College of Sciences كلية العلوم

Department of Mathematics قسم الرياضيات

First Makeup Mid Term Exam

Tuesday, October, 20th	MATH 487	Academic year 1443H
9-11	Complex analysis	First Semester

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Question I

A. Write the following function, $f\ (z)$, in terms of z, and simplify the result.

$f\ (z) = x^{\,2} - y^2 - 2y + i(2x - 2xy)$

B. Prove or disprove each of the following.

1) $|Im(1 + \bar{z}^2 + z^4)| \leq 4\ when\ |z| \leq \sqrt{2}.$

2) $argz_1\bar{z_2} = argz_1 - argz_2 \ \ where\ z_1 \neq 0\ and\ z_2 \neq 0$

3) $z\ is$ pure imaginary if and only if $\bar{z} = -z$

Question II

Use de Moivre's formula to derive the following trigonometric identity:

$$\sin 3\theta = 3\cos^2\theta \sin\theta - \sin^3\theta.$$

Question III

A. Evaluate each of the following in rectangular form:

i) $(1 - \sqrt{3}i)^{24}$

ii) $\left(\frac{-1}{8i}\right)^{\frac{1}{3}}$

c) Find all roots of the equation $z^2 - (3 - 2i)z + (1 - 3i) = 0$

A. Sketch the region onto which the sector $r \leq 3, 0 \leq \theta \leq \pi/4$ is mapped by the transformation $w = \frac{1}{\bar{z}}$

Sketch the following sets and determine which are <u>domains, closed or bounded</u>

(a) $|z - 2 + 5i| \leq \frac{1}{2}$;

(b) $Re\, z > \sqrt{3}$

(c) $arg\, z = -\frac{\pi}{4} + 2n\pi,\ n = 0, \mp 1, \mp 2, \ldots$

<u>Question IV</u>

A. Let $f(z) = \begin{cases} \frac{\bar{z}}{z}, & z \neq 0 \\ 0, & z = 0 \end{cases}$

1. Prove that $\lim_{z \to 0} f(z)$ doesn't exist.
2. Find the limit of this function elsewhere in $\mathbb{C}$.