EXERCISE SHEET #1

MATH 111 and MATH 106

Note: These exercises will be discussed in the tutorial lecture, but you are advised to solve the exercises at the end of each section in the book.

Chapter 4

Section 4.1

In problems $1-7$, find the most general antiderivative:

1. $\int dx$;
2. $\int xdx$
3. $\int adx$, where a is a constant;
4. $\int (ax+b)dx$, where a and b are constants;
5. $\int (1+x+x^2+x^3+x^4)dx$;
6. $\int \frac{x^3+x^2-x}{x^{3/2}}dx$;
7. $\int (2\sin x - 3\cos x)dx$.

In problems $8-12$, the derivative of a function and one point on its graph are given. Find the function:

8. $\frac{dy}{dx} = x^3+x^2-3, \qquad (1,5)$;
9. $\frac{dy}{dx} = 2x(x+1), \qquad (2,0)$;
10. $\frac{dy}{dx} = \sqrt[3]{x} + x - \frac{1}{3\sqrt[3]{x}}, \qquad (-1,-8)$;

11. $\dfrac{dy}{dx} = 13x^{15/18} - 3, \qquad (1, 14);$

12. $\dfrac{dy}{dx} = \cos x, \qquad \left(\dfrac{\pi}{6}, 4\right).$

Section 4.2

In problems $1 - 4$, evaluate the given sums:

1. $\sum_{k=1}^{8} 3^k;$

2. $\sum_{k=0}^{6} 1;$

3. $\sum_{i=2}^{5} \dfrac{i}{i+1};$

4. $\sum_{j=5}^{7} \dfrac{2j+3}{j-2};$

In problems $5 - 12$, write each sum using $\sum$ notation:

5. $1 + 2 + 4 + 8 + 16;$

6. $1 - 3 + 9 - 27 + 81 - 243;$

7. $\dfrac{2}{3} + \dfrac{3}{4} + \dfrac{4}{5} + \dfrac{5}{6} + \cdots + \dfrac{n}{n+1};$

8. $1 - \dfrac{1}{2!} + \dfrac{1}{3!} - \dfrac{1}{4!} + \dfrac{1}{5!} - \dfrac{1}{6!} + \dfrac{1}{7!};$

9. $1 + x^3 + x^6 + x^9 + x^{12} + x^{15} + x^{18} + x^{21};$

10. $-1 + \dfrac{1}{a} - \dfrac{1}{a^2} + \dfrac{1}{a^3} - \dfrac{1}{a^4} + \dfrac{1}{a^5} - \dfrac{1}{a^6} + \dfrac{1}{a^7} - \dfrac{1}{a^8} + \dfrac{1}{a^9};$

11. $\dfrac{1}{32}\left(\dfrac{1}{32}\right)^2 + \dfrac{1}{32}\left(\dfrac{2}{32}\right)^2 + \dfrac{1}{32}\left(\dfrac{3}{32}\right)^2 + \cdots + \dfrac{1}{32}\left(\dfrac{32}{32}\right)^2;$

12. $0.1\sin(0.1) + 0.1\sin(0.2) + 0.1\sin(0.3) + \cdots + 0.1\sin(1)$.

In problems $13 - 15$, three expressions are given, two of them being equal. Identify the one that does not equal the other two:

13. $\sum\limits_{k=0}^{7}(2k+1), \quad \sum\limits_{i=1}^{15} i, \quad \sum\limits_{j=2}^{9}(2j-3);$

14. $\sum\limits_{k=1}^{7} k^2, \quad \sum\limits_{j=0}^{6}(7-j)^2, \quad \sum\limits_{i=1}^{7}(7-i)^2;$

15. $\left(\sum\limits_{k=7}^{11} k\right)^4, \quad \sum\limits_{m=-11}^{-7} m^4, \quad \sum\limits_{n=7}^{11} n^4.$

Section 4.3

In problems $1 - 4$, approximate the area under the curve, on the given interval, using n rectangles and the evaluation rules (a) left endpoints, (b) midpoint,(c) right endpoint:

