

King Saud University – Department of Mathematics
Math 473 – Second Midterm Exam (1447-1)

Duration: 1 h 30 mn – Total: 25 points

1. [4 points] Find the curvature κ and the torsion τ for the curve

$$\alpha(t) = (t, t^2 + 1, t^3), \quad t \in \mathbb{R}.$$

2. [4 points]

- (a) Define what is a Bertrand curve.
- (b) Give an example of a Bertrand curve.
- (c) Define what is a spherical curve.
- (d) Give an example of a spherical curve.

3. [3 points]

In which of the following cases the mapping $X : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ define a simple surface?

- (a) $X(u, v) = (u, v, uv)$.
- (b) $X(u, v) = (u + u^2, v, v^2)$.

4. [8 points] Consider the simple surface M given by

$$X(u, v) = (u, v, u^2 - v^2).$$

- (a) (3 pts) Compute the first and second fundamental forms.
- (b) (2 pts) Find the equation of the tangent plane to M at $p = (2, 1, 3)$.
- (c) (3 pts) Find the area of the region $X(\mathcal{R})$, where $\mathcal{R} = \{(u, v) : 1 \leq u^2 + v^2 \leq 4\}$.

5. [6 points] Consider the cylinder of radius 1 parametrized by

$$X(u, v) = (\cos u, \sin u, v).$$

For the following curves, compute the geodesic and normal curvatures and precise those which are geodesics.

- (a) $\alpha(t) = X(u_0, t)$, where u_0 is a constant.
- (b) $\beta(t) = X(t, v_0)$, where v_0 is a constant.
- (c) $\gamma(t) = X(t, t)$.

Good luck!