College of Science.
Department of Mathematics

كلية العلوم
قسم الرياضيات

Second Midterm Exam
Academic Year 1442-1443 Hijri- SecondSemester

Exam Information معلومات الامتحان			
Course name	**Complex Analysis**		اسم المقرر
Course Code	**Math 487**		رمز المقرر
Exam Date	**2022-03-23**	**1443-08-20**	تاريخ الامتحان
Exam Time	**03: 00 PM**		وقت الامتحان
Exam Duration	**2 hours**	ساعتان	مدة الامتحان
Classroom No.	**GA 0 19**		رقم قاعة الاختبار
Instructor Name	**Haifa Tahlawi**		اسم استاذ المقرر

Student Information معلومات الطالب		
Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions:

- Your Exam consists of 6 PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
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تعليمات عامة:

- عدد صفحات الامتحان 6 صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف والساعات الذكية خارج قاعة الامتحان.
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هذا الجزء خاص بأستاذ المادة

This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1				
2				
3				
4				
5				
6				
7				
8				

EXAM COVER PAGE

QI	QII	QIII	QIV	Total

Question I

Prove or disprove each of the following:

1. If $f(z)$ is analytic and $Imf(z)$ is constant, then $f(z)$ is constant.

2. If $Re\ z_1 > 0$ and $\text{Re}\ z_2 > 0$, then

$$\text{Log}\left(\frac{z_1}{z_2}\right) = \text{Log}\, z_1 - \text{Log}\, z_2$$

3. $\log(-i)^2 = 2\log(-i)$ where $\log z$ here is the branch with $0 < \theta < 2\pi$.

Question II

A. Use the definition of analytic function to prove that $f(z) = |z|$ is nowhere differentiable.

B. Discuss the analyticity of each of the following functions and find its derivative, $f'(z)$, if it exists.

1. $f(z) = \frac{4\bar{z}+z}{5}$

2. $f(z) = \overline{\sinh z}$

3. $f(z) = \sqrt[3]{r}\, e^{i\theta/3}, \qquad 0 < \theta \leq 2\pi$

Question III

1. Prove that the function is $u = e^{-y} \sin x$, is harmonic in its domain and find its harmonic conjugate

2. Solve each of the following equations

a) $e^{2z+1} = i$

b) $\text{Log}(z^2 - 1) = \frac{i\pi}{2}$.

3. Prove that, $\cosh^2 z - \sinh^2 z = 1$.

Question IV

1. Find the domain of analyticity of the function, $f(z) = \text{Log}(\frac{z-1}{z})$.

2. Determine each of the following:

i) A branch of $\log z$ which is analytic at $z = -2$ and find its derivative there.

ii) A branch of $\log(1 - 4z)$ of which is analytic at $z = -2$ and find its derivative there.

Question IV

A. Find each of the following integrals:

$$\int_0^{\frac{\pi}{4}} e^{-2zt}dt \; =$$

$$\int_1^3 \left(\frac{1}{t} - i\right) dt =$$

$$\int_0^{\infty} e^{-it}dt$$