1. $y = 3x + 2, \quad [0, 3];$
2. $y = x + x^2, \quad [0, 1]$
3. $y = \dfrac{1}{2}x^2, \quad [0, 2];$
4. $y = 3x^3, \quad [-1, 0].$

Section 4.4

In problems $1 - 3$, answer *without computing the integrals*:

1. Explain why $\int\limits_4^6 \frac{x}{x+6}dx \le \int\limits_4^6 \frac{x}{10}dx;$

2. Is $\int\limits_1^2 x dx$ greater or smaller than $\int\limits_1^2 \sqrt{x} dx$?;

3. Show that $\int\limits_0^1 \sqrt{1+x^3} dx$ lies between 1 and $\sqrt{2}$.

In problems $4-9$, find upper and lower bounds for the given integrals:

4. $\int\limits_1^4 4\sqrt{x} dx$;

5. $\int\limits_1^8 7x^{1/3} dx$;

6. $\int\limits_1^9 \frac{1}{\sqrt{x}} dx$;

7. $\int\limits_2^3 (x^2+x^3) dx$;

8. $\int\limits_1^{100} \frac{1}{x} dx$;

9. $\int\limits_0^1 \frac{1}{1+x^2} dx$.

In problems $10-12$, find the value of c that satisfies the conclusion of the Integral Mean Value Theorem on the given interval:

10. $f(x) = 3\sqrt{x+1}$; $[-1,8]$, $\int_{-1}^8 f(x)\, dx = 54$.

11. $f(x) = x^2$, $[1,4]$, $\int_1^4 f(x)\, dx = 21$.

12. $f(x) = 3x^2 - 2x + 3$, $[-1,3]$; $\int_{-1}^3 f(x)\, dx = 32$.

In problems $13-14$, express as one integral:

13. $\int\limits_c^e f(x)\, dx + \int\limits_a^b f(x)\, dx - \int\limits_c^b f(x)\, dx - \int\limits_d^d f(x)\, dx$;

14. $\int_a^d f(x)\,dx - \int_t^b f(x)\,dx - \int_g^g f(x)\,dx - \int_m^d f(x)\,dx + \int_t^a f(x)\,dx.$

Section 4.5

In problems $1-13$, calculate the following integrals:

1. $\int_0^4 7dx;$
2. $\int_1^4 (7t-3)dt;$
3. $\int_0^5 s^3 ds;$
4. $\int_{-1}^1 x^3 dx;$
5. $\int_3^0 (2z+4)dz;$
6. $\int_{-1}^1 |x|dx;$
7. $\int_a^b x^2 dx.$
8. $\int_1^3 (x^3+3x+5)dx;$
9. $\int_a^b (c_1x^2+c_2x+c_3)dx;$
10. $\int_0^1 (1+x^8+x^{16}+x^{32})dx;$

11. $\int_{-a}^{a} x^{2n+1} dx$, where n is a positive integer and a is a real number;

12. $\int_{2}^{3} (x-1)(x+2) dx$;

13. $\int_{0}^{1} (x^{3/2} - x^{2/3})(x^{4/3} - x^{3/4}) dx$.

In problems $14 - 17$, calculate the derivative $F'(x)$. Then, evaluate $F'(x_0)$, for the given x_0:

14. $F(x) = \cos x \int_{3}^{\sin x} \frac{dt}{1+t^3}, \quad x_0 = 0;$

15. $F(x) = \int_{-x^3}^{x} \frac{s^2}{s^2+5} ds, \quad x_0 = 3;$

16. $F(x) = \ln|x| \int_{x^2+1}^{0} \frac{\sqrt{u-1}}{\sqrt{u+1}} du, \quad x_0 = 1;$

17. $F(x) = \int_{2e^x}^{x} \frac{1+2t-3t^2}{t^3+t^{7/9}} dt, \quad x_0 = 1.$

18. Let $f(x) = -\frac{1}{x^2}$ and $F(x) = \frac{1}{x}$. Are the following statements true or false?

    (a) $F'(x) = f(x)$;

    (b) $\int f(x) dx = F(x)$;

    (c) $\int f(x) dx = F(x) + \text{constant}$;

    (d) $\int_{-1}^{1} f(x) dx = F(1) - F(-1)$;

    (e) $\int_{-1}^{1} f(x) dx$ does not exist.

