College of Science.
Department of Mathematics

كلية العلوم
قسم الرياضيات

Second Midterm Exam
Academic Year 1447-1448 Hijri- Second Semester

Exam Information معلومات الامتحان			
Course name	مقدمة في الهندسة التفاضلية		اسم المقرر
Course Code	473ربض		رمز المقرر
Exam Date	2026-04-27	1447-11-10	تاريخ الامتحان
Exam Time	10: 00 AM		وقت الامتحان
Exam Duration	2 hours	ساعتان	مدة الامتحان
Classroom No.			رقم قاعة الاختبار
Instructor Name	هناء السديس		اسم استاذ المقرر

Student Information معلومات الطالب		
Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions:

- Your Exam consists of 6 PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
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تعليمات عامة:

- عدد صفحات الامتحان 6 صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف والساعات الذكية خارج قاعة الامتحان.
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هذا الجزء خاص بأستاذ المادة

This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1	C.L.O 1.1	IVA	/2	
2	C.L.O 1.2	IVB,C	/5	
3	C.L.O 2.1	Ii, IIi, IIIi	/6	
4	C.L.O 2.3	I(ii,iii), II(ii,iii), III(ii-v)	/12	
5				25
6				
7				
8				

EXAM COVER PAGE

Question	Ii	I(ii,iii)	IIi	II(ii,iii)	IIIi	III(ii-v)	IVA	IV(B,C)	Total
Mark									

Question I: [3.5+1+1=5.5 points]

Let $x: R^2 \to R^3$ be defined by

$$x(u,v) = (u+v, uv, u-v).$$

(i) Prove that x is a simple surface.
(ii) Compute the unit normal $\vec{n}(u,v)$.
(iii) Find the equation of the tangent plane at $x(1,0)$.

Question II: [1.25+1+1=3.25]

Let $x: R^2 \to R^3$ be a Monge patch defined by

$$x(u,v) = \left(u, v, \frac{1}{2}(u^2 - v^2)\right),$$

and $\beta: R \to R^3$, be defined by

$$\beta(t) = \left(\cosh t\,, \sinh t\,, \frac{1}{2}\right).$$

Then answer the following:

(i) Show that the curve $\beta(t)$ lies on the simple surface $x(u,v)$.

(ii) Use part (i) to find a tangent vector to the simple surface at the point $p = x(1,0)$.

(iii) Find the parametric curves of x at the point $x(1,2)$.

Question III: [1.25+2.5+2.5+1+2=9.25}

Let $x: (0,\infty) \times (-\pi,\pi) \to R^3$ be a simple surface defined by:

$$x(u,v) = (u\cos v\,, \mathrm{u}\sin v\,, \ln u).$$

(i) Show that x is a surface of revolution.

(ii) Compute the coefficients of the first fundamental form (g_{ij}) and its inverse $\left(g^{kl}\right)$.

iii) Compute the coefficients of the second fundamental form (L_{ij}).

(iv) Compute the Christoffel symbol Γ_{12}^{1} and Γ_{12}^{2}.

(v) For the unit speed curve $\beta(t) = x(1,t)$ lying on the simple surface compute the normal curvature k_n.

Question IV: [2+4+1=7points]

A. Define a coordinate transformation.

B. Let $x: U \rightarrow R^3$ be a simple surface, $f: V \rightarrow U$ be a coordinate transformation. Then prove that $y = x \circ f: V \rightarrow R^3$ is a simple surface with the same image as x.

C. If X is a tangent vector to a simple surface $x: U \to R^3$ at the point $p = x(a, b)$, $(a, b) \in U$ and $r \in R$ then prove that rX is a tangent vector to x at $p = x(a, b)$.

Good Luck 😊