College of Science.
Department of Mathematics

كلية العلوم
قسم الرياضيات

Final Exam
Academic Year 1441-1442 Hijri- First Semester

Exam Information معلومات الامتحان			
Course name	**Geometry and Topology**		**اسم المقرر**
Course Code	**Math570-1**		**رمز المقرر**
Exam Date	**2020-12-29**	**1442-05-14**	**تاريخ الامتحان**
Exam Time	**08: 00 AM**		**وقت الامتحان**
Exam Duration	**3 hours**	**ثلاث ساعات**	**مدة الامتحان**
Classroom No.	**G013**		**رقم قاعة الاختبار**
Instructor Name	**Norah AlSheheri**		**اسم استاذ المقرر**

Student Information معلومات الطالب		
Student's Name		**اسم الطالب**
ID number		**الرقم الجامعي**
Section No.		**رقم الشعبة**
Serial Number		**الرقم التسلسلي**

General Instructions:

- Your Exam consists of 10 PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
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تعليمات عامة:

- عدد صفحات الامتحان 10 صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف والساعات الذكية خارج قاعة الامتحان.
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هذا الجزء خاص بأستاذ المادة
This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1				
2				
3				
4				
5				
6				
7				
8				

EXAM COVER PAGE

Q1: Prove or disprove the following:

I. Let $X = \{(x, y): y = 0\} \cup \left\{(x, y): y = \frac{1}{x}, x > 0\right\}$ be a subset of $\mathbb{R}^2$ with standard topology. X is not connected.
   - o True
   - o False

II. Any subspace of a path connected space is path connected.
   - o True
   - o False

III. The component of any space is closed.

- o True
- o False

IV. If a space X has a basis consisting of connected subsets, then X is locally connected.

- o True
- o False

V. Any locally path connected space is path connected.

- o True
- o False

VI. Let $A = (\mathbb{R} \times 0) \cup ([0, \infty) \times \mathbb{R})$ be a subset of $\mathbb{R}^2$ with standard topology, and $f: A \to \mathbb{R}$ be a function defined by $f(x, y) = x$. f is a quotient function but neither open nor closed.

- o True
- o False

VII. Let X be a Hausdorff topological space and X^* is the partition of X into disjoint subsets whose union is X. Then X^* is Hausdorff.

- o True
- o False

VIII. $\mathbb{R}$ with discrete topology satisfies the first countability axiom but not the second.

- o True
- o False

IX. Every normal space is a regular space.

- o True
- o False

X. If $f: \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation ($n, m \in \mathbb{N}$), then for all $p \in \mathbb{R}^n$, $D_p f = f$.

- o True
- o False

Q2:

Let M be an n −dimensional manifold and N be non-empty open subset of M. Prove that N is an n −dimensional manifold.

Q3:

Let $f: S^2 \to \mathbb{R}$ defined by $f(x, y, z) = x + y + z$. Show that f is a smooth function at $p = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0)$ and find all partial derivatives of f at p.

Q4:

Let $f: M \to N$ and $g: N \to L$ be smooth functions. Prove that for $p \in M$, $g \circ f$ is a smooth function and,

$$d_p(g \circ f) = d_{f(p)}g \circ d_p f.$$

Q5:

Let $f: \mathbb{R}^4 \to \mathbb{R}^2$ be a function defined by $f(x, y, z, w) = (x + y, z - w)$. Show that f is a submersion.

Q6:

Let $f: M \to N$ be smooth function and $p \in M$. If $d_p f: T_p M \to T_{f(p)} N$ is an isomorphism. Prove that there exist open subsets U of p and V of $f(p)$, respectively such that $f: U \to V$ is a diffeomorphism.