19. Compute $\frac{d}{dx} \int_{3}^{x} (\frac{d}{dt} \cos t) dt$.

20. Prove that $\int_0^x \frac{t}{\sqrt{1+t^2}}dt + \frac{d}{dx}\int_x^{\tan x} \sqrt{1+t^2}dt = \sec^3 x - 1,\ -\frac{\pi}{2} < x < \frac{\pi}{2}$.

In problems 21-22, identify each sum as a Riemannian sum and evaluate the limit.

21. (a) $\lim_{n\to\infty} \frac{1}{n}[\sin\frac{\pi}{n} + \sin\frac{2\pi}{n} + ... + \sin\pi]$
(b) $\lim_{n\to\infty} \frac{1}{n}[\frac{1}{1+\frac{2}{n}} + \frac{1}{1+\frac{4}{n}} + ... + \frac{1}{3}]$

22. (a) $\lim_{n\to\infty} \frac{1}{n}[e^{\frac{4}{n}} + e^{\frac{8}{n}} + ... + e^4]$
(b) $\lim_{n\to\infty} \frac{4}{n}[\frac{2}{\sqrt{n}} + \frac{2\sqrt{2}}{\sqrt{n}} + ... + 2]$

Section 4.6

Evaluate the integrals:

1. $\int \sqrt{2x+1}dx;$

2. $\int \frac{x}{\sqrt{1-4x^2}}dx;$

3. $\int e^{5x}dx;$

4. $\int \sqrt{1+x^2}x^5dx;$

5. $\int\limits_1^2 \frac{dx}{(3-5x)^2};$

6. Suppose that f is continuous on $[-a, a]$.

   (a) Show that, if f is even, then $\int\limits_{-a}^{a} f(x)dx = 2\int\limits_0^a f(x)dx;$

   (b) Show that, if f is odd, then $\int\limits_{-a}^{a} f(x)dx = 0;$

   (c) Evaluate $\int\limits_{-2}^{2} (x^6+1)dx;$

(d) Evaluate $\int\limits_{-1}^{1} \frac{\tan x}{1+x^2+x^4}dx$.

Evaluate the integrals:

7. $\int e^x \sin e^x dx$;

8. $\int x^3(2+x^4)^5 dx$;

9. $\int (x+1)\sqrt{2x+x^2}dx$;

10. $\int \frac{(\ln x)2}{x}dx$;

11. $\int (1+\tan x)^5 \sec^2 x\mathrm{dx}$;

12. $\int \frac{\sin 2x}{1+\cos^2 x}dx$;

13. $\int\limits_{0}^{\pi/2} \cos x \sin(\sin x) dx$;

14. $\int\limits_{0}^{4} \frac{x}{\sqrt{1+2x}}dx$;

15. $\int\limits_{0}^{a} x\sqrt{x^2+a^2}dx \quad (a>0)$.

Section 4.7

In exercises $1-4$, compute Midpoint, Trapezoidal and Simpson's rule approximations by hand, for $n=4$:

1. $\int\limits_{0}^{1} (x^3+x)dx$;

2. $\int\limits_{1}^{3} \frac{1}{x}dx$;

3. $\int_{0}^{1} 5x^4 dx$;

4. $\int_{0}^{1} \cos x dx$.

Section 4.8

In exercises $1-4$, solve each equation for x:

1. $2\ln x = 1$;
2. $\ln(5-2x) = -3$;
3. $\ln x + \ln(x-1) = 1$;
4. $\ln(\ln x) = 1$.
5. $4xe^{-x^2} = |x|$.

In exercises $6-8$, find the exact value of each expression:

6. $\ln \frac{1}{e}$;
7. $\ln(\ln e^{e^{10}})$;
8. $e^{-2\ln 5}$.

In exercises $9-11$, express the given quantity as a single logarithm:

9. $\log 5 + 5\log 3$;
10. $\ln(a+b) + \ln(a-b) - 2\ln c$;

11. $\ln(1+x^2)+\dfrac{1}{2}\ln x-\ln\sin x$.

