

King Saud University / Department of Mathematics
Math 473 – First Midterm Exam (1447-1)

Duration: 1 h 30 mn – Total: 25 points

Instructions: Answer **all** questions. Show all steps and justify your answers. Calculators are not allowed.

1. (10 points) Let $\gamma(s) = (\frac{1}{\sqrt{2}} \cos s, \frac{1}{\sqrt{2}} \cos s, \sin s)$ with $s \in \mathbb{R}$.
 - (a) (3 pts) Prove that γ is a unit-speed curve.
 - (b) (4 pts) Find the Frenet-Serret apparatus of γ .
 - (c) (3 pts) Compute the curvature and torsion of γ .

 2. (6 points) Let $\alpha(t) = (3 \cos t, 3 \sin t, 4t)$ with $t \in \mathbb{R}$.
 - (a) (2 pts) Compute the length of α from $t = 0$ to $t = 5$.
 - (b) (3 pts) Find the reparametrization of the curve α by arc length.
 - (c) (1 pts) Show that α is a helix.

 3. (3 points) Find the equation of the normal plane to the curve $\alpha(t) = (3 \cos t, 3 \sin t, 4t)$ at the point $p = \alpha(\frac{\pi}{4})$.

 4. (6 points)
 - (a) (4 pts) Let α be a unit-speed curve defined on the interval I . Show that if β is an involute of α , then $\beta(s) = \alpha(s) + (c - s)T_\alpha(s)$, for some constant c .
 - (b) (2 pts) Find an involute of the curve $\alpha(t) = (3 \cos t, 3 \sin t, 4t)$, $t \in \mathbb{R}$.
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Good luck!