College of Science.
Department of Mathematics

كلية العلوم
قسم الرياضيات

First Midterm Exam
Academic Year 1447-1448 Hijri- Second Semester

Exam Information معلومات الامتحان			
Course name	مقدمة في الهندسة التفاضلية		اسم المقرر
Course Code	473 ريض		رمز المقرر
Exam Date	2026-04-08	1447-10-20	تاريخ الامتحان
Exam Time	10: 00 AM		وقت الامتحان
Exam Duration	2 hours	ساعتان	مدة الامتحان
Classroom No.	5G3		رقم قاعة الاختبار
Instructor Name	هناء السديس		اسم استاذ المقرر

Student Information معلومات الطالب		
Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions: **تعليمات عامة:**

- Your Exam consists of 6 PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
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- عدد صفحات الامتحان 6 صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف والساعات الذكية خارج قاعة الامتحان.
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هذا الجزء خاص بأستاذ المادة

This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1	C.L.O 1.1	I A	/2	
2	C.L.O 1.2	IV	/4	
3	C.L.O 1.3	III (ii)	/2	
4	C.L.O 2.1	I B, II B	/5	
5	C.L.O 2.2	II A	/4	25
6	C.L.O 2.3	III(i, iii, iv)	/8	
7				
8				

EXAM COVER PAGE

Question	IA	IB	IIA	IIB	III (i,iii,iv)	III(ii)	IIIC	IV	Total
Mark									

Question I: [2+3=5 points]

A. Define a regular curve.

B. Decide which of the following curves is regular, justify your answer.

(i) $\alpha: (0, \infty) \to R^3$, defined by $\alpha(t) = (1, \ln t\,, t^3)$.

(ii) $\beta: R \to R^3$, defined by $\beta(t) = (cos\, t\,,\ t^2, t - e^t)$.

Question II: [4+2=6 points]

A. Reparametrize the regular curve α by its arc length, where $\alpha: (-1,1) \to R^3$ is defined by

$$\alpha(t) = \left(\frac{t}{\sqrt{2}}, \sqrt{1-t^2}, \frac{t}{\sqrt{2}} \right).$$

B. Let α be a unit speed curve with Frenet-Serret apparatus $\{k, \tau, T, N, B\}$. Prove that the **tangent spherical image** of α is a constant curve if and only if α is a straight line.

Question III: [5.5+2+1.25+1.25= 10 points]

Let $\alpha: R \to R^3$ be a unit speed curve defined by

$$\alpha(s) = \frac{1}{\sqrt{3}}(\sin s + \cos s, \cos s - \sin s, s).$$

Then answer the following:

(i) Compute its Frenet-Serret apparatus.

(ii) Deduce that the curve α a helix. Find its axis and pitch.

(iii) Find the equation of the rectifying plane to $\alpha(s)\ at\ s = 0$.

(iv) Write the vector $v = (1,0,1)$ in terms of T, N, and B.

Question IV: [4 points]

A. Let $\alpha(s)$ be a unit speed curve with Frenet Serret apparatus $\{T, N, B, k, \tau\}$. For $k \neq 0$. Prove that if the image of α lies in a plane, then the binormal vector field B is a constant vector.

B. Prove that if $\alpha(s)$ is a unit speed curve with $k \neq 0, \tau \neq 0,$ lying on a sphere, then

$$\tau\rho = -\left[\frac{\rho'}{\tau}\right]'.$$

(Hint: Recall the radius r is given by $r^2 = \rho^2 + (\rho'\sigma)^2$, where ρ is the radius of curvature and σ is the radius of torsion).

Good Luck