FINAL EXAMINATION, SEMESTER I, 2025-26

Dept. Math., College of Science, KSU

Math: 107 Full Mark: 40 Time: 3 Hours

Note: You are NOT allowed to use calculator

Q1. [4+3+2=9] (a) (i) Find inverse of the matrix A by the elementary matrix method

$$A = \begin{bmatrix} 3 & -1 & 2 \\ 2 & -3 & 1 \\ 1 & -2 & 1 \end{bmatrix}$$

(ii) Using the inverse of A, solve the following system of linear equations:

$$3x - y + 2z = 0$$
$$2x - 3y + z = 1$$
$$x - 2y + z = 2.$$

(iii) Using A^{-1} (as you obtained in (i)), find $adj(A)$ (adjoint of the matrix A.)

(b) If

$$B = \begin{bmatrix} \sqrt{5}\lambda - 1 & -2 \\ -1 & \sqrt{5}\lambda + 1 \end{bmatrix},$$

find all values of λ such that $\det(B) = \det\left(\begin{bmatrix} -2 \\ \lambda \end{bmatrix} \begin{bmatrix} -1 & \lambda^2 \end{bmatrix}\right)$.

(c) Use Cramer's Rule to solve the system only for c: $a+b+c=6$ $a+b+2c=9$ $2a+b+c=7$.

Q2. [2+2+3=7] (a) Find the volume of the parallelepiped (box) having $\mathbf{a} = 3\mathbf{i} - 5\mathbf{j} + \mathbf{k}$, $\mathbf{b} = 2\mathbf{j} - 2\mathbf{k}$, and $\mathbf{c} = 3\mathbf{i} + \mathbf{j} + \mathbf{k}$ as adjacent edges.

(b) Find the distance between the two planes given by $3x - y + 2z = 6$ and $6x - 2y + 4z = -4$.

(c) Find the equation of the plane containing the points $P(2,1,1)$, $Q(0,4,1)$, and $R(-2,1,4)$.

Q3. [3+3+3=9] (a) Describe the curve defined by the vector valued function $\mathbf{r}(t) = \langle 1, 2\cos t, 2\sin t\rangle$; what is its domain? Find its curvature at $t = -\pi$.

(b) Find parametric equations for the tangent line to the curve $C: x = 8t + 2$, $y = 2t^3 - 1$, $z = -5t^2 + 3$ at the point $(10, 1, -2)$.

(c) Find the tangential and normal components of acceleration of a moving particle with position vector given by $\mathbf{r}(t) = 3t\mathbf{i} - t\mathbf{j} + t^2\mathbf{k}$.

Q4. [3+3+3+3+3=15] (a) If $w = f(x,y)$, where $x = r\cos\theta$, $y = r\sin\theta$, show that $(\frac{\partial w}{\partial x})^2 + (\frac{\partial w}{\partial y})^2 = (\frac{\partial w}{\partial r})^2 + \frac{1}{r^2}(\frac{\partial w}{\partial \theta})^2$

(b) Let $f(x,y) = x^3y^2$. Find the directional derivatives of f at the point $P(-1,2)$ in the direction of the vector $\mathbf{a} = 4\mathbf{i} - 3\mathbf{j}$.

(c) Find the equations of the tangent plane and the normal line to the surface $z = \sin x \cos y$ at $(\frac{\pi}{2}, \pi, -1)$.

(d) If $f(x,y) = -x^3 + 4xy - 2y^2 + 1$, find the local extrema and saddle points of f, if any.

(e) Use Lagrange multiplier to find the minimum value of $f(x,y,z) = 2x^2 + y^2 + 3z^2$ subject to the constraint $2x - 3y - 4z = 49$.