In exercises $12-15$, evaluate the integrals:

12. $\int\limits_3^{10}\dfrac{x}{x^2-4}dx$;

13. $\int\left(\dfrac{1-x}{x}\right)^2 dx$;

14. $\int\dfrac{t^2}{1+t^3}dt$;

15. $\int\dfrac{e^{\sqrt{x}}}{\sqrt{x}}dx$;

16. $\int\dfrac{1}{\sqrt{x}(\sqrt{x}+1)}dx$.

In exercises $17-19$, evaluate the derivative:

17. $\dfrac{d}{dx}\log\left(\dfrac{2x^3}{x^4+2}\right)$;

18. $\dfrac{d}{dx}\ln\sqrt{x^4+3}$;

19. $\dfrac{d}{dx}\ln(x(x+1)^{7/2}\sin^2 3x)$;

20. $\dfrac{d}{dx}\{5^{3x-4}+\log_3\left|\frac{1-x^2}{2-4x^3}\right|\}$.

Hyperbolic and Inverse Hyperbolic functions

Evaluate the integrals;

1. $\int \sinh 4x dx$;
2. $\int \operatorname{csch} 3x \coth 3x dx$;
3. $\int x^2 \cosh(x^3 + 4) dx$;
4. $\int \cosh 2x \sinh 3x dx$;
5. $\int \cosh^2 3x dx$;
6. $\int \operatorname{sech}^2 7x dx$;
7. $\int e^2 \sinh 3x dx$;
8. $\int \operatorname{sech} x dx$;
9. $\int_3^4 \frac{1}{\sqrt{x^2 - 4}} dx$;
10. $\int_1^2 \frac{1}{\sqrt{x^2 + 2x}} dx$;
11. $\int_0^1 \frac{1}{\sqrt{x^2 + 2x + 5}} dx$;
12. $\int_{-1}^1 \frac{1}{\sqrt{x^2 + 6x + 8}} dx$;
13. $\int_4^5 \frac{x + 1}{\sqrt{x^2 - 9}} dx$;
14. $\int_1^2 \frac{1}{\sqrt{x^2 + x}} dx$;
15. $\int_{-1}^0 \frac{1}{\sqrt{2x^2 + 4x + 7}} dx$;

16. $\int\limits_1^2 \frac{1}{x\sqrt{4-x^2}}dx.$

17. $\int \tanh^2 x dx;$

18. $\int \sinh^4 x dx;$

In problems $20-27$ find $f'(x)$,

19. $f(x) = \sqrt{x}\tanh\sqrt{x};$

20. $f(x) = \frac{\coth x}{\cot x};$

21. $f(x) = \sinh^2(\cos x);$

22. $f(x) = \sinh^{-1} e^x;$

23. $f(x) = \text{sech}^{-1}\sqrt{1-x};$

24. $f(x) = \sqrt{\cosh^{-1} x};$

25. $f(x) = \tanh^{-1}(x^2-1);$

26. $f(x) = \coth^{-1}\left(\frac{x}{x+1}\right).$

In problems 28−35, verify the formula:

27. $\sinh 2x = 2\sinh x\cosh x;$

28. $\cosh 2x = \cosh^2 x + \sinh^2 x$

29. $\sinh(x+y) = \sinh x\cosh y + \cosh x\sinh y;$

30. $\cosh(x+y) = \cosh x\cosh y + \sinh x\sinh y;$

31. $\tanh(x+y) = \frac{\tanh x+\tanh y}{1+\tanh x\tanh y};$

32. $\cosh x + \sinh x = e^x;$

33. $(\cosh x + \sinh x)^n = \cosh nx + \sinh nx$,for every positive integer n;

34. $(\cosh x - \sinh x)^n = \cosh nx - \sinh nx$,for every positive integer n.

Chapter 6:

Calculus, Early Transcendental Functions pages 509-560

6.1: 7,12,14,20,22,29,30,32,35,39.

6.2: 15,18,28,30,33,36,39,40.

6.3: 15,17,20,21,24,26,29,30,33.

6.4: 4,7,8,17,21,24,28,30,32,37.

6.6: 6,8,11,16,26,32,49,52.