarrow$ O is countable. 1

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