# FINAL EXAMINATION, SEMESTER II, 2025

Dept. Math., College of Science, KSU

Math: 107 Full Mark: 40 Time: 3 Hours

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**Q1**. [4+4+4=12]

(a) Find the real numbers $x$, $y$, and $z$ such that the matrix

$$A = \begin{bmatrix} 2 & x-y+z & x+y+z \\ 0 & 5 & 0 \\ 0 & 4x+2y+z & 7 \end{bmatrix}$$

is symmetric.

(b) Determine the invertibility of the matrix

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

and then find the matrix $B$ satisfying the equation $BA = A^2 + 3A$.

(c) Let

$$\begin{bmatrix} 3 & 1 & \lambda^2-6 & \lambda-3 \\ 1 & 2 & 1 & 0 \\ 2 & 3 & 1 & -1 \end{bmatrix}$$

be the augmented matrix of a system of linear equations. Find the value of $\lambda$ that makes the linear system inconsistent.

**Q2**. [4+4+4=12]

Let $A(1,1,0)$, $B(-1,2,1)$, $C(0,2,-1)$, and $D(3,1,1)$ be four points in the space.

(a) Find the equation of the plane passing through $A$, $B$, and $C$.

(b) Find the area of the triangle $ABC$.

(c) Find the volume of the box $ABCD$.

**Q3.** [4+4+4=12]

(a) Let $C$ be the curve with parametric equations: $x = \sin t + \cos t$, $y = (\sin t)e^t$, $z = \cos t$. Find parametric equations for the tangent line to $C$ at $P(1,0,1)$.

(b) If $\mathbf{r}(t) = \langle (t-1)^2, 2t, \ln t\rangle$, $(\frac{1}{2} \le t \le 2)$ is the position vector of a moving point $P$, then find its velocity, speed and acceleration at $t = 1$.

(c) Find the tangential and normal components of acceleration for the position vector given by $\mathbf{r}(t) = \langle \cos t, \sin t, e^{-t}\rangle$ at $t = 0$.

**Q4**. [2+2+3+4+3=14]

(a) If $\omega = e^{-x}\cos y + e^{-y}\cos x$, then show that $\frac{\partial^2\omega}{\partial x^2} + \frac{\partial^2\omega}{\partial y^2} = 0$.

(b) If $u = \ln(x+y)$ with $x = e^{-2t}$ and $y = t^3 - t^2 + 5$, use chain rule to find $\frac{du}{dt}$.

(c) Find the directional derivative of $f(x,y) = x^2 - 5xy + 3y^2$ at the point $P(3,-1)$ in the direction of the vector $\mathbf{a} = \mathbf{i} + \mathbf{j}$.

(d) If $f(x,y) = x^3 + 3xy - y^3$, find the local extrema and saddle points of $f$.

(e) Use Lagrange multipliers to find the minimum value of $f$ where $f(x,y,z) = x^2 + y^2 + z^2$ subject to the constraint $x - y + z = 1$.